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Four-body potential in multiquark states

R. C. Warner and G. C. Joshi
School of Physics, University of Melbourne, Parkville, Victoria, Australia, 3052
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We present a detailed analysis of the first member of a previously derived hierarchy of instanton-generated many-body potentials for multiquark states. Comparison with the two-body instanton-generated potential in the heavy-quark framework is made in the T-baryonium system.

I. INTRODUCTION

In a recent paper,[1] hereinafter referred to as WJ, we investigated the contribution of instantons to the interaction potentials present in a particular family of multiquark configurations. We found that in the heavy-quark framework there are, as might be expected from considerations of color structure, many-body potentials as well as the two-body potential found by Callan *et al.*[2]

In WJ we derived a simple rule, developed by induction, for constructing the one-instanton effective Hamiltonian in the static quark limit, for the members of a multiquark family, which we term the "color tree" family from the tree-graph appearance of the usual stringlike diagrams of color structure. (See, for example, Fig. 1.) This is the family obtained through building up from the meson by replacing a particle at each step by a color antisymmetrized (color triplet) pair of antiparticles; the sequence resulting is meson, baryon, T baryonium, $(4Q)\bar{Q}$ state, dibaryon, and $(\bar{Q}\bar{Q})Q\bar{Q}(QQ)$ states, etc. We also considered the M-baryonium state. A hierarchy of many-body potentials results as one moves to more complicated states with richer color structure.

In this paper we present an analysis of the first of the many-body potentials—the four-body term—in the dilute-instanton-gas picture. This is the only many-body potential encountered in the baryonia (T and M), $(4Q)\bar{Q}$, and dibaryon states of current experimental[4] and theoretical[5,6] interest. We considered that a detailed analysis of the first member of the hierarchy was thus warranted, as the importance of these more complicated potential terms needed to be resolved. Two agreeable possibilities existed. These many-body interactions might strongly influence orientation and structure of multiquark states; alternatively they might be unimportant, allowing a further simplification of the effective Hamiltonian from WJ into the residue of two-body potentials from the many-body terms. This would accord well with some phenomenological approaches.[7] To give a concrete view of the relative importance of the four-body potential we also consider it in the context of T baryonium (Fig. 1).

In Sec. II we examine the asymptotic behavior of the four-body potential while Sec. III contains detailed calculations for specific configurations. In Sec. IV we indicate the effect of the four-body term in T baryonium. Further discussion and conclusions follow in Secs. V and VI.

II. THE FOUR-BODY POTENTIAL–ITS ASYMPTOTICS

From WJ the four-body potential involving four particles in two pairs (1,2 and 3,4) due to the presence of a dilute instanton gas is

$$V_4(\{\vec{x}_i\}) = 2\int \frac{d\rho}{\rho^2} D(\rho)\, W_4(\{\vec{x}_i/\rho\})\,, \tag{1}$$

where the integration is over instanton scale size ρ, $D(\rho)$ the density of the instantons of size ρ is

$$D(\rho) = x^6 e^{-x} \tag{2}$$

with

$$x = 8\pi^2/g^2(\rho)\,,$$

and where, following Callan *et al.*,[2] we have introduced a dimensionless potential $W_4(\{\vec{x}_i/\rho\})$. Using dimensionless units $\vec{y}_i = \vec{x}_i/\rho$, $\vec{r}/\rho \to \vec{r}$,

$$W_4(\{\vec{y}_i\}) = -\frac{4}{N}\int d^3r\left\{\cos[\pi f(\vec{y}_1,\vec{r})]\cos[\pi f(\vec{y}_2,\vec{r})] + \frac{(\vec{y}_1-\vec{r})\cdot(\vec{y}_2-\vec{r})}{|\vec{y}_1-\vec{r}|\,|\vec{y}_2-\vec{r}|}\sin[\pi f(\vec{y}_1,\vec{r})]\sin[\pi f(\vec{y}_2,\vec{r})] - 1\right\}\begin{Bmatrix}\vec{y}_1\to\vec{y}_3\\ \vec{y}_2\to\vec{y}_4\end{Bmatrix}, \tag{3}$$

where

$$f(\vec{y}_i,\vec{r}) = \frac{|\vec{y}_i-\vec{r}|}{(1+|\vec{y}_i-\vec{r}|^2)^{1/2}} \tag{4}$$

and N is the square of the normalization factor of the multiquark-state color projection operator. [This is strictly correct for the T baryonium, $(4Q)\bar{Q}$, and dibaryon states. In M baryonium the four-body potential appears with various weights and signs, and in states such as $(\bar{Q}\bar{Q})Q\bar{Q}(QQ)$ of the tree family a six-particle interaction is present which has four-body elements of the form of Eq. (3) but with different weights. For details see Secs. IV and V of WJ.]

As the behavior of $g^2(\rho)$, the strong coupling, is not known at all scale lengths we follow Callan *et al.*, in concentrating our attention on the structure of the dimensionless potential W_4. The two-body dimensionless potential of Callan *et al.* has the closely related form

$$W_2(\vec{y}_1,\vec{y}_2) = -\tfrac{2}{3}\int d^3r\left\{\cos[\pi f(\vec{y}_1,\vec{r})]\cos[\pi f(\vec{y}_2,\vec{r})] + \frac{(\vec{y}_1-\vec{r})\cdot(\vec{y}_2-\vec{r})}{|\vec{y}_1-\vec{r}|\,|\vec{y}_2-\vec{r}|}\sin[\pi f(\vec{y}_1,\vec{r})]\sin[\pi f(\vec{y}_2,\vec{r})] - 1\right\}. \tag{5}$$

Before proceeding to a detailed numerical analysis of the four-body potential for some specific spatial configurations we indicate its asymptotic behavior. When all four particles are close together (near the origin for convenience), $|\vec{y}_i| \ll 1$, we can expand the integrand, and defining $\vec{s}=\vec{y}_1-\vec{y}_2$, $\vec{t}=\vec{y}_3-\vec{y}_4$ we find, after some algebra [Eqs. (66)–(71) in WJ],

$$W_4(\vec{s},\vec{t}) = -\frac{4}{N}s^2t^2 4\pi[A+\tfrac{1}{2}B+(\hat{s}\cdot\hat{t})^2B], \tag{6}$$

where

$$A=\int_0^\infty r^2dr\left\{\frac{1}{12r^4}\sin^4\left[\frac{\pi r}{(1+r^2)^{1/2}}\right]+\frac{\pi^2}{6r^2(1+r^2)^3}\sin^2\left[\frac{\pi r}{(1+r^2)^{1/2}}\right]\right\} \tag{7}$$

and

$$B=\tfrac{2}{15}\int_0^\infty r^2dr\left\{\frac{1}{2r^2}\sin^2\left[\frac{\pi r}{(1+r^2)^{1/2}}\right]-\frac{\pi^2}{2(1+r^2)^3}\right\}^2. \tag{8}$$

We see that A and B are both positive indicating that W_4 is repulsive when all four particles are close together. It is to be noted that in leading order there is no dependence on separation between the two pairs. If either pair of particles is superimposed then the four-body potential term vanishes regardless of the coordinates of the other pair.

Evaluation of the integrals indicates that spatial orientational information may prove elusive as $A=0.709$ while $B=0.013$. The second simple asymptotic limit cited in WJ occurs when all interparticle distances are large. Then from WJ

$$W_4 \sim -\frac{\pi^2}{N}\frac{K}{2}\Bigg[\left(\frac{(\vec{y}_3-\vec{y}_1)}{|\vec{y}_3-\vec{y}_1|^3}-\frac{(\vec{y}_4-\vec{y}_1)}{|\vec{y}_4-\vec{y}_1|^3}\right)^2+\left(\frac{(\vec{y}_1-\vec{y}_3)}{|\vec{y}_1-\vec{y}_3|^3}-\frac{(\vec{y}_2-\vec{y}_3)}{|\vec{y}_2-\vec{y}_3|^3}\right)^2$$
$$+\left(\frac{(\vec{y}_1-\vec{y}_4)}{|\vec{y}_1-\vec{y}_4|^3}-\frac{(\vec{y}_2-\vec{y}_4)}{|\vec{y}_2-\vec{y}_4|^3}\right)^2+\left(\frac{(\vec{y}_3-\vec{y}_2)}{|\vec{y}_3-\vec{y}_2|^3}-\frac{(\vec{y}_4-\vec{y}_2)}{|\vec{y}_4-\vec{y}_2|^3}\right)^2\Bigg], \tag{9}$$

where

$$K=\int d^3r\left\{1+\cos\left[\frac{\pi r}{(1+r^2)^{1/2}}\right]\right\} \tag{10}$$

$$=-4\pi\frac{\pi^2}{3}\left[\frac{\pi}{2}J_0(\pi)+\tfrac{1}{2}J_1(\pi)\right] \tag{11}$$

$$=13.874\,. \tag{12}$$

This K integral appears in the analysis of the two-body potential Eq. (5) for large separations performed by Callan *et al.*[2] Here the potential is attractive.

A further simply analyzed case is that in which the particles of each pair are separated but the two pairs are superimposed, e.g., $\vec{y}_1=\vec{y}_3$, $\vec{y}_2=\vec{y}_4$. In this case the integrand of Eq. (3) for W_4 is just the square of that in Eq. (5) for W_2. Naturally as separation of the pairs is not involved in Eq. (6) the small-separation approximation is as before with $\vec{s}=\vec{t}$. The results are naturally reminiscent of the analysis of Eq. (5) by Callan *et al.*[2]; the potential approaches a constant as separation increases:

$$W_4(s) \sim -\frac{4}{N}8\pi\int_0^\infty r^2dr\left\{1+\cos\left[\frac{\pi r}{(1+r^2)^{1/2}}\right]\right\}^2$$
$$-\frac{4}{N}\frac{4\pi}{s^4}\int_0^\infty r^2dr\left(\frac{\pi^2}{6}\sin^2\left[\frac{\pi r}{(1+r^2)^{1/2}}\right]-\left\{1+\cos\left[\frac{\pi r}{(1+r^2)^{1/2}}\right]\right\}\frac{\pi^2}{2}\cos\left[\frac{\pi r}{(1+r^2)^{1/2}}\right]\right),$$

where $\vec{s}=\vec{y}_4-\vec{y}_3=\vec{y}_2-\vec{y}_1$ and $\vec{y}_1=\vec{y}_3$, $\vec{y}_4=\vec{y}_2$. In addition,

$$W_4(s)\sim\frac{4}{N}\frac{4\pi^3}{3}\left[J_1(2\pi)+2J_1(\pi)+2\pi J_0(\pi)+2\pi J_0(2\pi)\right]-\frac{4}{N}\frac{2\pi^3}{s^4}\left[\tfrac{2}{9}\pi^2J_1(2\pi)+\tfrac{4}{9}\pi^3J_0(2\pi)+\frac{\pi^2}{6}J_1(\pi)+\frac{\pi^3}{6}J_0(\pi)\right] \quad (13)$$

$$\sim -\frac{4}{N}7.058-\frac{4}{N}90.89\frac{1}{s^4}\,. \quad (14)$$

With these asymptotic descriptions in mind, we now move to a detailed numerical evaluation of W_4 for some specific orientations.

III. DETAILED CALCULATIONS–SPECIFIC CONFIGURATIONS

For intermediate particle separations it is necessary to evaluate the integral in Eq. (3) numerically. Clearly with four particles it is necessary to restrict the configurations analyzed. We have investigated in detail two configurations of particle pairs which appear appropriate to the comparison of the relative importance of two- and four-body potentials in the specific case of T baryonium considered in Sec. IV.

The two configurations which we term "planar" and "crossed" are shown in Figs. 2 and 3, respectively. In the latter case the two pairs are oriented at right angles. The two parameters are d, the spacing of the elements of the pairs, and l, the pair separation.

Figure 4 shows $(N/4)$ W_4 for the planar configuration plotted as a function of d and l. The graph shows that as $d\to\infty$ at finite l W_4 approaches a constant:

$$W_4\to-\frac{4}{N}2\int d^3r\{1+\cos[\pi f(0,\vec{r})]\}\{1+\cos[\pi f(\vec{l},\vec{r})]\}. \quad (15)$$

In Fig. 5 $(N/4)$ W_4 is displayed for the crossed configuration. Here as $d\to\infty$ W_4 tends to zero in accordance with the asymptotic form from Eq. (9), as there are no particles remaining in close proximity.

IV. *T* BARYONIUM

With the numerical evaluation of W_4 in Sec. III we are ready to compare the two- and four-body contributions for the two configurations of Figs. 2 and 3, which are reasonable T-baryonium structures. From Eq. (61) of WJ the complete dimensionless potential takes the form

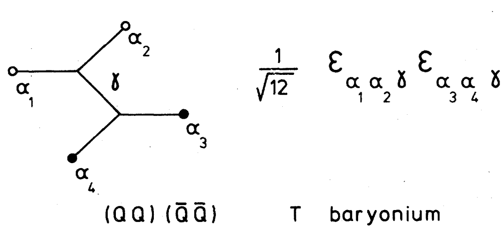

FIG. 1. The color-string diagram for T baryonium, together with its normalized color projection operator. (Greek color indices take values 1, 2, 3.) Each string junction represents a Levi-Civita tensor in the projection operator.

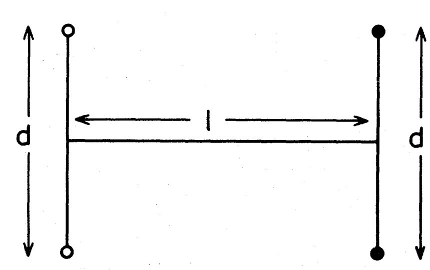

FIG. 2. "Planar" configuration of four particles in two pairs described by separations d,l.

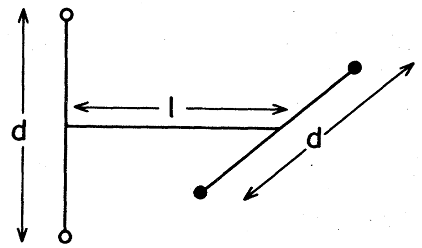

FIG. 3. "Crossed" configuration of four particles in two pairs described by separations d, l. The two pairs are oriented at right angles.

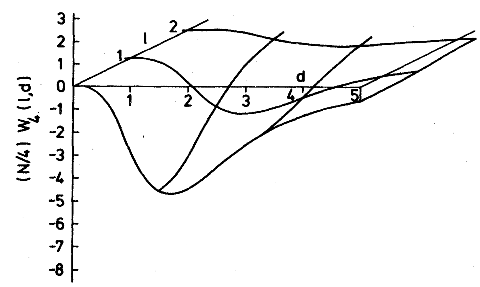

FIG. 5. Graph of $(N/4)W_4(l,d)$ for the crossed configuration of Fig. 3.

$$W_{qq\bar{q}\bar{q}}(\vec{y}_1, \vec{y}_2, \vec{y}_3, \vec{y}_4) = \frac{1}{4}\sum_{\substack{i=1,2\\ j=3,4}} W_2(\vec{y}_i, \vec{y}_j) + \frac{1}{2}W_2(\vec{y}_1, \vec{y}_2) + \frac{1}{2}W_2(\vec{y}_3, \vec{y}_4) + W_4^{(N=12)}(\vec{y}_1, \vec{y}_2; \vec{y}_3, \vec{y}_4)\,. \quad (16)$$

It should be stressed that the two-body terms linking the quarks at $\vec{y}_1, \vec{y}_2$ and those linking antiquarks at $\vec{y}_3, \vec{y}_4$ were found in the course of reducing the derived many-body potentials of T baryonium to true many-body form.

For the planar configuration

$$W(l,d) = \frac{1}{2}W_2(l) + \frac{1}{2}W_2((l^2+d^2)^{1/2}) + W_2(d) + W_4^{\text{planar}}(l,d)\,, \quad (17)$$

while for the crossed configuration

$$W(l,d) = W_2((l^2+d^2/2)^{1/2}) + W_2(d) + W_4^{\text{crossed}}(l,d)\,. \quad (18)$$

We have computed the two-body potential as well and in Figs. 6 and 7 we display for planar and crossed configurations, at $l=0$, the potentials with (curve b) and without (curve a) the four-body terms. From Figs. 4 and 5 it is clear that the difference decreases as l increases. The correction is localized in the crossed case but due to the asymptotic constant value, the four-body potential lowers the large d value of $W(0,d)$ in the planar configuration by approximately 8%.

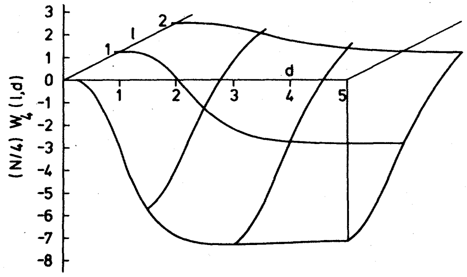

FIG. 4. Graph of $(N/4)W_4(l,d)$ for the planar configuration of Fig. 2.

V. DISCUSSION

It is clear from the previous sections that the four-body dimensionless potential terms can be quite appreciable for certain configurations. We were interested to observe that the magnitude of the four-body term can be large compared to that of the $O(1/m_q^2)$ corrections to the two-body instanton-generated potential which were investigated by Callan *et al.*[2] and by Aragão de Carvalho.[3] While it is clear that our four-body static potentials must eventually dominate over two-body $O(1/m_q^2)$ terms for sufficiently heavy quarks, it is interesting that for a quark mass of 5 GeV the maximum of the spin-spin potential of Callan *et al.*, which is 11.25 $(2m_1m_2)^{-1}\langle\sigma_1\sigma_2\rangle$, becomes 0.224 $\langle\sigma_1\sigma_2\rangle$ while the four-body potential has a maximum depth of approximately 2.4 for T-baryonium in the planar configuration. While the four-body potentials can be sizable, compared with the corrections to the

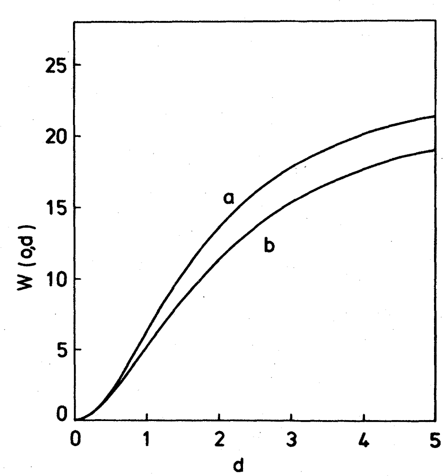

FIG. 6. Graphs of $W(l=0,d)$ for the planar configuration of T baryonium: curve a without the W_4 contribution, curve b complete [Eq. (17)].

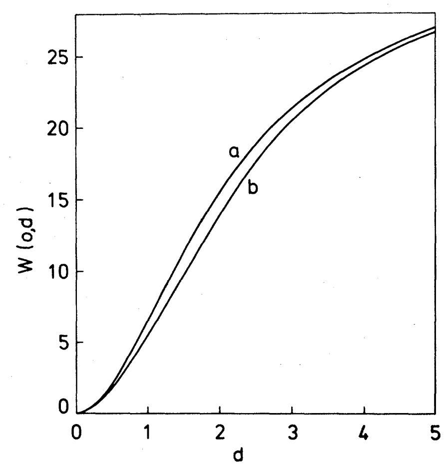

FIG. 7. Graphs of $W(l=0,d)$ for the crossed configuration of T baryonium: curve a without the W_4 contribution, curve b complete [Eq. (18)].

static two-body potential, it appears from Figs. 6 and 7 that the two-body potential terms in Eqs. (17) and (18) may provide a reasonable approximation for the T-baryonium system. This should not be considered to devalue the work in WJ since the form of these equations with their combinations of two-body terms emerged from the reduction of many-body terms encountered in calculating the potentials of the multiquark state.

It is also amusing to note that the two-body term weightings in Eqs. (17) and (18) match the QCD-inspired phenomenological potential

$$V(\{\vec{x}_i\}) = \sum_{i>j}\sum_a \tfrac{1}{4}\lambda_i^a\lambda_j^a V(\vec{x}_i - \vec{x}_j)\,, \tag{19}$$

where the expectation value is taken over the color part of the wave function of the multiquark configuration, in which some theoretical interest has been shown.[7] (For antiquarks $\lambda_i \to -\lambda_i^*$.) McDougall[8] has shown that it is possible to carry out the scale-size integrations for the potentials of Callan *et al.*[2] by making assumptions about the behavior of $g^2(\rho)$ for large scale sizes and about the range of validity of the one-loop calculations of 't Hooft[9] on instanton quantum effects for large coupling.

While the results are very interesting, especially the suggestion that the role of the two-body instanton-generated potential is very large, we prefer to compare our many-body potential with the two-body ones at the dimensionless level to avoid these extra assumptions. Naturally the real potential will be of a somewhat different shape from the dimensionless one after enfolding with the instanton-size distribution. For example, with

$$W_2\left(\frac{x}{\rho}\right) \sim a\left(\frac{x}{\rho}\right)^2, \quad \frac{x}{\rho} \ll 1 \tag{20a}$$

$$\sim b + c\left(\frac{\rho}{x}\right), \quad \frac{x}{\rho} \gg 1\,, \tag{20b}$$

then we find the relative strengths for $V_2(x)$ renormalized:

$$V_2(x) \sim x^2 a \int d\rho \frac{D(\rho)}{\rho^4} \text{ as } x \to 0 \tag{21a}$$

$$\sim b \int d\rho \frac{D(\rho)}{\rho^2} + \frac{c}{x}\int d\rho \frac{D(\rho)}{\rho} \text{ as } x \to \infty\,, \tag{21b}$$

showing how the harmonic term is more sensitive to small-scale-size instantons.

VI. CONCLUSION

The four-body potential term due to instantons which we derived previously for the family of multiquark states of current interest has been analyzed in detail and shown to be appreciable (in T baryonium in particular) in comparison with the heavy-quark expansion of the two-body interactions.

While the many-body terms in the multiquark potential are appreciable in certain configurations, it appears that the overall features of multiquark states are controlled by the two-body static limit terms, with many-body and $O(1/m_q^2)$ term corrections. The two-body terms in the multiquark state potential emerge from the many-body analysis of our previous paper and agree with the forms commonly in use in multiquark calculations.[7]

ACKNOWLEDGMENT

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