Superconvergence sum rules and charmed-baryon couplings

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Superconvergence sum rules have been obtained for the invariant amplitudes of the elastic as well as inelastic and exotic as well as nonexotic scattering processes involving pions and charmed baryons. The assumption of saturating these sum rules by the known low-lying states is then exploited to determine the pion–charmed-baryon coupling strengths.

I. INTRODUCTION

Recent experimental discoveries of charmed baryons[1] in electron-positron annihilation and in neutrino interactions as well as in photoproduction experiments tempt us to study the hadronic parameters of these states. These states have so far been detected with charm + 1. The lowest-mass state is an isospin-singlet state C_0^+ (with $J^P=\frac{1}{2}^+$ and mass 2.26 GeV). Next states are isospin-triplet states C_1 (with $J^P=\frac{1}{2}^+$ and mass as 2.43 GeV) and C_1^* (with $J^P=\frac{3}{2}^+$ and mass 2.48 GeV), respectively.[2] The latter states are resonant states and the experiments indicate their decay into πC_0^+ states. However, the experimental data are still incomplete and preliminary. It is, therefore, desirable to predict theoretically the hadronic parameters of these states so that in the future when the experimental data become available on the hadronic production of these states, they can be experimentally verified. Recently we have used[3] Adler's consistency conditions for the elastic as well as inelastic pion–charmed-baryon scatterings to evaluate the charmed-baryon couplings. The purpose of this paper is to obtain and use superconvergence sum rules for various meson–charmed-baryon processes for determining charmed-baryon couplings with the pion. This investigation will thus lend an alternative theoretical test for our previous findings.

Much literature[4-5] exists on the practical utilization of superconvergence sum rules based on the saturation with the low-lying states in the narrow-width approximation. This approximation becomes indispensable because many of the processes for which superconvergence relations are written are not accessible to direct experiments at the present time. In the past, such an attitude has often resulted in useful algebraic relations among the masses and coupling constants. Such relations are linear in the squares of the coupling constants for elastic scattering and/or in products of coupling constants for inelastic processes. In this paper, we obtain superconvergence sum rules for the crossing-antisymmetric invariant amplitudes of the following processes:

$$\pi^+ + C_1^0 \to \pi^- + C_1^{++}, \tag{1}$$

$$\pi^+ + C_1^0 \to \pi^- + C_1^{*++}, \tag{2}$$

$$\pi + C_0 \to \pi + C_0, \tag{3}$$

$$\pi + C_0 \to \pi + C_1^*. \tag{4}$$

Processes (1) and (2) are exotic exchange reactions but (3) and (4) are nonexotic processes. These sum rules when saturated with low-lying known charmed states yield algebraic relations which can be solved to determine the coupling constants $g_{\pi C_0 C_1}$, $g_{\pi C_0 C_1^*}$, $g_{\pi C_1 C_1}$, $g_{\pi C_1 C_1^*}$, and $g_{\pi C_1^* C_1^*}$.

II. SUM RULES

If a certain crossing-odd invariant amplitude $F(\nu)$ satisfies an unsubtracted dispersion relation at $t=0$ and behaves asymptotically for large ν as $\nu^{-1-\epsilon}$, with $\epsilon>0$, it must satisfy the superconvergence relation,[5]

$$\int_{-\infty}^{\infty} \mathrm{Im}F(\nu)d\nu = 0\,, \tag{5}$$

where $\nu=(s-u)/4m$ and s, t, and u are the Mandelstam variables. Such sum rules were obtained by several workers[6-7] for exotic exchange processes by assuming that the possible t-channel Regge trajectories with $I=2$ for such processes have negative intercept at zero-momentum transfer (as is suggested by the absence of low-lying meson states with such quantum numbers). So for the meson baryon elastic process (1) we can write the relation as follows[8]:

$$\int_{-\infty}^{\infty} \mathrm{Im}B^{(2)}(\nu, t=0)d\nu = 0\,, \tag{6}$$

where B is the usual invariant amplitude. Here the superscript 2 denotes the value of the t-channel isospin.

A large number of sum rules can be constructed for processes involving particles with higher spin, for example, processes (2) and (4). The analysis of sum rules for such processes is rather difficult in certain cases due to the existence of a large number of independent amplitudes. For process (2), if q_1 (p_1) and q_2 (p_2) are the four-momenta of initial and final pion (baryon), respectively, we can write the four invariant amplitudes as[9]

$$\bar{C}^*_{1\mu}(p_2)M_\mu C_1(p_1) = \bar{C}^*_{1\mu}(p_2)[(a_1P_\mu + a_2Q_\mu) + i\gamma\cdot Q(b_1P_\mu + b_2Q_\mu)]\gamma_5 C_1(p_1)\,, \tag{7}$$

where $P=(p_1+p_2)/2$, $Q=(q_1+q_2)/2$, $C^*_{1\mu}$, is the Rarita-Schwinger wave function for the spin-$\frac{3}{2}$ particle C_1^*, and C_1 is the Dirac spinor for the particle C_1. Then from the high-energy behavior of these amplitudes we can safely write the following superconvergence sum rules[9]:

$$\int_{-\infty}^{\infty} \mathrm{Im}a_2^{(2)}(\nu,0)d\nu = 0\,, \tag{8}$$

$$\int_{-\infty}^{\infty} \mathrm{Im}b_1^{(2)}(\nu,0)d\nu = 0\,. \tag{9}$$

Similarly we get for the process (4) the following superconvergence relation:

$$\int_{-\infty}^{\infty} \mathrm{Im}b_2^{(1)}(\nu,0)d\nu = 0\,, \tag{10}$$

where $\nu=(s-u)/2(m_C+m_{C^*})$. Several other nontrivial higher-moment superconvergence relations can be derived. However, they are less reliable than the zero-moment sum rules which we have written above. This is because in the simple saturation scheme, higher-moment sum rules are more sensitive to a number of resonances lying in the higher-energy region. In our cases, since the higher-energy region lies still unexplored for the new charmed-baryon resonances with charm+1, we hope the above relations (6) and (8)–(10) are more reliable in the saturation with low-lying states.

For the process (3), we write finite-energy sum rules since no scattering amplitude satisfies the required asymptotic bound in this case and hence the condition for superconvergence is not fulfilled. In order to get such sum rules, we truncate the dispersion integral at some energy ω, so that the integrand $\mathrm{Im}B(\nu)$ in (5) may be replaced by its asymptotic form when $\nu \geq \omega$ and we get[10]

$$\int_0^\omega \mathrm{Im}B(\nu,t)d\nu = \sum_i \frac{\beta_i(t)\omega^{\alpha_i+1}}{\alpha_i+1}\,, \tag{11}$$

where $\beta_i(t)$ is the residue function and $\alpha_i(t)$ is the Regge trajectory. Among the Regge poles for the process (3), only f and ϵ can be considered. However, Renner and Zerwas[11] have found that their contributions are very small for $\pi\Lambda\to\pi\Lambda$ scattering. Assuming that these Regge poles give negligibly small contributions for the process (3) also, we get the sum rule as follows:

$$\int_0^\omega \mathrm{Im}B^{(0)}(\nu,0)d\nu = 0\,. \tag{12}$$

Equation (12) is only an approximate relation. However, in the absence of experimental data, we find it useful for getting a relation between the decay widths of C_1^* and C_1 into the $C_0\pi$ channel. We would then be able to compare this relation with the relations found elsewhere.[3,12]

III. CALCULATION AND DISCUSSION

We saturate the sum rules (6)–(10) by the known charmed-baryon pole-term contributions. The effective Lagrangians used are given as[9]

$$\begin{aligned}
\mathcal{L}_{\pi C_1C_1} &= g_{\pi C_1C_1}\bar{C}_1\gamma_5 C_1\pi\,,\\
\mathcal{L}_{\pi C_1C_1^*} &= \frac{g_{\pi C_1C_1^*}}{m_\pi}\bar{C}^*_{1\mu}C_1\partial_\mu\pi\,,\\
\mathcal{L}_{\pi C_1^*C_1^*} &= g_{\pi C_1^*C_1^*}\bar{C}^*_{1\mu}\gamma_5C^*_{1\mu} + \frac{g'_{\pi C_1^*C_1^*}}{m_\pi^2}\bar{C}^*_{1\mu}\gamma_5C^*_{1\nu}\partial_\mu\partial_\nu\pi\,.
\end{aligned} \tag{13}$$

The pertinent isospin crossing relations are

$$A_t^{(I=2)} = \tfrac{1}{3}A_s^{(0)} - \tfrac{1}{2}A_s^{(1)} + \tfrac{1}{6}A_s^{(2)}\,, \tag{14}$$

for exotic exchange reactions and

$$A_t^{(I=1)} = -A_s^{(1)} \tag{15}$$

for process (3). We thus get for (6), (8), (9), and (10) the following relations between the coupling constants, respectively:

$$\frac{1}{3}g_{\pi C_0C_1}{}^2 - \frac{1}{2}g_{\pi C_1C_1}{}^2 - \frac{1}{2}\frac{g_{\pi C_1C_1^*}^{2}}{m_\pi^2}\left[\frac{4}{3}m_{C_1}{}^2 + \frac{m_{C_1}}{3m_{C_1^*}}(m_{C_1}{}^2+m_{C_1^*}^2-m_\pi^2) - \frac{1}{6m_{C_1^*}^2}(m_{C_1}{}^2+m_{C_1^*}^2-m_\pi^2)^2\right]=0\,, \tag{16}$$

$$\begin{aligned}
&\frac{1}{3}\frac{g_{\pi C_0C_1^*}g_{\pi C_0C_1}}{m_\pi}\left(m_{C_0}+\frac{m_{C_1^*}-m_{C_1}}{2}\right) - \frac{1}{2}\frac{g_{\pi C_1C_1^*}g_{\pi C_1C_1}}{m}\left(\frac{m_{C_1}+m_{C_1^*}}{2}\right)\\
&\quad+\frac{1}{2}\frac{g_{\pi C_1^*C_1^*}g_{\pi C_1C_1^*}}{m_\pi}\frac{1}{6m_{C_1^*}^2}[3m_{C_1^*}^3+4m_{C_1^*}^2m_{C_1}+2m_{C_1^*}m_{C_1}{}^2+m_{C_1}{}^3-m_\pi^2(3m_{C_1^*}+m_{C_1})]\\
&\qquad+\frac{1}{2}\frac{g'_{\pi C_1^*C_1^*}g_{\pi C_1C_1^*}}{m_\pi^3}\frac{m_\pi^2}{12m_{C_1^*}^2}[6m_{C_1^*}^3+7m_{C_1^*}^2m_{C_1}+2m_{C_1^*}m_{C_1}{}^2+m_{C_1}{}^3-m_\pi^2(3m_{C_1^*}+m_{C_1})]=0\,,
\end{aligned} \tag{17}$$

$$\frac{1}{3}\frac{g_{\pi C_0 C_1^*} g_{\pi C_0 C_1}}{m_\pi} - \frac{1}{2}\frac{g_{\pi C_1 C_1^*} g_{\pi C_1 C_1}}{m_\pi} + \frac{1}{2}\frac{g_{\pi C_1^* C_1^*} g_{\pi C_1 C_1^*}}{m_\pi}\frac{1}{3m_{C_1^*}^2}(5m_{C_1^*}^2 + m_{C_1^*}m_{C_1} - m_{C_1}^2 + m_\pi^2) - \frac{1}{2}\frac{g'_{\pi C_1^* C_1^*} g_{\pi C_1 C_1^*}}{m_\pi^3}\frac{m_\pi^2}{6m_{C_1^*}^2}(4m_{C_1^*}^2 - m_{C_1^*}m_{C_1} + m_{C_1}^2 - m_\pi^2) = 0\,, \tag{18}$$

$$\frac{g_{\pi C_1 C_1^*} g_{\pi C_0 C_1^*}}{m_\pi} + \frac{g_{\pi C_0 C_1^*} g_{\pi C_1^* C_1^*}}{m_\pi}\frac{1}{3m_{C_1^*}^2}(m_{C_1^*}^2 - m_{C_1^*}m_{C_1} + m_{C_1}^2 - m_\pi^2) + \frac{g'_{\pi C_1^* C_1^*} g_{C_0 C_1^*}}{m_\pi^3}\frac{m_\pi^2}{6m_{C_1^*}^2}(4m_{C_1^*}^2 - m_{C_1^*}m_{C_1} + m_{C_1}^2 - m_\pi^2) = 0\,. \tag{19}$$

Similarly saturating the sum rule (12) with the known charmed-baryon resonances C_1 and C_1^* in the narrow-width approximation we get the following relation between the decay widths:

$$\Gamma_{C_1^*} - 5.35\Gamma_{C_1} = 0\,, \tag{20}$$

where Γ_{C_1} ($\Gamma_{C_1^*}$) is the decay width of C_1 (C_1^*) into πC_0^+ states.

We find that Eq. (20) compares favorably with our previous result[3] using the Adler consistency condition and also with the result obtained by Lee, Quigg, and Rosner.[12] Using $\Gamma_{C_1^*} = 20$ MeV,[3,12] we get $\Gamma_{C_1} = 3.7$ MeV. We can, therefore, evaluate the following two coupling constants:

$$\frac{g_{\pi C_0 C_1^*}^2}{4\pi} = 0.15 \tag{21}$$

and

$$\frac{g_{\pi C_0 C_1}^2}{4\pi} = 51.56\,. \tag{22}$$

In our previous paper,[3] we obtained $g_{\pi C_0 C_1}^2/4\pi = 58.3$. Now using Eqs. (21) and (22) we solve the relations (16)–(19) and get the following set of coupling constants:

$$\frac{g_{\pi C_1 C_1}^2}{4\pi} = 24.56\,, \tag{23}$$

$$\frac{g_{\pi C_1 C_1^*}^2}{4\pi} = 0.024\,, \tag{24}$$

$$\frac{g_{\pi C_1^* C_1^*}^2}{4\pi} = 16.37\,, \tag{25}$$

$$\frac{g'^2_{\pi C_1^* C_1^*}}{4\pi} = 6.33\,. \tag{26}$$

In our previous paper,[3] the reported values were $g_{\pi C_1 C_1}^2/4\pi = 28.42$, $g_{\pi C_1 C_1^*}^2/4\pi = 0.034$, and $g_{\pi C_1^* C_1^*}^2/4\pi = 1.43$. We thus find that the values of all the hadronic coupling constants reported here agree well with our previous values except for $g_{\pi C_1^* C_1^*}$. The reason for this difference in $g_{\pi C_1^* C_1^*}$ may possibly be that we neglected the $g'_{\pi C_1^* C_1^*}$ coupling in our previous calculation.

In conclusion, the superconvergence and finite-energy sum rules form a powerful tool for getting the algebraic relations among masses and coupling constants on being saturated with poles and resonances in the narrow-width approximation. Such relations treat stable particles and resonant states on an equal footing. However, great care is required in selecting the set of superconvergence relations to be used in a simple saturation scheme. For example, we find that the inclusion of higher-moment sum rules together with the relations reported above modifies the values of the couplings in an arbitrarily large way. Moreover, the saturation scheme becomes questionable when we have only two or three known low-lying states. However, no consistent prescription has been followed in deciding the number of low-lying states required to saturate the sum rules.[13] In principle, one should take an infinite number of poles and resonances to saturate the sum rules in order to avoid the difficulties regarding the violation of locality when nontrivial currents in a current commutator are involved.[14] In our case, the lack of experimental information on the existence of other new states in the higher-energy region and the agreement achieved in the values of the couplings evaluated in two different theoretical schemes prompt us to conclude that C_0, C_1, and C_1^* are the only low-lying charmed states with the quark combination *cud*. Finally we hope that these values of the coupling constants will be of much help in studying the strong-interaction features of these new charmed states from both theoretical and experimental points of view.

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