Casimir effect and topological mass

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The analogy between field theory at a finite temperature and field theory in periodically identified flat space-time is discussed and used to deduce the Casimir effect. The renormalization of a twisted scalar field with a $\lambda\phi^4$ self-interaction in periodically identified flat space-time is also discussed.

I. INTRODUCTION

Although the Casimir effect[1] for a free field theory has been known for some time, it is only recently that the effects of interactions have begun to be examined.[2-4] In particular, the effect of a ϕ^4 self-interaction on the vacuum energy and the self-energy of a massless scalar field in a flat space-time which is given the non-Minkowskian topology of $S^1 \times R^3$ by making a periodic identification[5] in one of the spatial coordinates has been obtained.[2,3] It is found that a field which is massless at the tree-graph level can develop a mass as a consequence of both the nontrivial topology and the self-interaction; hence, such a phenomenon might be dubbed topological mass generation.

The purpose of this note is to discuss the analogy between this case and field theory at a finite temperature. We shall also show that a twisted scalar field[6] with a $\lambda\phi^4$ self-interaction is renormalizable to order λ^2 where the counterterms in the Lagrangian are identical to those for a scalar field in Minkowski space-time.

II. THE CASIMIR EFFECT AND FIELD THEORY AT A FINITE TEMPERATURE

In quantum field theory at a finite temperature the partition function may be defined as the following functional integral[7,8]:

$$Z = \int d[\phi] \exp\left[\int d^4x \mathcal{L}(x)\right], \tag{1}$$

where the usual rotation of the time axis has been performed so that the metric is now a positive-definite Euclidean metric, and where $d[\phi]$ represents a measure on the space of classical fields which enter into the Lagrangian, $\mathcal{L}(x)$. We may use the standard formula for the energy from statistical mechanics[9]:

$$E = -\frac{\partial}{\partial\beta} \ln Z. \tag{2}$$

We shall deal first with an ordinary, free, massless scalar field. The functional integration in Eq. (1) is taken over all fields which are periodic in Euclidean time with period β, where β is the inverse temperature, and which satisfy periodic boundary conditions on the walls of a box whose sides are assumed to be of lengths L_1, L_2, L_3. If we take the large-box limit (specifically $L_1, L_2, L_3 \gg \beta$), then we will obtain an expression for Z which is valid at a finite temperature in ordinary Minkowski space-time. If we take the limit $\beta, L_2, L_3 \gg L_1$, then we will obtain an expression for Z which is valid at zero temperature in a flat space-time with the topology $S^1 \times R^3$, where X^1 is the spatial coordinate which has been given the periodic identification. Furthermore, it then follows from Eq. (1) that the expressions obtained for Z in these two cases will be identical upon the interchange of β and L_1. This allows us to deduce the Casimir effect from the thermodynamic result for the partition function.

For a free, massless, real scalar field the partition function is well known from thermodynamics to be given by[10]

$$\ln Z = \frac{\pi^2 L_1 L_2 L_3}{90\beta^3}. \tag{3}$$

(We choose units such that $\hbar = c = 1$.) As mentioned previously, this result holds only in the large-box limit. To obtain the partition function in the Casimir case we merely interchange β and L_1 to give

$$\ln Z = \frac{\pi^2 \beta L_2 L_3}{90 L_1^3}. \tag{4}$$

Using Eqs. (2) and (4), the energy in the Casimir case is

$$E = -\frac{\pi^2 L_2 L_3}{90 L_1^3}, \tag{5}$$

which is the correct result for a scalar field satisfying periodic boundary conditions.[2]

Consider now a real scalar field with a mass term and a ϕ^4 self-interaction. The Lagrangian which enters into Eq. (1) is

$$\mathcal{L}(x) = -\tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \tfrac{1}{2}m_B^2\phi^2 - \frac{\lambda_B}{4!}\phi^4, \tag{6}$$

where m_B is the bare mass, and λ_B is the bare coupling constant. The Feynman rules at a finite temperature are discussed in Ref. 7. It is evident from Eqs. (1) and (6) that the equivalence described above between field theory at a finite temperature and field theory in a periodically identified flat space-time extends to the interacting case as well, so that the Feynman rules in the periodically identified flat space-time will follow from those of Ref. 7 upon the replacement of β with L_1.

The renormalization of field theory at a finite temperature has been discussed by Kislinger and Morley,[11] where a procedure is described for the renormalization of finite-temperature Green's functions to all orders in perturbation theory. An explicit demonstration to order λ^2 is given for $\lambda\phi^4$ scalar field theory at finite temperature in Minkowski space-time. The mass and coupling-constant counterterms are shown to be temperature-independent to this order. Using the mathematical equivalence argued above, it then follows that the corresponding theory in periodically identified flat space-time is renormalizable to order λ^2. It is also shown in Refs. 8 and 11 that there is a temperature-dependent one-loop contribution to the $(\text{mass})^2$ which, upon the interchange of β and L_1, gives an L_1-dependent $(\text{mass})^2$ of

$$\tfrac{1}{2}\lambda_R \int \frac{d^3k}{(2\pi)^3}\,(k^2+m_R{}^2)^{-1/2}\{\exp[L_1(k^2+m_R{}^2)^{1/2}]-1\}^{-1} \tag{7}$$

in periodically identified flat space-time. The λ_R and m_R which appear in Eq. (7) are the renormalized values of the bare constants appearing in Eq. (6).

Ford and Yoshimura[3] considered only the massless theory, $m_R=0$, in which case it is easy to see that Eq. (7) reduces to their result of $\lambda_R/4!\,L_1{}^2$. The method of calculation which was used in Ref. 3 was first-order perturbation theory where the divergences were dealt with by ζ-function regularization. No renormalization was performed using this procedure since all of the divergences disappeared; however, it has been emphasized by Kay[4] that this will not hold true if the fields are massive.

III. RENORMALIZATION OF THE TWISTED SCALAR FIELD

The notion of twisted fields was introduced by Isham.[6] A very brief description of this idea is included here for completeness; however, the reader should refer to Ref. 6 for the full details.

Since a real scalar field defined on some space-time manifold M assigns a real number to each point of the manifold, we could regard the field as the cross section of the product vector bundle whose bundle space is $M\times R^1$, whose base space is M, and whose fiber is R^1. Isham noted that one could equally well define a real scalar field as the cross section of some nonproduct vector bundle with a base space of M and a fiber of R^1, since locally the bundle space is still $M\times R^1$. He called such scalar fields twisted scalar fields, and the bundles twisted vector bundles. The number of inequivalent twisted vector bundles which is allowed for a given space-time is determined by the space-time topology. In the case of Minkowski space-time only one type of scalar field is allowed, corresponding to the usual notion of a scalar field. If however, we take the manifold M to be a flat space-time in which a periodic identification is made in one of the spatial coordinates, then there are two types of scalar fields allowed. In addition to the usual scalar field, which is periodic in the identified coordinate, there is also a twisted scalar field which is antiperiodic in the identified coordinate.

The Feynman rules for a scalar field satisfying antiperiodic boundary conditions at finite temperature follow in a manner which is similar to those discussed in Ref. 7 for fermions. The rules for a twisted scalar field in a flat space-time, in which the X^1 coordinate is periodic with period L_1, will then follow from these rules upon the interchange of β and L_1. They will then be the usual momentum-space rules except that $k^1=(\pi/L_1)\times(2n+1)$, where n takes on all integral values, is discrete, and where $\int d^4k/(2\pi)^4$ is replaced with

$$\sum_{n=-\infty}^{+\infty}\frac{1}{L_1}\int\frac{d^3k}{(2\pi)^3}\,.$$

We shall usé dimensional regularization[12] to deal with divergent Feynman integrals, where $\int d^3k/(2\pi)^3$ is replaced by $\mu^{4-\omega}\int d^{\omega-1}k/(2\pi)^{\omega-1}$ and then analytic continuation is performed to a neighborhood of $\omega=4$. Here μ is the unit of mass[13] which is introduced so that the coupling constant remains dimensionless in ω dimensions. The summation formula[11]

$$\sum_{n=-\infty}^{+\infty}\frac{1}{L_1}f\left(\frac{\pi}{L_1}(2n+1)\right)=\int_{-\infty}^{+\infty}\frac{dk_1}{2\pi}f(k_1)-\int_{-\infty+i\epsilon}^{\infty+i\epsilon}\frac{dk_1}{2\pi}\,(e^{-iL_1k_1}+1)^{-1}[f(k_1)+f(-k_1)] \tag{8}$$

is used to convert the summations which occur in Feynman integrals into contour integrals. The second contour in Eq. (8) is closed in the upper half of the complex k_1 plane which encloses any poles of $f(k_1)+f(-k_1)$.

Define $\Sigma(p)$ to be the sum of all one-particle-irreducible self-energy graphs, where p denotes the four-momentum of the external line. The contribution to $\Sigma(p)$ of order λ_R comes from Fig. 1(a); the contributions of order $\lambda_R{}^2$ arise from Figs. 1(b)–1(e). In Fig. 1 a black dot denotes a coupling-constant counterterm, while a cross denotes a mass counterterm. The contributions of order $\lambda_R{}^2$ to the vertex function $\Gamma^{(2)}(p_1,p_2,p_3,p_4)$ are shown in Fig. 2, where p_1,p_2,p_3,p_4 label the external momenta.

The Lagrangian is given in Eq. (6), and we express the bare quantities in terms of the renormalized ones as[13,14]

$$\lambda_B=\mu^{4-\omega}\left[\lambda_R+\sum_{\nu=1}^{\infty}\sum_{j=\nu}^{\infty}\frac{a_{\nu j}}{(\omega-4)^\nu}\lambda_R{}^j\right], \tag{9}$$

$$m_B{}^2=m_R{}^2\left[1+\sum_{\nu=1}^{\infty}\sum_{j=\nu}^{\infty}\frac{b_{\nu j}}{(\omega-4)^\nu}\lambda_R{}^j\right]. \tag{10}$$

The field renormalization constant is expressed as

$$Z=1+\sum_{\nu=1}^{\infty}\sum_{j=\nu}^{\infty}\frac{c_{\nu j}}{(\omega-4)^\nu}\lambda_R{}^j. \tag{11}$$

Here $a_{\nu j}$, $b_{\nu j}$, $c_{\nu j}$ are coefficients to be chosen such that the renormalized two-point and four-point functions are finite as we let ω tend to 4. Specifically,[14] they are to be chosen such that

$$\lim_{\omega\to4} Z[p^2+m_B{}^2-\Sigma(p)], \tag{12}$$

$$\lim_{\omega\to4}\mu^{4-\omega}Z^2[-\lambda_B+\Gamma^{(2)}(p_1,p_2,p_3,p_4)+O(\lambda_R{}^3)] \tag{13}$$

are both finite. Unlike the situation in Minkowski space-time,[15] we have no guarantee that the coefficients are mass independent or that they will be L_1 independent; this remains to be shown.

The contribution to $\Sigma(p)$ which is of order λ_R is contained in Fig. 1(a) and is

$$\tfrac{1}{2}(-\lambda_R)\mu^{4-\omega}\sum_{n=-\infty}^{+\infty}\frac{1}{L_1}\int\frac{d^{\omega-1}k}{(2\pi)^{\omega-1}}\times\left[k^2+\left(\frac{\pi}{L_1}(2n+1)\right)^2+m_R{}^2\right]^{-1}.$$

Using the summation formula, Eq. (8), and expanding about $\omega=4$ gives

$$-\frac{\lambda_R m_R{}^2}{16\pi^2}(\omega-4)^{-1}-\frac{\lambda_R m_R{}^2}{32\pi^2}\left[\gamma-1+\ln\left(\frac{m_R{}^2}{4\pi\mu^2}\right)\right]+\tfrac{1}{2}\lambda_R\int\frac{d^3k}{(2\pi)^3}\,\omega_k{}^{-1}(e^{L_1\omega_k}+1)^{-1}, \tag{14}$$

where $\omega_k=(k^2+m_R{}^2)^{1/2}$. Since the divergent part is the same as in Minkowski space-time, the coefficients of order λ_R which appear in Eqs. (10) and (11) are the same as those in Minkowski space-time.[14] It is also seen from Eq. (14) that there is a negative L_1-dependent (mass)^2 term of

$$-\tfrac{1}{2}\lambda_R\int\frac{d^3k}{(2\pi)^3}(k^2+m_R{}^2)^{-1/2}\{\exp[L_1(k^2+m_R{}^2)^{1/2}]+1\}^{-1}. \tag{15}$$

In the case of a massless theory, Eq. (15) reduces to $-\lambda_R/48L_1{}^2$.

Letting $[p_1,(\pi/L_1)(2l+1)]$, $[p_2,(\pi/L_1)(2m+1)]$ label the incoming momenta, Fig. 2(a) gives a contribution of

$$\tfrac{1}{2}(-\lambda_R)^2\mu^{4-\omega}\sum_{n=-\infty}^{+\infty}\frac{1}{L_1}\int\frac{d^{\omega-1}k}{(2\pi)^{\omega-1}}\left[k^2+\left(\frac{\pi}{L_1}(2n+1)\right)^2+m_R{}^2\right]^{-1}\times\left[(k-p_1-p_2)^2+\left(\frac{\pi}{L_1}(2n+1)-\frac{\pi}{L_1}(2l+1)-\frac{\pi}{L_1}(2m+1)\right)^2+m_R{}^2\right]^{-1}$$

to the four-point vertex function. Using Eq. (8) to deal with the summation, the divergent part of this is seen to be $-(\lambda_R{}^2/16\pi^2)(\omega-4)^{-1}$ which is independent of the external momenta; thus, Figs. 2(b) and 2(c) have an identical divergent part. The divergent part of the vertex function to order $\lambda_R{}^2$ is then $-(3\lambda_R{}^2/16\pi^2)\times(\omega-4)^{-1}$ which is the same as in Minkowski space-time. It then follows that to second order in λ_R the coefficients $a_{\nu j}$ appearing in Eq. (9) are the same as those which occur for ϕ^4 theory in Minkowski space-time.[14]

The contributions to $\Sigma(p)$ of order $\lambda_R{}^2$ are shown in Figs. 1(b)–1(e). Figure 1(b) gives

$$\tfrac{1}{4}(-\lambda_R)^2(\mu^2)^{4-\omega}\sum_{n=-\infty}^{+\infty}\frac{1}{L_1}\sum_{m=-\infty}^{+\infty}\frac{1}{L_1}\int\frac{d^{\omega-1}k}{(2\pi)^{\omega-1}}\frac{d^{\omega-1}q}{(2\pi)^{\omega-1}}\left[k^2+\left(\frac{\pi}{L_1}(2n+1)\right)^2+m_R{}^2\right]^{-2}\left[q^2+\left(\frac{\pi}{L_1}(2m+1)\right)^2+m_R{}^2\right]^{-1}.$$

After using Eq. (8) for each sum, the divergent part of this expression may be seen to be

$$-\frac{\lambda_R{}^2m_R{}^2}{256\pi^4}(\omega-4)^{-2}-\frac{\lambda_R{}^2m_R{}^2}{512\pi^4}\left[2\gamma-1+2\ln\left(\frac{m_R{}^2}{4\pi\mu^2}\right)\right](\omega-4)^{-1}+\frac{\lambda_R{}^2m_R{}^2}{64\pi^2}(\omega-4)^{-1}\int\frac{d^3k}{(2\pi)^3}(k^2)^{-1}\omega_k{}^{-1}(e^{L_1\omega_k}+1)^{-1}+\frac{\lambda_R{}^2}{32\pi^2}(\omega-4)^{-1}\int\frac{d^3k}{(2\pi)^3}\omega_k{}^{-1}(e^{L_1\omega_k}+1)^{-1}. \tag{16}$$

Figure 1(c) makes a contribution of $\Sigma(p)$ of

$$\tfrac{1}{2}(-\delta\lambda)\mu^{4-\omega}\sum_{n=-\infty}^{+\infty}\frac{1}{L_1}\int\frac{d^{\omega-1}k}{(2\pi)^{\omega-1}}\left[k^2+\left(\frac{\pi}{L_1}(2n+1)\right)^2+m_R{}^2\right]^{-1},$$

where $\delta\lambda=-(3\lambda_R{}^2/16\pi^2)(\omega-4)^{-1}$ is the coupling-constant counterterm to order $\lambda_R{}^2$ which follows from above. The divergent part of this expression is

$$\frac{3\lambda_R{}^2m_R{}^2}{256\pi^4}(\omega-4)^{-2}+\frac{3\lambda_R{}^2m_R{}^2}{512\pi^4}(\omega-4)^{-1}\left[\gamma-1+\ln\left(\frac{m_R{}^2}{4\pi\mu^2}\right)\right]-\frac{3\lambda_R{}^2}{32\pi^2}(\omega-4)^{-1}\int\frac{d^3k}{(2\pi)^3}\omega_k{}^{-1}(e^{L_1\omega_k}+1)^{-1}. \tag{17}$$

Figure 1(d) gives a contribution to $\Sigma(p)$ of

$$\tfrac{1}{2}(-\lambda_R)(-\delta m^2)\mu^{4-\omega}\sum_{n=-\infty}^{+\infty}\frac{1}{L_1}\int\frac{d^{\omega-1}k}{(2\pi)^{\omega-1}}\left[k^2+\left(\frac{\pi}{L_1}(2n+1)\right)^2+m_R{}^2\right]^{-2},$$

where $\delta m^2=-(\lambda_R m_R{}^2/16\pi^2)(\omega-4)^{-1}$ is the mass counterterm to order λ_R which follows from above. The divergent part of this expression is

$$\frac{\lambda_R{}^2m_R{}^2}{256\pi^4}(\omega-4)^{-2}+\frac{\lambda_R{}^2m_R{}^2}{512\pi^4}(\omega-4)^{-1}\left[\gamma+\ln\left(\frac{m_R{}^2}{4\pi\mu^2}\right)\right]-\frac{\lambda_R{}^2m_R{}^2}{64\pi^2}(\omega-4)^{-1}\int\frac{d^3k}{(2\pi)^3}(k^2)^{-1}\omega_k{}^{-1}(e^{L_1\omega_k}+1)^{-1}. \tag{18}$$

Figure 1(e) makes a contribution to $\Sigma(p)$ of

$$\tfrac{1}{6}(-\lambda_R)^2(\mu^2)^{4-\omega}\sum_{n=-\infty}^{+\infty}\frac{1}{L_1}\sum_{m=-\infty}^{+\infty}\frac{1}{L_1}\int\frac{d^{\omega-1}q}{(2\pi)^{\omega-1}}\left[k^2+\left(\frac{\pi}{L_1}(2n+1)\right)^2+m_R{}^2\right]^{-1}$$
$$\times\left[q^2+\left(\frac{\pi}{L_1}(2m+1)\right)^2+m_R{}^2\right]^{-1}\left[(k+q-p)^2+\left(\frac{\pi}{L_1}(2n+1)+\frac{\pi}{L_1}(2m+1)-\frac{\pi}{L_1}(2l+1)\right)^2+m_R{}^2\right]^{-1}$$

where $[p,(\pi/L_1)(2l+1)]$ is the external four-momentum. Due to the overlapping divergence this graph is the only nontrivial one to evaluate. The divergent part may be shown to be

$$-\frac{\lambda_R{}^2m_R{}^2}{256\pi^4}(\omega-4)^{-2}+\frac{\lambda_R{}^2}{3072\pi^4}(\omega-4)^{-1}\left[p^2+6m_R{}^2-12(\gamma-1)m_R{}^2-12m_R{}^2\ln\left(\frac{m_R{}^2}{4\pi\mu^2}\right)\right]$$
$$+\frac{\lambda_R{}^2}{16\pi^2}(\omega-4)^{-1}\int\frac{d^3k}{(2\pi)^3}\omega_k{}^{-1}(e^{L_1\omega_k}+1)^{-1}, \tag{19}$$

where p^2 is the square of the four-momentum of the external line.

We see that each of the graphs in Figs. 1(b)–1(e) separately contains L_1-dependent divergences. In order to obtain the total contribution of order $\lambda_R{}^2$ to the self-energy we must add Eqs. (16)–(19). When this is done it may be seen that all of the L_1-dependent divergences cancel and we are left with

$$+\frac{\lambda_R{}^2m_R{}^2}{128\pi^4}(\omega-4)^{-2}+\frac{\lambda_R{}^2m_R{}^2}{512\pi^4}(\omega-4)^{-1}+\frac{\lambda_R{}^2}{3072\pi^4}(\omega-4)^{-1}p^2,$$

which is the same as the divergence that one obtains for a scalar field in Minkowski space-time. As a result, all of the counterterms to order $\lambda_R{}^2$ are the same as those for an ordinary scalar field

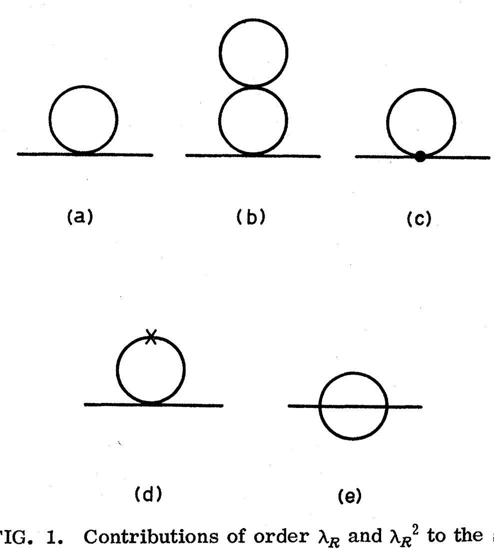

FIG. 1. Contributions of order λ_R and $\lambda_R{}^2$ to the self-energy.

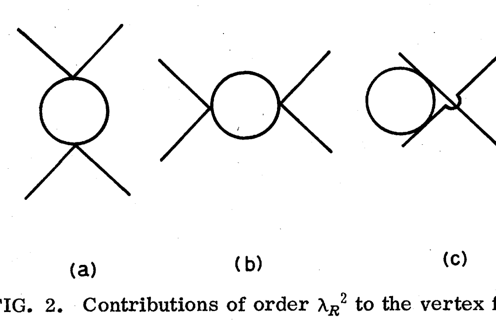

FIG. 2. Contributions of order $\lambda_R{}^2$ to the vertex function.

in Minkowski space-time, namely,[14]

$$\lambda_B = \mu^{4-\omega}\left[\lambda_R - \frac{3\lambda_R^{\,2}}{16\pi^2}(\omega-4)^{-1} + O(\lambda_R^{\,3})\right],$$

$$m_B^{\,2} = m_R^{\,2}\left[1 - \frac{\lambda_R}{16\pi^2}(\omega-4)^{-1} + \frac{\lambda_R^{\,2}}{128\pi^4}\left[(\omega-4)^{-2} + \tfrac{5}{24}(\omega-4)^{-1}\right] + O(\lambda_R^{\,3})\right],$$

$$Z = 1 + \frac{\lambda_R^{\,2}}{3072\pi^4}(\omega-4)^{-1} + O(\lambda_R^{\,3}).$$

Thus, we have shown that there is a mass-independent and L_1-independent renormalization procedure, at least to order $\lambda_R^{\,2}$, for a twisted scalar field with a ϕ^4 self-interaction in a periodically identified flat space-time.

IV. COMMENTS

In principle, there is nothing to stop us from extending the calculation of Sec. III to higher orders in λ_R; however, in practice it proves very difficult to do. In addition to the problem of overlapping divergences which is present in the higher loop graphs, one also requires more terms in the Laurent expansion about $\omega = 4$ of the integrals which occur.

Finally, we remark that a convenient approach to problems such as those considered in Refs. 2 and 3 is a calculation of the effective potential.[16] In addition to giving us the vacuum energy density and the radiative corrections to the mass, the effective potential also allows us to discuss possible symmetry breaking by searching for its minima. We have completed a calculation of the one-loop effective potential for a real, massless scalar field with a ϕ^4 self-interaction in the cases of periodically identified flat space-time for both twisted and untwisted scalar fields, Minkowski space-time in the presence of flat parallel conducting plates, and in the Einstein static universe. Furthermore, as a consequence of the renormalizability discussed above, the contribution of the two-loop effective potential to the vacuum energy density has been found. (The complete expression for the two-loop effective potential appears to be very difficult to obtain.) Except in the case of Minkowski space-time in the presence of parallel conducting plates, we find agreement with Refs. 2 and 3. In this latter case we find a finite contribution of $\lambda_R/18432L_1^{\,4}$ to the vacuum energy density in contrast to Ford[2] who obtains an infinite result, and a constant $(\text{mass})^2$ term of $\lambda_R/96L_1^{\,2}$ in contrast to Ford and Yoshimura[3] who obtain a result which is both negative and spatially dependent. A complete discussion of this calculation and of symmetry breaking will be reported in another paper.

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