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Gravity theories with propagating torsion

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We study the propagators for a large class of gravity theories having a nonzero, metric-compatible torsion. The theories are derivable from a Lagrangian containing all possible invariants quadratic or less in the torsion and Riemann curvature tensors, except that invariants are dropped if they do not contribute to the propagator in the linearized limit. Therefore, the torsion in these theories is, in general, a propagating field rather than one which vanishes outside matter. We study the constraints imposed on the propagator by the requirement that the theory have no ghosts or tachyons. In particular, we find that the addition of a spin-2^+ torsion multiplet does not remove the spin-2^+ ghost contributed by higher-derivative terms (Riemann curvature-squared terms). We discuss the phenomenology of theories with propagating torsion. The torsion must couple to spins with coupling constants much smaller than the electromagnetic fine-structure constant, or the force between two macroscopic ferromagnets, due to torsion exchange, would be huge, far greater than the familiar magnetic force due to photon exchange. We briefly discuss the phenomenology of propagating torsion "potentials." Theories involving such potentials have been proposed recently by several authors.

I. INTRODUCTION

In standard gravity theory, the torsion (equal to the antisymmetric part of the affinity $\Gamma^{\lambda}{}_{[\mu\nu]}$) is zero. Starting with Cartan in 1924, many authors have suggested that torsion should be nonzero and should be coupled to the intrinsic spin density of matter, so that the spin part of the Poincaré group can change the geometry of space-time, just as the energy-momentum part does.[1]

Historically, torsion was usually introduced into gravity theory via a kind of "minimal" substitution: In the Lagrangian for gravity plus matter, replace the standard (Christoffel) affinity $\{^{\lambda}{}_{\mu\nu}\}$ by the Cartan affinity $\Gamma^{\lambda}{}_{\mu\nu}$ everywhere. Then vary with respect to the metric and (say)

$$B^{\lambda}{}_{\mu\nu} \equiv \Gamma^{\lambda}{}_{\mu\nu} - \{^{\lambda}{}_{\mu\nu}\}\,, \tag{1.1}$$

where $B^{\lambda}{}_{\mu\nu}$ is the torsion-dependent part of the affinity. Since the standard gravity Lagrangian is just the curvature scalar R, the procedure is to replace $R[\{\ \}] \to R[\{\ \}+B]$, then vary. The new "field equations" for B turn out to be algebraic equations, not differential equations, however. [This happens because R is only linear in the first derivative of B; hence the Lagrangian does not contain any $(\partial B)^2$ "kinetic" terms for the B field.] These algebraic equations can be solved, and they predict that B is not an independent field but rather a known function of the metric and the matter degrees of freedom. In fact, the theory collapses to Einstein's theory, except for some unobservable corrections to the matter Lagrangian.

Matters might have ended there, except that there are reasons for considering a gravity Lagrangian which is not strictly linear in R. First, gauge-theoretic ideas suggest that R^2 terms (terms bilinear in curvature) should be present.[2,3] Second, adding R^2 terms can make the theory renormalizable (although so far unitarity suffers).[4] Third, of course, there is the desire to give torsion a more standard dynamics. Thus Gregorash and Skinner have written down the most general Lagrangian containing terms quadratic in the torsion fields, and have derived the corresponding field equations.[5]

If one wishes to go beyond the standard order-R gravity Lagrangian, the simplest way to do so is to apply the minimal-substitution procedure to an $R+R^2$ gravity Lagrangian. We did this in a previous paper (hereafter referred to as I) with results that were not entirely satisfactory.[6] We wish to describe those results of I, in order to motivate our decision to go beyond the minimal-substitution framework in the present paper. Since we wished to check the renormalizability and high-energy behavior of the theory in I, we derived the propagator, i.e., the inverse of the wave operator for the linearized limit of the theory. We anticipated very good high-energy behavior for the gravitation propagator, leading to renormalizability, because the propagator P in torsion-free $R+R^2$ theory goes as $1/k^4$ at high energies.[4] However, we found only $P \to$ constant behavior, too slow a falloff to bring about renormalizability.

How did adding torsion to an $R+R^2$ theory alter the high-energy behavior of the propagator so drastically? To answer this question in full detail, one would have to wade through the full calculation in I; however, the following simple model gives the essence of the answer. We write down two wave equations (in k space), linking two fields g and B to each other and to their matter sources T and S:

$$(\Lambda k^2 - M)B + \Lambda k^2 g = S\,, \tag{1.2a}$$

$$\Lambda k^2 B + (\Lambda k^2 + M) g = T\,. \tag{1.2b}$$

B and G are scalar fields, but may be thought of as stand-ins for tensor components of $B^{\lambda}{}_{\mu\nu}$ and $g_{\mu\nu}$ linkded by the field equations of I. Λ and M are constants, functions of the coupling and mass parameters occurring in the Lagrangian. If the B terms and Eq. (1.2a) were absent, one would get a $1/k^2$ propagator (inverse wave operator) for g. However, with the B terms present we must write Eqs. (1.2a) and (1.2b) in matrix form, and the propagator is the inverse of this matrix:

$$P \propto \begin{bmatrix} \Lambda k^2 - M & \Lambda k^2 \\ \Lambda k^2 & \Lambda k^2 + M \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \Lambda k^2 + M & -\Lambda k^2 \\ -\Lambda k^2 & \Lambda k^2 - M \end{bmatrix} (-M^2)^{-1}\,. \tag{1.3}$$

This has high-energy behavior $P \to k^2$; just as in I, we have gained four powers of k by adding the B field. By working backwards from the final answer (1.3), we can discover how this happened. In inverting a matrix, one must divide through each element of the matrix by a determinant. Matrix (1.3) would be expected to have a determinant of order k^4, because each element is order k^2. However, the determinant turns out to be order k^0 because of delicate cancellations between the Λk^2 and M terms. These cancellations in turn may be traced to the peculiar form of the original wave equations (1.2a) and (1.2b). The same mass parameter M occurs in both equations, and all four order-k^2 terms contain the same parameter Λ. This pattern of constants is very special; it can arise only if, in constructing the Lagrangian for this theory, we restrict ourselves to very special linear combinations of the available invariants:

$$\mathcal{L} = \tfrac{1}{2}\Lambda \partial_\mu (g+B)\partial^\mu (g+B) + \tfrac{1}{2}(g^2 - B^2)M\,. \tag{1.4}$$

Analogously, when we adopted the minimal-substitution procedure in I, we restricted ourselves to a very special choice of invariants.

If one is willing to ignore the quantum behavior and consider only the classical limit, minimal-substitution theory has some interesting features.[7] However, clearly minimal substitution does not guarantee acceptable high-energy behavior or orthodox dynamics. [Note that the "propagator" (1.3) does not propagate: It has no poles in k space, which means that in configuration space the B and g fields vanish outside sources.]

In the present paper we abandon the minimal-substitution principle and derive the propagator for the most general Lagrangian quadratic or less in torsion and curvature tensors. (See Secs. II and III.) This is quite easy to do (despite the presence of 17 invariants in the Lagrangian) because we can use a powerful projection operator machinery developed in I. The high-energy behavior of the propagator is no longer anomalous.

In Sec. IV we write down the constraints that the propagator must satisfy to be free of ghosts and tachyons. If there is only one particle of a given spin-parity present in a theory, one can tell that it is not ghostly or tachyonic easily, by inspecting its kinetic and mass terms in the Lagrangian. If there are two particles of the same spin-parity present and they couple (as is the case for $J^P = 2^+, 1^\pm, 0^+$ in the present theory) then the kinetic and mass terms in the Lagrangian form a 2×2 matrix. This matrix must be inverted to obtain the propagator, as at Eq. (1.3); then one can check for ghosts and tachyons by examining the residues and locations of the propagator poles. In general these poles and residues now depend in a complicated way on all of the parameters occurring in the original Lagrangian. We were especially interested in checking for absence of ghosts in the 2^+ sector after a 2^+ torsion multiplet is added; in higher-derivative gravity without torsion, a ghost always occurs in the 2^+ sector. Unfortunately, we have verified from the formulas of Sec. IV that the addition of torsion does not change this situation: At least one residue remains ghostly. Section IV also summarizes the difficulties one encounters even if one drops the higher derivatives, ignores renormalizability, and attempts to construct a purely classical theory of propagating spin-two torsion multiplets.

In Sec. V we study the phenomenology of torsion, emphasizing theories which do not use the minimal-substitution principle. (Hehl *et al.* have reviewed the phenomenology of the minimal-substitution theory with gravity Lagrangian linear in R.[1]) It suffices to consider a linearized theory, since no large-spin source exists for torsion, analogous to the large mass sources existing for gravity. (We do not consider possible early-universe effects.) A linearized theory is completely defined by its propagator (which we already have) plus a choice of torsion-to-matter coupling. We choose a coupling modeled on the electromagnetic one, and arrive at useful estimates of spin-spin forces mediated by torsion.

There is another way to get propagating torsion fields, while retaining the traditional linear-in-R Lagrangian plus minimal-substitution framework. One introduces derivatives into the Lagrangian by writing the torsion as the derivative of a potential,[8,9] for instance[8]

$$B^{\lambda}{}_{\mu\nu} \equiv \phi^{\lambda}{}_{\mu;\nu}\,. \tag{1.5}$$

The B^2 terms in R then become $(\partial\phi)^2$ kinetic terms for ϕ. Two points are worth making about these "potential" models. First, their phenomenology is quite different from that of models where the $(\partial B)^2$ terms are introduced "by hand" (see Sec. V). Second, models proposed up to now in the literature appear to be ghost-free; however, future models should be presumed ghostly until proven otherwise; i.e., the pattern of signs in the B^2 terms is quite random, so that there is a 50 percent chance the $(\partial\phi)^2$ kinetic term will have the wrong sign.[10]

II. THE LAGRANGIAN

As in I, we choose vierbein and contortion fields V_a^α and $B^\lambda{}_{\mu\nu}$ as our independent basis set to construct the Lagrangian, because the propagator will be very easy to calculate if we use the projection operators already worked out for these fields in I. $B^\lambda{}_{\mu\nu}$ is the torsion-dependent part of the Cartan connection $\Gamma^\lambda{}_{\mu\nu}$, defined at Eq. (1.1). Our connection $\Gamma^\lambda{}_{\mu\nu}$ is metric compatible, which implies $B_{\lambda\mu\nu} = -B_{\mu\lambda\nu}$. The tensor $B^\lambda{}_{\mu\nu}$ is not identical to the torsion; but B is nonzero if and only if the torsion is, because the following equations are invertible:

$$2\Gamma^\lambda{}_{[\mu\nu]} = B^\lambda{}_{\mu\nu} - B^\lambda{}_{\nu\mu}\,. \tag{2.1}$$

We now write down all possible invariants which contain a part bilinear in vierbein and/or contortion fields; these are the only ones which can contribute to the propagator. (However, we ignore a possible cosmological term.) We use the conventions[11]

$$R^\lambda{}_{\mu\alpha\beta} = \partial_\alpha \{{}^\lambda{}_{\mu\beta}\} - \partial_\beta \{{}^\lambda{}_{\mu\alpha}\} + \cdots\,, \tag{2.2a}$$

$$R_{\alpha\beta} = R^\lambda{}_{\alpha\lambda\beta}\,, \tag{2.2b}$$

$$R = R^\lambda{}_{\alpha\lambda\beta} g^{\alpha\beta}\,, \tag{2.2c}$$

$$V = \det V_a^\alpha\,. \tag{2.3}$$

Then

$$\begin{aligned} V\mathfrak{L} = {} & -R/2\kappa + \beta_{1R} R^2 + \beta_{2R} R_{\alpha\beta} R^{\alpha\beta} + \beta_{1RB} R (B^\alpha{}_{\beta\gamma;\alpha} - B^\alpha{}_{\beta\alpha;\gamma}) g^{\beta\gamma} + \beta_{2RB} R^\alpha{}_\beta (B^\beta{}_{\mu\nu;\alpha} - B^\beta{}_{\mu\alpha;\nu}) g^{\mu\nu} \\ & + \beta_1 B^\mu{}_{\nu\alpha;\beta} B^\mu{}_{\nu\alpha;\beta} + \beta_2 (B^\mu{}_{\nu\alpha;\beta} B^\mu{}_{\nu\beta;\alpha} + B^\mu{}_{\nu\alpha;\alpha} B^\mu{}_{\nu\beta;\beta}) + \beta_3 (B^\mu{}_{\nu\alpha;\beta} B^\alpha{}_{\beta\mu;\nu} + B^\mu{}_{\nu\alpha;\nu} B^\alpha{}_{\beta\mu;\beta}) \\ & + \beta_4 (B^\mu{}_{\nu\alpha;\beta} B^\mu{}_{\alpha\nu;\beta}) + \beta_5 (B^\mu{}_{\nu\alpha;\beta} B^\mu{}_{\beta\nu;\alpha} + B^\mu{}_{\nu\alpha;\alpha} B^\mu{}_{\beta\nu;\beta}) + \beta_6 (B^\mu{}_{\nu\alpha;\beta} B^\mu{}_{\beta\alpha;\nu} + B^\mu{}_{\nu\alpha;\nu} B^\mu{}_{\beta\alpha;\beta}) \\ & + \beta_7 B^\mu{}_{\nu\mu;\alpha} B^\lambda{}_{\nu\lambda;\alpha} + \beta_8 (B^\mu{}_{\nu\mu;\alpha} B^\lambda{}_{\alpha\lambda;\nu} + B^\mu{}_{\nu\mu;\nu} B^\lambda{}_{\alpha\lambda;\alpha}) + \beta_9 (B^\mu{}_{\nu\mu;\alpha} B^\lambda{}_{\nu\alpha;\lambda} + B^\mu{}_{\nu\mu;\lambda} B^\lambda{}_{\nu\alpha;\alpha}) \\ & + \mu_M{}^2 (B P_{BM} B) + \mu_A{}^2 (B P_{BA} B) + \mu_T{}^2 (B P_{BT} B)\,. \end{aligned} \tag{2.4}$$

$\kappa = 8\pi G$, so that the Newtonian limit comes out correctly. Repeated covariant or contravariant indices in the β_1–β_9 terms are understood to be contracted with appropriate $g_{\mu\nu}$'s. The projection operators P_{Bi} $(i = T, M, A)$ in the mass terms break the torsion up into its trace (T) plus two other irreducible tensors having mixed (M) or antisymmetric (A) behavior under interchange of indices.[6] The six indices on P_{Bi} and the three on B have been suppressed.

One could construct invariants in addition to those included in Eq. (2.4), but they always turn out to equal invariants already included, plus a total derivative or a term which is trilinear or higher in the fields, therefore does not contribute to the propagator. For instance the term

$$R^\mu{}_{\nu\alpha\beta} B^\mu{}_{\nu\alpha;\beta}\,, \tag{2.5}$$

is proportional to the β_{2RB} term. (Proof: Integrate by parts convariantly to transfer the β index from B to R and use Bianchi identities.) Similar arguments rule out

$$(B^\alpha{}_{\beta\alpha;\nu} + B^\alpha{}_{\beta\nu;\alpha}) R^{\nu\beta}\,. \tag{2.6}$$

Again, one could generate six seemingly new terms from the β_i terms with $i = 2, 3, 5, 6, 8, 9$, by changing the plus sign inside each set of parentheses to a minus sign. For instance from β_2 one could obtain

$$\beta_2'(B^\mu{}_{\nu\alpha;\beta} B^\mu{}_{\nu\beta;\alpha} - B^\mu{}_{\nu\alpha;\alpha} B^\mu{}_{\nu\beta;\beta})\,. \tag{2.7}$$

This term turns out to be equal to a total derivative plus a term trilinear or higher in the fields. [Proof: apply covariant integration by parts to the β derivative on the second term in Eq. (2.7); interchange the α and β derivatives, which generates a trilinear term of order RB^2; apply integration by parts to the α derivative.]

III. THE PROPAGATOR

As in I, we now drop all but the bilinear terms in the Lagrangian ($\mathfrak{L} \to \mathfrak{L}^{(2)}$) and transform to momentum space. Using the covariant projection operators developed in I, we break contortion and vierbein tensors B and V up into multiplets having a definite spin-parity $J^P = 2^\pm$, $1^\pm$, or $0^\pm$, so that $\mathfrak{L}^{(2)}$ decomposes into mutually disjoint sectors. The propagator is then (-1) times the inverse of $\mathfrak{L}^{(2)}$.

The $J^P = 2^-, 0^-$ sectors of $\mathfrak{L}^{(2)}$ contain one multiplet each, while the $J^P=2^+, 1^\pm, 0^+$ sectors contain two multiplets each. When written as a 2×2 matrix, each of the latter sectors has the form

$$\mathfrak{L}^{(2)}=\begin{bmatrix} (\Lambda_{11}k^2+a_{11}/\kappa)P_{11} & \Lambda_{12}k^2\sqrt{f}P_{12} \\ \Lambda_{12}k^2\sqrt{f}P_{21} & (\Lambda_{22}k^2+a_{22}/\kappa)fP_{22} \end{bmatrix}. \tag{3.1}$$

The projection operators P_{ij} are defined in I; the constants Λ_{ij}, a_{ii}, and f for each sector are presented in Table I.

[Note: To aid in dimensional analysis, we have renormalized our vierbein basis slightly, at Eq. (3.1); we have switched from V_a^α to $V_a^\alpha/\sqrt{\kappa}$ as basis, so that B and $V/\sqrt{\kappa}$ both have the same dimension, 1/length. Because of this switch, there are some extra factors of κ in the f column of Table I; also, every matrix element in Eq. (3.1) has the same dimension, $(\text{length})^2$.]

Equation (3.1) is easily inverted to give the propagator P (note the projection operators are normalized by $P_{ij}P_{jk}=\delta_{ik}P_{ii}$):

$$P=(-1/\det\mathfrak{L}^{(2)}) \times\begin{bmatrix} (\Lambda_{22}k^2+a_{22}/\kappa)fP_{11} & -\Lambda_{12}k^2\sqrt{f}P_{12} \\ -\Lambda_{12}k^2\sqrt{f}P_{21} & (\Lambda_{11}k^2+a_{11}/\kappa)P_{22} \end{bmatrix}. \tag{3.2}$$

We shall analyze the poles and residues of this Green's function in the next section. Before doing this, let us also record the propagators for the two 1×1 sectors, which contain torsion multiplets having $J^P=0^-$ and 2^-:

$$P(0^-)=-[\mu_A{}^2-(\beta_1-\beta_4)k^2]^{-1}P_B(0^-), \tag{3.3}$$

$$P(2^-)=-[\mu_M{}^2-(\beta_1+\tfrac{1}{2}\beta_4)k^2]^{-1}P_B(2^-). \tag{3.4}$$

IV. THE 2^+ SECTOR OF THE PROPAGATOR

We now write down the constraints which propagator (3.2) must obey if it is to be free of tachyons and ghosts. We do so using the 2^+ sector as an example; the constraints for the $1^\pm$ sector will look slightly different because the function f is slightly different [see Eq. (3.2) and Table I].

The poles of the 2^+ propagator are given by the zeros of the $\det\mathfrak{L}^{(2)}$ factor in Eq. (3.2):

$$\det\mathfrak{L}^{(2)}=[k^4\det\Lambda+(\Lambda_{11}a_{22}+\Lambda_{22}a_{11})k^2/\kappa +a_{11}a_{22}/\kappa^2]k^2\kappa, \tag{4.1}$$

$$\det\Lambda\equiv\Lambda_{11}\Lambda_{22}-\Lambda_{12}{}^2. \tag{4.2}$$

The factor of k^2 in Eq. (4.1) produces the usual zero-mass graviton pole. In addition, the quantity in square brackets in Eq. (4.1) can produce as many as two massive poles. This double-pole structure is expected. We have added $J^P=2^+$ torsion fields, which should give rise to one massive pole. In addition, we have added order-R^2 terms containing more than two derivatives acting on $g_{\mu\nu}$. These additional derivatives should give rise to a second massive pole, since an order-R^2 higher-derivative theory is equivalent to a two-derivative theory containing one auxiliary massive field.[12]

The condition for no tachyons is that $\det\mathfrak{L}^{(2)}$ have no negative roots. That is, we must have

TABLE I. Constants occurring in each sector of the Lagrangian of Eq. (3.1). The first column gives the spin-parity of the sector. The second column specifies whether the sector links a B multiplet to another B multiplet or to a V (vierbein) multiplet. In the latter case, the 2 index in Eq. (3.1) always refers to the vierbein multiplet.

J^P	Links	a_{11}	a_{22}	f	Λ_{11}	Λ_{22}	Λ_{12}
0^+	B to V	$\mu_T{}^2\kappa$	-1	$k^2\kappa$	$-(\beta_1+\beta_3+\beta_4/2+\beta_6 +3\beta_7/2+3\beta_8)$	$2(6\beta_{1R}+2\beta_{2R})$	$(2)^{1/2}(3\beta_{1RB}+\beta_{2RB})$
1^+	B to B	$\mu_A{}^2\kappa$	$\mu_M{}^2\kappa$	1	$+(-\beta_1-2\beta_2/3 +2\beta_3/3+\beta_4 +2\beta_5/3-2\beta_6/3)$	$-\beta_1-4\beta_2/3+\beta_3/3-\beta_4/2 -2\beta_5/3-\beta_6/3$	$(2)^{1/2}(-2\beta_2-\beta_3-\beta_5+\beta_6)/3$
1^-	B to B	$\mu_M{}^2\kappa$	$\mu_T{}^2\kappa$	1	$-(\beta_1+4\beta_2/3 +\beta_4/2+2\beta_5/3 +2\beta_6/3)$	$-(\beta_1+2\beta_2/3+\beta_4/2 +\beta_5/3+\beta_6/3+3\beta_7/2)$	$(-4\beta_2/3-2\beta_5/3-2\beta_6/3 +\beta_9)/(2)^{1/2}$
2^+	B to V	$\mu_M{}^2\kappa$	$\frac{1}{2}$	$k^2\kappa$	$-(\beta_1+\beta_3+\beta_4/2+\beta_6)$	β_{2R}	$\beta_{2RB}/(2)^{3/2}$

$$\kappa m_\pm^2 \equiv \{-(a_{22}\Lambda_{11}+a_{11}\Lambda_{22}) \pm [(a_{22}\Lambda_{11}+a_{11}\Lambda_{22})^2 - 4a_{11}a_{22}\det\Lambda]^{1/2}\}/2\det\Lambda > 0 . \quad (4.3)$$

The condition for no ghosts is that each element of the diagonalized propagator must be of the form $Q/(-k^2+m^2)$ with $Q \geq 0$. In computing the signs of residues, one must keep in mind that the projection operators P_{ij} in Eq. (3.2) reduce to +1 or −1 at poles, according to whether the sector contains multiplets of even or odd intrinsic parity. At the $k^2=0$ pole it is easy to see that the quantity a_{22}/κ determines the residue, hence

$$a_{22}/\kappa = 1/2\kappa > 0 \quad (4.4)$$

must be satisfied in order to avoid a ghost. Of course we must also impose Eq. (4.4) in order that the potential in the Newtonian limit have the correct sign. At the massive poles, we do not have to diagonalize the matrix (3.2) in order to obtain the residues. We need only note that at a pole

$$\det(\text{residue matrix}) = \det\mathcal{L}^{(2)} = 0 , \quad (4.5)$$

where "residue matrix" is the matrix (3.2) with a factor of $(-1)/\det\mathcal{L}^{(2)}$ deleted. From Eq. (4.5), if the residue matrix were diagonalized, one of the eigenvalues would be zero. Since the residue matrix is only 2×2, the remaining nonzero eigenvalue must equal the trace of the residue matrix. Hence we can find the residue at the pole just by taking the trace of Eq. (3.2); we do not need to diagonalize. After taking into account some factors contributed by $\det\mathcal{L}^{(2)}$, we find, for the residue at $k^2 = m_\pm^2$,

$$\text{residue} = \pm \frac{[(\Lambda_{11}+\Lambda_{22}m_\pm^2\kappa)m_\pm^2\kappa + (a_{11}+a_{22}m_\pm^2\kappa)]}{[(a_{22}\Lambda_{11}+a_{11}\Lambda_{22})^2 - 4a_{11}a_{22}\det\Lambda]^{1/2} m_\pm^2\kappa} . \quad (4.6)$$

A negative residue would imply a ghost, hence

$$(\Lambda_{11}+\Lambda_{22}m_+^2\kappa)m_+^2\kappa + (a_{11}+a_{22}m_+^2\kappa) \geq 0 , \quad (4.7a)$$

$$(\Lambda_{11}+\Lambda_{22}m_-^2\kappa)m_-^2\kappa + (a_{11}+a_{22}m_-^2\kappa) \leq 0 . \quad (4.7b)$$

In the Introduction, we stated that the higher derivatives contribute a ghost which is not removed by coupling in a 2^+ torsion multiplet. We now prove this statement, by showing that no choice of parameters will satisfy the constraints (4.3), (4.4), (4.7a), and (4.7b). Since the proof involves straightforward algebra, we give only an outline of the proof, to demonstrate that we have considered all possible choices for the magnitudes and signs of the parameters.

(1) Assume $\det\Lambda(2^+) \neq 0$. (If $\det\Lambda = 0$, the high-energy falloff in the 2^+ sector is not rapid enough to produce renormalizability.)

(2) Assume $a_{22}(2^+) > 0$. [See Eq. (4.4).]

(3) Either (a) $a_{11}(2^+) < 0$; or (b) $a_{11}(2^+) > 0$; or (c) $a_{11}(2^+) = 0$. Consider each alternative in turn.

(3a) If $a_{11}(2^+) < 0$, then $\mu_M^2 < 0$, and the 2^- propagator, Eq. (3.4), will have a ghost or tachyon pole unless $(\beta_1 + \frac{1}{2}\beta_4) = 0$. If $(\beta_1 + \frac{1}{2}\beta_4) = 0$, 2^- exchange will be nonrenormalizable. Abandon alternative (3a) and try alternative (3b).

(3b) If $a_{11}(2^+) > 0$, then either $m_+^2 = m_-^2$ or $m_+^2 \neq m_-^2$. If $m_+^2 = m_-^2$, one shows easily $\Lambda_{12}(2^+) = 0$; graviton and torsion multiplets then decouple, and the decoupled theory is known to contain ghosts.[4] If $m_+^2 \neq m_-^2$, the mass constraints require $\Lambda_{22} < 0$, while the residue constraints require $\Lambda_{22} > 0$. Abandon alternative (3b) and try alternative (3c).

(3c) If $a_{11}(2^+) = 0$, one of the $m_\pm^2$ vanishes, suggesting that there is now a torsion pole at $k^2 = 0$, in addition to the usual graviton pole. There are now two residue constraints at $k^2 = 0$, rather than the one constraint Eq. (4.4). The propagator, Eq. (3.2), becomes $(-1/k^2)$ times a 2×2 matrix of residues, and there must be two constraints to ensure that both eigenvalues of this matrix are ≥ 0. It is easy to convince oneself that a 2×2 symmetric matrix will have eigenvalues ≥ 0 only if both diagonal elements are ≥ 0. Thus constraint (4.4), $a_{22}/\kappa > 0$, continues to hold, because a_{22}/κ determines the sign of one of the diagonal elements. (The case $a_{22}/\kappa = 0$ is excluded because the propagator would have a $1/k^4$ pole.) We shall not need any other $k^2 = 0$ constraint. If we now examine the massive pole, we find it is necessarily a ghost or tachyon. We have now exhausted all three alternatives (3a)–(3c), and the theorem is proved: Adding a 2^+ torsion multiplet does not remove the higher-derivative ghost.

We could consider dropping the higher derivatives ($\Lambda_{22} = \Lambda_{12} = 0$). The theory would be nonrenormalizable but perhaps acceptable classically. However, there are a number of theorems in the literature which state, essentially, that it is impossible to construct a ghost- and tachyon-free theory involving spin $\geq \frac{3}{2}$, if the Lagrangian contains ≤ 2 derivatives and the spinning particle is massive.[13,14,15] If the particle is massless, one can construct a satisfactory linearized theory, but the extension to a fully covariant theory does not always go through.[16] Because of these theoretical difficulties, we will assume in the next section that all exchanged torsion multiplets are spin ≤ 1.

V. PHENOMENOLOGY OF TORSION EXCHANGE

Gravity has never been quantized, but it is satisfactory as a classical theory. In the present section we shall accept a torsion theory if it is satisfactory in the classical limit, even though there may be difficulties when the theory is quantized. Thus we exclude theories with spin-two propagating torsion, because the difficulties (with ghosts, tachyons, and covariance) mentioned in the previous section all affect the classical theory. Conversely, we do not exclude theories with spin ≤ 1 propagating torsion even though their quantum theory may be unsatisfactory. [Possible difficulties with quantization arise as soon as the torsion multiplet is coupled to matter. It is natural to couple torsion to the fermion axial-vector current, since one wants the B field coupled to spin, but this current may develop triangle anomalies which spoil renormalization.[17] Also, in those theories where one replaces B by $\partial\phi$, as at Eq. (1.2), the matter-to-ϕ coupling becomes a derivative coupling which is probably nonrenormalizable.]

We assume that the interaction Lagrangian $\mathcal{L}_I$ containing the torsion-to-matter coupling has the following form (all tensor indices are suppressed):

$$\mathcal{L}_I = g\bar{\psi}O\psi B\,. \tag{5.1}$$

That is, (i) the interaction contains a coupling which has the dimensions of a charge ($g^2/4\pi\hbar c$ is dimensionless); (ii) the interaction couples each fermion field ψ to the contortion field B [defined at Eq. (1.1)] via an operator O which is probably $\gamma_\mu\gamma_5$; however our order-of-magnitude estimates below will not be affected if O is γ_μ. Note that if the reducible tensor $B^\lambda{}_{\mu\nu}$ is split into irreducible components and the part containing spin $2^\pm$ is discarded, the remaining two irreducible tensors form a polar four-vector and an axial four-vector.[18] (iii) $\mathcal{L}_I$ contains no derivatives. In potential models [$B\to\partial\phi$; see Eq. (1.2)] (iii) is replaced by (iii)′: $\mathcal{L}_I$ contains one derivative and ϕ is a massless particle. To be consistent with the minimal-substitution philosophy of these models, we shall also assume that ϕ couples with strength $g=1$.

In addition to a coupling, we also need a propagator. If B is massless, with kinetic term $\Lambda(\partial B)^2$ in the Lagrangian, then the propagator will be $1/\Lambda k^2$ in k space or $1/4\pi r\Lambda$ in configuration space. Since the force will be long ranged, initially we consider a macroscopic source where we can benefit from the cumulative effect of as large a number of (aligned) spins as possible, i.e., a ferromagnet. The potential between two spins in two such sources would then be Coulomb-type:

$$V_{12}(B) \sim g^2\langle\sigma_1\rangle\,\frac{1}{4\pi\Lambda r}\,\langle\sigma_2\rangle\,. \tag{5.2}$$

(We have taken $O=\gamma_\mu\gamma_5$; for $O=\gamma_\mu$ replace $\langle\sigma_i\rangle$ by $\langle 1\rangle$.) Assuming a Λ of order unity and a ($g^2/4\pi\hbar c$) of order 10^{-2}, we find this force would be huge, far larger than the magnetic force between the same two spins:

$$V_{12}(\gamma) \sim e^2\langle\sigma_1\rangle\,\frac{1}{4\pi r^3}\,\langle\sigma_2\rangle\left(\frac{\hbar}{mc}\right)^2, \tag{5.3}$$

where the last factor comes from the Bohr magnetons.

If we perform the same analysis for a potential model, $B\to\partial\phi$, the kinetic terms contain a factor $1/\kappa$, since they originate in the R term in the gravity Lagrangian, which is multiplied by $1/\kappa$. Hence the ϕ propagator is gravitylike, equal to $\kappa/4\pi r$ in configuration space. We anticipate that the factor of $1/r$ will be turned into $1/r^3$ after it is acted upon by the two derivatives of the ϕ field, one from each vertex. Compare the magnetic dipole interaction Eq. (5.3), which also has one derivative at each vertex. [The vertex responsible for interaction (5.3) is the $\bar{\psi}\sigma_{\mu\nu}\psi F_{\mu\nu}$ part of the Gordon decomposition of the electromagnetic current.] We find

$$V_{12}(\phi) \sim \kappa m^2\langle\sigma_1\rangle\,\frac{1}{4\pi r^3}\,\langle\sigma_2\rangle\left(\frac{\hbar}{mc}\right)^2. \tag{5.4}$$

We have multiplied and divided through by factors of fermion mass, to aid comparison with Eq. (5.3). Evidently the $V_{12}(\phi)$ potential is too weak to be readily observable, in contrast to the potential $V_{12}(B)$, which is too strong.

Can we make the too strong potential (5.2) weaker by giving the exchanged multiplet a mass? Torsion forces would then become weak for the same reason that intermediate-vector-boson forces are weak in β decay: The exchange is short range. There appears to be no satisfactory way to give the B multiplet a mass. We cannot invoke spontaneous symmetry breaking and a Higgs-Kibble mechanism, because there is no internal symmetry to be broken.[19] We cannot put in a mass "by hand" without creating a ghost. [For example, suppose we take the mass parameter $\mu_A{}^2\neq 0$ in Lagrangian (2.4); this is a natural choice because it makes the axial-vector part of the B multiplet massive; and the axial-vector part couples to matter via $O=\gamma_\mu\gamma_5$ in Eq. (5.1). The propagator for the fourth component of this axial vector is given by Eq. (3.3); from that equation we must take $\mu_A{}^2>0$ to avoid a 0^- ghost. As for the space components of the four-vector, they are propagated by the 1^+ sector. When $\mu_A{}^2>0$, the parameter a_{11} in this sector must be positive, from Table I. Furthermore, we must take $a_{22}=0$ in the 1^+ sector, since we do not want any spin-two torsion multiplets; from Eq. (3.4) a_{22} is essentially the mass of the 2^- multiplet. It is now straightforward to apply the methods of the preceding section and show that with $a_{11}>0, a_{22}=0$, the 1^+ sector has at least one ghost or tachyon.] We conclude that giving the B multiplet a mass is out and that g^2 in Eq. (5.2) must be very small if it is nonzero. Using $r=1m$ and an electron mass in Eqs. (5.2) and (5.3) we find g^2/e^2 must be less than $(\hbar/mc)^2 \sim 10^{-25}$. This limit from macroscopic experiments with ferromagnets is so stringent that it may rule out the possibility of detecting a torsion interaction of the type (5.2) via precision microphysics experiments with elementary particles.

Next we turn to the potential model interaction, Eq. (5.4). Since this interaction is too weak, we ask whether there are ways to make the effective coupling $\kappa(m)^2$ larger rather than smaller. Kaempffer notes that κ as conventionally defined includes a proportionality constant, the ratio of active to passive gravitational mass; if there is a different proportionality constant for the ratio of active to passive intrinsic spin, then κ must be replaced by a different constant κ' in Eq. (5.4), allowing a stronger coupling that would be easier to see.[10] The phenomenology of ϕ exchange merits further study, and we intend to return to it in a future publication.

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[1]For references to the early literature on torsion, see F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, Rev. Mod. Phys. 48, 393 (1976). Strictly speaking, torsion equals $\Gamma^\lambda{}_{[\mu\nu]}$ (and torsion vanishes in standard gravity) only if we work in "holonomic" coordinates. In standard gravity, torsion may exist but is always a coordinate artifact rather than an independent field.

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[10]F. A. Kaempffer [Ref. 8 and Gen. Relativ. Gravit. 7, 327 (1976)] proposes a less rigid potential model in which the coefficient of the $(\partial\phi)^2$ term is not completely determined by the minimal-substitution procedure. In such a theory, ghosts would not be a worry, of course.

[11]Our other conventions are metric signature $=-2$, semicolons denote covariant derivatives with respect to the Christoffel affinity, not the $\Gamma^\lambda{}_{\mu\nu}$ affinity of Eq. (2.1). Note this last convention plus Eq. (2.2a) are a change from the notations used in Ref. 6.

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[18]The irreducible tensor containing spin $2^\pm$ also contains two additional multiplets having spins $1^\pm$. It may be possible to design a covariant gauge invariance which will remove the dangerous $2^\pm$ parts of this multiplet, while preserving the $1^\pm$ parts. If so, then the propagators in the $1^\pm$ sector would be 2×2 matrices, and the spin-spin potentials could contain both $1/r$ and $e^{-\lambda r}/r$ terms. For phenomenological purposes such a potential should be replaced by its longest-ranged component.

[19]We are perhaps being overly conservative here, since of course we could insert a local U(1) coupling, say $|(\partial_\mu - igB^\nu{}_{\mu\nu})H|^2$, $H=$ Higgs scalar, then break the U(1) symmetry. Here we have moved far from the original idea of Cartan, since we are not coupling the torsion to the intrinsic spin of matter. It has been pointed out that such a coupling would be more natural in a theory with a local scale invariance. In such a theory, the Christoffel symbols must be modified; they acquire a torsion. (Metric compatibility is also affected.) If the torsion arose from local scale invariance rather than spin effects, a coupling to scalars would not seem so forced. See for example, Lee Smolin, Nucl. Phys. (to be published).