Net baryon number and *CP* nonconservation with unified fields

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We derive the net baryon density of the universe which emerged, as a result of departure from thermal equilibrium in the early universe, from the dynamics of boson decays to fermions as prescribed by unified theories of weak-electromagnetic interactions. It is observed that at least two independent scalar-boson multiplets coupled to fermions are required to generate net baryon number. Numerical estimates based on exact formulas in the simplest SU(5) model and reasonable estimates of undetermined parameters (but essentially model-independent for order-of-magnitude estimates) are made and found to be compatible with the range of observational estimates of baryon density.

The observed baryon-antibaryon imbalance of the universe has been attributed to various sources.[1] In essentially all proposed scenarios, departure from equilibrium in the early universe is seen as the cause of the small but significant net baryon-number density. Adopting this picture, we present an explicit determination of the net baryon-number density of the universe, based on grand unified models in elementary particle theory[2] in the context of the standard cosmological description of the universe.[3] The critical ingredients are the existence of interactions which violate baryon-number (B) conservation, charge-conjugation (C) symmetry, and C plus parity (CP) conservation, added to cosmological expansion of the universe, which pulls interactions out of equilibrium. All of these are provided in the SU(5) grand unified model,[4] which we use for explicit evaluations since it is the simplest such model. Other models possessing B-, C-, and CP-nonconserving forces can be used; results in such models will differ from ours at most by small numbers representing differences in numbers of channels, or differences in parametrization of known effects.

The picture we envision here is that the earliest universe was in a highly collapsed state, with a radius $<10^{-9}$ cm. Thus typical particle separations were 10^{-33} cm or less with interaction energies about 10^{19} GeV or more. Consequently, the baryon-number-violating interactions of unified gauge theories[2] will have reached equilibrium because they are characterized by the energy scale of SU(5) breaking, which is about 10^{15} GeV or less.[5] Such a period of equilibrium will eliminate any possible explanation of today's observed baryon excess[6] as a boundary condition, since particle distributions will be governed only by the Boltzmann factor which is the same for particle and antiparticle. As the universe expands, its density[7] decreases, interparticle distances increase, and the typical energy scale of particle interaction drops. All of these effectively reduce reaction rates. Indeed, interactions involving heavy particles will disappear at energies of the order of those particle masses. It seems to be established that baryon-number-violating scattering processes are not critical in determining when equilibrium ceases; instead, decay and inverse decay processes for the heavy baryon-number-violating mesons are the relevant processes.[8] Thus, baryon number can begin to appear when these inverse decay processes cease and these mesons go out of equilibrium.

It should be noted that, although these bosons carry net color and thus are strongly interacting, nevertheless in this cosmological era strong interactions are not strong (due to asymptotic freedom). Hence, unless confinement is relevant, strong-interaction effects between bosons can reasonably be neglected. Even if confinement is relevant, producing an appreciable number of boson pair states which enhance annihilation rates relative to decays, nevertheless meson-quark states will be significantly more numerous, due to the Boltzmann factor ($M_F=0$, whereas $M\sim T$) and the greater multiplicity of quark flavors. Thus the overall picture for meson interaction rates will be little changed by confinement; each meson will merely be accompanied by a spectator quark, rather than being free. Also, at these high energies the constituents of these states will be essentially free, and a free-particle treatment should be a reasonable approximation. In either

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case we thus find that neglecting strong interactions and confinement will be a reasonable approximation.

We assume the standard SU(5) formalism, with three families of fermions; each includes a left-handed $\underline{10}$ and a right-handed $\underline{\bar{5}}$, plus antiparticles. At much lower energies these three families will break up and mix into mass eigenstates, yielding the currently observed (or presumed) six flavors of quark color triplets, three charged leptons, and three neutrinos. In the cosmological era we are considering, these fermions are all massless; their only interactions are their gauge couplings and Yukawa couplings. The gauge couplings can be taken diagonal (in family or flavor); CP nonconservation occurs naturally in the Yukawa couplings to scalar fields.[9] (Low-energy CP nonconservation is seen in the weak-interaction gauge couplings due to the redefinitions necessary to diagonalize the observed fermion mass matrix.) Thus the only possible CP-nonconserving couplings will be the Yukawa vertices and any scalar-particle self-couplings.

Although decays of the heavy gauge bosons [those which receive masses from SU(5) breaking] do violate B conservation, enumeration of possible diagrams shows that they do not in fact have any CP-nonconserving part at the one-loop level, while a two-loop CP-nonconserving part will be down by α/π relative to the scalar contribution. Hence, we are concerned only with the (Higgs) scalar-boson decays. The relevant couplings are given by the Lagrangian terms

$$\mathcal{L}_I = \Psi^{\dagger\alpha\beta}\gamma^0 f_i \phi_{i\alpha}\Psi_\beta + \tfrac{1}{4}\epsilon^{\alpha\beta\gamma\delta\epsilon}\Psi^T_{\alpha\beta}\gamma^0 h_i \phi_{i\gamma}\Psi_{\delta\epsilon} + \text{H.c.}, \tag{1}$$

where Greek indices are SU(5) indices running from 1 to 5, Roman indices label different scalar multiplets, and f_i and h_i are matrices acting on the suppressed family index of the fermion multiplets. At least two scalar multiplets, with different f_i and h_i, must exist, or the contribution which we calculate will also vanish at the one-loop level. [It is interesting to note that two scalar multiplets coupled to fermions seem to be needed to obtain realistic physical mass relations in SU(5) models, and for Weinberg's scheme[10] of CP nonconservation in Higgs propagators.]

Some or all of the scalar multiplets here could alternatively be taken to be 45-plets under SU(5), rather than 5-plets. In this case $\phi_{i\alpha}$ is replaced by $\phi^{\gamma}_{i\alpha\beta}$ for these multiplets, where the extra two indices are used instead of contracting two indices elsewhere in the term. The same diagrams will occur, since the 45-plet terms couple to the same particle types as the 5-plets and no others; the index relations of possible channels will be altered, but the effect on our results will only be to change combinatoric counting factors. (This option can be limited by a hypothesis of asymptotic freedom for the unified SU(5) theory. That hypothesis, plus the known existence of three fermion families, plus an adjoint representation of scalars [to break SU(5) → SU(3) × SU(2) × U(1); these cannot couple to known fermions] limits the Higgs sector to two 45's and up to 3 quintets, or one 45 and up to 27 quintets, or no 45's and up to 51 quintets; three or more 45's are excluded.) However, we will ignore this possibility for the remainder of this paper.

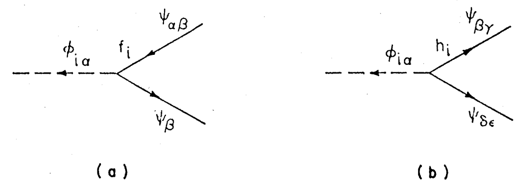

FIG. 1. Diagrams for the principal decay modes of a Higgs scalar. (a) Decay into $\underline{5}$ and $\underline{\overline{10}}$ fermions via an f vertex. (b) Decay into two $\underline{10}$ fermions via an h vertex.

The total decay rates for the scalar mesons are essentially given by the tree diagrams for the decays into two fermions, Fig. 1. There are two diagrams corresponding to the two Yukawa vertices, with inequivalent final states. Evaluating the diagrams and summing over all final-state variables to obtain a total rate, we obtain the decay width for particle $\phi_{i\alpha}$,

$$\Gamma_{i\alpha} = \frac{M_{i\alpha}}{16\pi}\,(2\,\mathrm{Tr} f_i f_i^\dagger + 3\,\mathrm{Tr} h_i h_i^\dagger)\,. \tag{2}$$

The leading CP-nonconserving contributions come from the (one-loop) triangular diagrams of Fig. 2 and specifically from interference cross terms between these and the diagrams of Fig. 1.

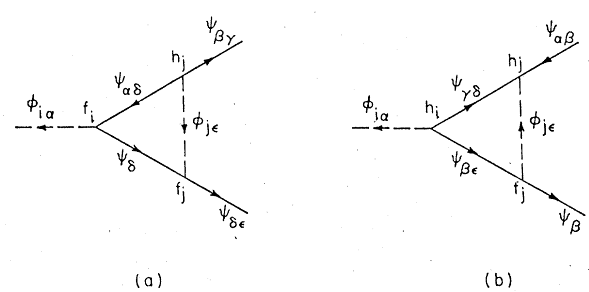

FIG. 2. Diagrams for the leading CP-nonconserving decay amplitudes. (a) Decay into two $\underline{10}$ fermions starting with an f vertex. (b) Decay into $\underline{5}$ and $\underline{\overline{10}}$ fermions starting with an h vertex.

The triangle diagrams share a common kinematic structure (see the Appendix), and involve the same CP-nonconserving parameter dependence. Thus, we find that although the sum over all final states of the CP-nonconserving partial widths $\delta\Gamma$ vanishes (as required by CPT invariance), the sum of $B\delta\Gamma$ does not vanish:

$$\sum B\,\delta\Gamma = -\frac{M_{i\alpha}}{8\pi}\,\frac{1}{16\pi}\sum_j T_{ij}\,F\left(\frac{M_j^2}{M_i^2}\right), \tag{3}$$

where

$$F(x) = 1 - x\ln(1+1/x), \tag{4}$$

$$T_{ij} = \mathrm{Im}\,\mathrm{Tr} f_i f_j^\dagger h_i^\dagger h_j, \tag{5}$$

and M_j is the mass of the heavy components of $\phi_{j\alpha}$. The baryon number generated per decay of a heavy component of $\phi_{i\alpha}$ is thus given by

$$\begin{aligned} n_i &= \frac{1}{\Gamma_i}\sum B\,\delta\Gamma \\ &= -\frac{1}{8\pi}\,(2\,\mathrm{Tr} f_i f_i^\dagger + 3\,\mathrm{Tr} h_i h_i^\dagger)^{-1}\sum_j T_{ij}\,F\left(\frac{M_j^2}{M_i^2}\right). \end{aligned} \tag{6}$$

We can obtain a numerical estimate of n_i by assuming that the matrices f_i and h_i are all of similar magnitude. Note, however, that we cannot assume $f_i = f_j$, $h_i = h_j$; in that case the antisymmetry of T_{ij} leads to its vanishing, and consequently no baryon-number generation. This also excludes the possibility of B generation when only one Higgs scalar appears in the theory. We also assume that the phase of the matrix trace is similar in magnitude to the CP-nonconserving phases of observed processes, about 10^{-3}. Subject to these assumptions, the magnitudes of f and h can then be determined by the fact that when $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ symmetry is eventually broken, these same Yukawa vertices will generate the observed fermion mass spectrum (see the Appendix). If we also take $M_i \simeq M_j$ and average over scalar types we obtain

$$n \simeq \frac{1}{8\pi}\,\tfrac{2}{3}\times 10^{-3} m_b^{\,2}(\sqrt{2}G_F)(1-\ln 2) \sim 10^{-8}\text{–}10^{-9}, \tag{7}$$

where m_b is the b-quark mass. Essentially the same order-of-magnitude estimate will appear in any B-generating models, since this involves only the known CP-nonconservation strength, 10^{-3}, quark masses, and the weak-interaction strength G_F, plus the universal kinematic factors $1/8\pi$ and $(1-\ln 2)$; only the $\frac{2}{3}$ factor is specific to SU(5), and no reasonable model is likely to replace it with a significantly larger or smaller number.

The density of each type of Higgs particle in equilibrium, relative to the density of photons, will be given by a Boltzmann factor $\exp(-M/T)$. Since these heavy mesons will go out of equilibrium at $T \simeq M$,[11] we estimate that this factor will be 10^{-1} to 10^{0}. The present baryon-to-photon density ratio will be given by that number, multiplied by n, the baryon number per decay, and by the number of meson types decaying (at least 12, counting colors and antiparticles), and divided by the increase in photons (about 10^2) due to later annihilations and to various noncosmological processes (stellar radiation, for instance). We thus arrive at a present baryon-to-photon ratio of order 10^{-9}–10^{-10}. While the actual value depends significantly on parameters which we cannot determine at present, this estimate shows that reasonable approximations can lead to results that are compatible with the present range of observational estimates.

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APPENDIX

1. The triangle integral

The loop integration for either triangle diagram is, up to overall factors,

$$I = \int \frac{d^4Q}{(2\pi)^4}\,\frac{\gamma\cdot(Q+p_1)}{(Q+p_1)^2}\,\frac{\gamma\cdot(Q-p_2)}{(Q-p_2)^2}\,\frac{1}{Q^2-m^2},$$

where m is the exchanged meson mass, since all fermions are massless in this cosmological era. Also, since the final-state fermions are massless, the Dirac equation can be applied to eliminate $\gamma\cdot p_1$ and $\gamma\cdot p_2$ from I, leaving

$$I = \int \frac{d^4Q}{(2\pi)^4}\,\frac{Q^2}{(Q+p_1)^2(Q-p_2)^2(Q^2-m^2)}.$$

Since we will retain only the CP-nonconserving part of the decay width, we find that only the real part of I contributes. Because of the Wick rotation into Euclidean space, the leading behavior (logarithmic divergence) of I is pure imaginary; ReI is infinity-free. (This can also be seen in another way. Net baryon generation is observable, and this class of theories is renormalizable. Hence, if there are any infinities in the calculation, they must be canceled by contributions from counterterms. But at this order in perturbation theory, counterterms occur only in tree diagrams and tree diagrams cannot contribute net CP-nonconservation.)

We used dimensional regularization to express I and found by straightforward manipulations

$$\text{Re}I = \frac{-1}{16\pi}\left[1 - \frac{m^2}{M^2}\ln\left(1 + \frac{M^2}{m^2}\right)\right]$$
$$= -\frac{1}{16\pi}F\left(\frac{m^2}{M^2}\right),$$

where $M^2 = 2p_1 \cdot p_2$ is the external meson mass squared.

We may also note that explicit enumeration shows that exchanges of the doublet components of the Higgs multiplets have zero coefficient in $\sum B\Gamma$, while decays of doublet components are possible only to $B=0$ states. Thus baryon-number generation is contributed solely by triplet-Higgs exchange in triplet-Higgs decays, without regard to relative masses. [$SU(3) \times SU(2) \times U(1)$ symmetry is, of course, assumed in this enumeration.]

2. The matrices *f* and *h*

Consideration of the breaking of $SU(2) \times U(1)$ symmetry by these fields leads to the relations

$$\sum f_i \langle H^i \rangle = -\sqrt{2}M_d ,$$
$$\sum h_i \langle H^i \rangle = -M_u ,$$

where M_u is the mass matrix for the up-series quarks (u,c,t) and M_d is the mass matrix for the down-series quarks (d,s,b) which is also the charged-lepton mass matrix. [Both are in the original generation-space basis; our freedom of redefinition will let us take M_d real and diagonal, but M_u is arbitrary except that it is symmetric (as are the h_i).]

With only two matrix equations to relate at least four independent coupling matrices, additional assumptions must be made if a specific estimate is to be obtained. We assume that each F_i is of order $\sqrt{2}M_d/\langle H\rangle$ and that each h_i is of order $M_u/\langle H\rangle$, where $\langle H\rangle$ is the vacuum expectation value magnitude required to give observed weak interactions, $\langle H\rangle \sim (\sqrt{2}G_F)^{-1/2}$. We also assume that the CP-nonconserving phases combine to give an imaginary part to the trace that is of order 10^{-3} times the leading real terms. The leading real terms are given by neglecting mixing angles and lighter quarks, thus, $M_u \sim m_t, M_d \sim m_b$. Combining these assumptions, we obtain

$$T_{ij} \simeq 10^{-3}(\sqrt{2}m_b)^2(m_t)^2(\sqrt{2}G_F)^2 ,$$

while

$$2\,\text{Tr}ff^\dagger + 3\,\text{Tr}hh^\dagger \simeq [2(\sqrt{2}m_b)^2 + 3(m_t)^2](\sqrt{2}G_F)$$
$$\simeq (3m_t{}^2)(\sqrt{2}G_F) ,$$

which combine to give Eq. (7) in the text.

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[5]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 45 (1974). See also A. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B135, 66 (1978); D. Ross, *ibid.* B140, 1 (1978).

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[11]We are considering masses of the order of 10^{15} GeV; the reactions most relevant for such particles are inverse decays, whose rate equals the cosmological expansion rate at a time not much after the time when $T=M$. (Toussaint *et al.*, Ref. 1, for instance, give relevant rate estimates.)