Current anticommutators on the null plane and their applications

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With use of the Deser-Gilbert-Sudarshan representation, the connected diagonal matrix elements of current anticommutators taken between two stable one-particle states on the null plane are derived. It is shown that the assumption necessary to restrict to the null plane in this case is equivalent to that in the case of current commutators. Following Dicus, Jackiw, and Teplitz, we derive sum rules from these anticommutators. We investigate some of them, and obtain, in the case of pion-nucleon or kaon-nucleon scattering, the quantitative relation between the sea-quark distribution in the nucleon and the Pomeron.

I. INTRODUCTION

The Deser-Gilbert-Sudarshan (DGS) representation,[1] which incorporates both causality and spectrum conditions, has been of great value in the investigation of one-particle matrix elements of current commutators.[2,3] Recently the author has postulated the validity of current anticommutators restricted to the null plane with a very brief discussion of its theoretical origin.[4,5] The purpose of this paper is to provide a full discussion with the use of the DGS representation and the current commutators on the null plane derived formally through the neutral-vector-gluon model.[6] Since our concern here is the weak and electromagnetic currents, the results obtained hold true formally in the SU(3) color gauge theory (quantum chromodynamics).[7] In Sec. II the main ideas are illustrated with scalar currents to avoid the kinematical complexity due to vector currents and to obtain a good insight into the issue. A case of conserved vector currents is discussed in Sec. III. In Sec. IV the method is applied to pion-nucleon[5] or kaon-nucleon scattering with use of the partially conserved axial-vector current hypothesis (PCAC).[8] These results are summarized in Sec. V.

II. SCALAR-CURRENT MODEL

First we consider the DGS representation[1,2] of the current commutator taken between two scalar one-particle states of momentum p,

$$\langle p|[J_a(x),J_b(0)]|p\rangle_c = \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta\, h^{ab}(\lambda^2,\beta)\exp(i\beta p\cdot x) \times i\Delta(x,\lambda^2)\,, \tag{2.1}$$

where c means to take the connected part, the scalar current $J_a(x)$ is defined as

$$J_a(x)=\varphi^\dagger(x)\tau_a\varphi(x)\,, \tag{2.2}$$

and $i\Delta(x,\lambda^2)$ is defined as

$$i\Delta(x,\lambda^2)=\frac{1}{(2\pi)^3}\int d^4k\,\exp(-ik\cdot x)\epsilon(k^0)\delta(k^2-\lambda^2)\,. \tag{2.3}$$

We define the null-plane commutator of the complex scalar field in the usual way as[9,10]

$$[\varphi^\dagger(x),\varphi(0)]\big|_{x^+=0}=-\tfrac{1}{4}i\epsilon(x^-)\delta(\vec{x}^\perp)\,. \tag{2.4}$$

Then we obtain

$$[J_a(x),J_b(0)]\big|_{x^+=0} = -\tfrac{1}{4}i\delta(\vec{x}^\perp)\epsilon(x^-)[\delta_{ab}S_0(x|0)+i\epsilon_{abc}A_c(x|0)]\,, \tag{2.5}$$

where

$$S_0(x|0)=\varphi^\dagger(x)\varphi(0)+\varphi^\dagger(0)\varphi(x)\,,$$
$$A_a(x|0)=\varphi^\dagger(x)\tau_a\varphi(0)-\varphi^\dagger(0)\tau_a\varphi(x)\,. \tag{2.6}$$

Under a suitable assumption, the right-hand side of Eq. (2.1) on the null plane becomes

$$-\tfrac{1}{4}i\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\,\exp(i\beta p\cdot x)h^{ab}(\lambda^2,\beta)\delta(\vec{x}^\perp)\epsilon(x^-)\,. \tag{2.7}$$

Since the left-hand side of Eq. (2.1) is given by Eq. (2.5), we obtain

$$\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\,\exp(i\beta p^+x^-)\,h^{ab}(\lambda^2,\beta) = \langle p|\delta_{ab}S_0(x|0)+i\epsilon_{abc}A_c(x|0)|p\rangle_c\big|_{x^+=\vec{x}^\perp=0}\,. \tag{2.8}$$

Following Cornwall,[3] we consider next the generalization of the DGS representation in the case of a stable one-particle matrix element given by

$$\langle p|\{J_a(x),J_b(0)\}|p\rangle_c = \int_0^\infty d\lambda^2\int_{-1}^1 d\beta\,\exp(i\beta p\cdot x)h^{ab}(\lambda^2,\beta)\Delta^{(1)}(x,\lambda^2)\,, \tag{2.9}$$

where $\Delta^{(1)}(x,\lambda^2)$ is defined as

$$\Delta^{(1)}(x,\lambda^2)=\frac{1}{(2\pi)^3}\int d^4k\,\exp(-ik\cdot x)\delta(k^2-\lambda^2). \tag{2.10}$$

Then, again under a suitable assumption, we obtain the null-plane restriction of Eq. (2.9) as

$$-\frac{1}{2\pi}\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\,\exp(i\beta p\cdot x)h^{ab}(\lambda^2,\beta)\delta(\vec{x}^\perp)\ln|x^-|, \tag{2.11}$$

where[11]

$$\ln|x^-|=-\tfrac{1}{2}\int_{-\infty}^\infty \frac{da}{|a|}\exp(-iax^-). \tag{2.12}$$

Therefore, with the use of Eq. (2.8), we obtain

$$\langle p|\{J_a(x),J_b(0)\}|p\rangle_c|_{x^+=0}=-\frac{1}{2\pi}\delta(\vec{x}^\perp)\ln|x^-|\langle p|[\delta_{ab}S_0(x|0)+i\epsilon_{abc}A_c(x|0)]|p\rangle_c|_{x^+=0}. \tag{2.13}$$

Equation (2.13) will be obtained if we use Wick's decomposition theorem on the null plane, $x^+=0$, as

$$\begin{aligned}\langle p|[\varphi^\dagger(x)\tau_a\varphi(x)\varphi^\dagger(0)\tau_b\varphi(0)+\varphi^\dagger(0)\tau_b\varphi(0)\varphi^\dagger(x)\tau_a\varphi(x)]|p\rangle_c&=-\frac{1}{2\pi}\delta(\vec{x}^\perp)\ln|x^-|\langle p|[\delta_{ab}S_0(x|0)+i\epsilon_{abc}A_c(x|0)]|p\rangle_c\\&\quad+\langle p|[:\varphi^\dagger(x)\tau_a\varphi(x)\varphi^\dagger(0)\tau_b\varphi(0):\\&\quad+:\varphi^\dagger(0)\tau_b\varphi(0)\varphi^\dagger(x)\tau_a\varphi(x):]|p\rangle_c.\end{aligned} \tag{2.14}$$

Since at $x^+=0$, $x^2=2x^+x^- -\vec{x}^{\perp 2}=-\vec{x}^{\perp 2}\le 0$, the second term on the right-hand side of Eq. (2.14) does not contribute to the connected matrix element and we obtain Eq. (2.13). However, the second term does contribute to the connected part in the timelike region, and it is not clear what kind of an assumption is necessary to restrict to the null plane. The usefulness of the derivation with use of the DGS representation lies in this point, together with the treatment of the Schwinger terms. Now we investigate the assumption necessary to restrict Eqs. (2.1) or (2.9) to the null plane. First we define the Fourier transform of Eqs. (2.1) or (2.9) as

$$H_{ab}=\frac{1}{(2\pi)^3}\int d^4x\,\exp(iq\cdot x)\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\,\exp(i\beta p\cdot x)h^{ab}(\lambda^2,\beta)\int d^4k\,\exp(-ik\cdot x)\delta(k^2-\lambda^2)f(k^+), \tag{2.15}$$

where $f(k^+)=\epsilon(k^+)$ in the case of Eq. (2.1) and 1 in the case of Eq. (2.9). Since our concern here is not the mathematical problem of defining restrictions of a commutator to the null plane,[12] we interpret $\delta(x^+)$ in the following sense[13]:

$$I_{ab}=\lim_{\Lambda\to\infty}\int_{-\infty}^\infty dq^-\exp\left(-\frac{(q^-)^2}{\Lambda^2}\right)H_{ab}. \tag{2.16}$$

Then we obtain

$$I_{ab}=2\pi\lim_{\Lambda\to\infty}\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\,\exp\left[-\frac{1}{4(q^++\beta p^+)^2\Lambda^2}[\lambda^2-2\beta p^-(q^++\beta p^+)+(\vec{q}^\perp+\beta\vec{p}^\perp)^2]^2\right]h^{ab}(\lambda^2,\beta)\frac{f(q^++\beta p^+)}{2|q^++\beta p^+|}, \tag{2.17}$$

except at $q^++\beta p^+=0$.[14] From Eq. (2.17) we see that the part which prevents the interchange between the $\Lambda\to\infty$ limit and the integration over λ^2 is concerned with the behavior of $h^{ab}(\lambda^2,\beta)$ in the region $\lambda^2\ge O(\Lambda)$. Thus we divide the λ^2 integration as

$$\int_0^{\alpha\sqrt{\Lambda}} d\lambda^2\cdots+\int_{\alpha\sqrt{\Lambda}}^\infty d\lambda^2\cdots, \tag{2.18}$$

where α is some positive constant. The second term of Eq. (2.18) is rewritten as

$$2\pi\lim_{\Lambda\to\infty}\sqrt{\Lambda}\int_\alpha^\infty d\lambda'^2\int_{-1}^1 d\beta\,\exp\left[-\frac{\lambda'^2}{4(q^++\beta p^+)^2\Lambda}\left(1+\frac{(\vec{q}^\perp+\beta\vec{p}^\perp)^2-2\beta p^-(q^++\beta p^+)}{\sqrt{\Lambda}\,\lambda'^2}\right)^2\right]h^{ab}(\sqrt{\Lambda}\,\lambda'^2,\beta)\frac{f(q^++\beta p^+)}{2|q^++\beta p^+|}, \tag{2.19}$$

where $\lambda'^2=\lambda^2/\sqrt{\Lambda}$. Then, if the $h^{ab}(\lambda^2,\beta)$ is bounded and, moreover, decreasing as $\lambda^{-2-\epsilon}$ at large λ^2, where ϵ is some positive constant, Eq. (2.19) becomes zero. Further, since $h^{ab}(\lambda^2,\beta)$ is bounded we obtain

$$\left|\int_0^{\alpha\sqrt{\Lambda}} d\lambda^2\left[\exp\left(-\frac{m}{\Lambda^2}(\lambda^2+n)^2\right)-1\right]h^{ab}(\lambda^2,\beta)\right|\le\int_0^{\alpha\sqrt{\Lambda}} d\lambda^2\left|\left[\exp\left(-\frac{m}{\Lambda^2}(\lambda^2+n)^2\right)-1\right]h^{ab}(\lambda^2,\beta)\right|=O\left(\frac{1}{\sqrt{\Lambda}}\right), \tag{2.20}$$

where m and n are the appropriate constants. Since Eq. (2.20) becomes zero in the $\Lambda\to\infty$ limit, the interchange between the $\Lambda\to\infty$ limit and the λ^2 integration in the first term of Eq. (2.18) is allowed. Finally, I_{ab} becomes

$$2\pi\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\, h^{ab}(\lambda^2,\beta)\frac{f(q^+ +\beta p^+)}{2|q^+ +\beta p^+|}. \tag{2.21}$$

Equation (2.21) is exactly the Fourier transform of Eqs. (2.1) or (2.9) restricted to the null plane and corresponds to that of Eqs. (2.7) or (2.11). Therefore Eqs. (2.7) and (2.11) hold under the same assumption. Now we derive the sum rule: By defining C_{ab} as

$$C_{ab}(p\cdot q,q^2)=\int d^4x\,\exp(iq\cdot x)\langle p|\{J_a(x),J_b(0)\}|p\rangle_c, \tag{2.22}$$

we integrate over q^-, change the variable from q^- to $\nu=p\cdot q$, and assume that we may interchange performing the ν integration and setting $q^+=0$.[15] This assumption is clear if we interpret the q^- integration in the sense of Eq. (2.16). Moreover, in the case of the commutator, this problem is well known as the class II graph problem.[15-18] Then in the case of the anticommutator the problem is also regarded as the class II graph problem as was expected previously.[4] In other words, though both the symmetric and the antisymmetric parts of h^{ab} contribute to Eq. (2.21), if we set $q^+=0$ before the β integration one of them does not contribute, depending on commutators or anticommutators. Therefore we need the condition for the lost part to ensure the validity of the interchange of setting $q^+=0$ and the β integration, and we obtain

$$\int_0^\infty d\nu\, C_{\langle ab\rangle}(\nu,-\vec{q}^{\perp 2})=-\tfrac{1}{2}\delta_{ab}\int_{-\infty}^\infty d\alpha \ln|\alpha|S_0(\alpha,0), \tag{2.23}$$

where α is defined as p^+x^-, the matrix element of the bilocal current is defined as

$$\begin{aligned}\langle p|S_0(x|0)|p\rangle_c&=S_0(p\cdot x,x^2),\\ \langle p|A_c(x|0)|p\rangle_c&=A_c(p\cdot x,x^2),\end{aligned} \tag{2.24}$$

and the symmetric part of C_{ab} is defined as

$$C_{\langle ab\rangle}(\nu,-\vec{q}^{\perp 2})=\tfrac{1}{2}[C_{ab}(\nu,-\vec{q}^{\perp 2})+C_{ba}(\nu,-\vec{q}^{\perp 2})]. \tag{2.25}$$

We express the $\delta_{ab}S_0(\alpha,0)$ by the Fourier-transform formula of Dicus, Jackiw, and Teplitz[15] as

$$\delta_{ab}S_0(\alpha,0)=\frac{1}{2\pi}\int_{-1}^1\frac{d\omega}{\omega}\exp(i\omega\alpha)F_{\langle ab\rangle}(\omega), \tag{2.26}$$

where ω is defined as $-q^2/2\nu$ and $F_{ab}(\omega)$ is the Bjorken scaling limit of $W_{ab}(\nu,q^2)$ similarly defined as Eq. (2.22) in the case of the current commutator. Note that $C_{ab}(\nu,q^2)$ and $W_{ab}(\nu,q^2)$ are the same quantity in the $\nu\geq 0$ region. By substituting Eqs. (2.12) and (2.26) into Eq. (2.23), we obtain

$$\int_0^\infty d\nu\, C_{\langle ab\rangle}(\nu,-\vec{q}^{\perp 2})=\tfrac{1}{2}\,\mathrm{P}\int_0^1\frac{d\omega}{\omega^2}F_{\langle ab\rangle}(\omega), \tag{2.27}$$

where P means to take the principal value.

III. APPLICATION TO CONSERVED VECTOR CURRENTS

According to the discussion in Sec. II, we start with the connected spin-averaged nucleon matrix element given by

$$C^{\mu\nu}_{ab}(x)=\langle p|[J^\mu_a(x),J^\nu_b(0)]|p\rangle_c, \tag{3.1}$$

where the conserved vector current $J^\mu_a(x)$ is defined as

$$J^\mu_a(x)=\bar{q}(x)\gamma^\mu\tfrac{1}{2}\lambda_a q(x). \tag{3.2}$$

The simplest DGS representation of Eq. (3.1) is[2]

$$\begin{aligned}C^{\mu\nu}_{ab}(x)=\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\{&(\partial^\mu\partial^\nu-\Box g^{\mu\nu})[h^{ab}_1(\lambda^2,\beta)+ip\cdot\partial g^{ab}_1(\lambda^2,\beta)]\\ &+[-\Box p^\mu p^\nu+p\cdot\partial(p^\mu\partial^\nu+p^\nu\partial^\mu)-g^{\mu\nu}(p\cdot\partial)^2]h^{ab}_2(\lambda^2,\beta)\}\exp(i\beta p\cdot x)i\Delta(x,\lambda^2),\end{aligned} \tag{3.3}$$

where the Born pole term is included. Note that this pole should be separately considered when we try to restrict Eq. (3.3) to equal time.[2,13] Intuitively, this fact will be explained as follows: At the Born pole, $2\nu+q^2$ is zero and we can freely change $\Box$ into $-2ip\cdot\partial$ in Eq. (3.3). However, $\Box$ contains the time derivative twice, while $p\cdot\partial$ contains it once. Since with respect to time the even derivative of $\Delta(x)$ at $x^0=0$ is zero but the odd one is not, we cannot have a definite result. The situation is different in the case of the null plane, since both $\Box$ and $p\cdot\partial$ contain ∂^+ once, which plays the role of the time derivative, and $\Delta(x)$ at

$x^+=0$ is not zero. Now the current commutator on the null plane derived formally through the neutral-vector-gluon model is given by[6]

$$\delta(x^+)[J_a^+(x), J_b^\nu(0)] = i\{s^{+}{}_{\beta}{}^{\nu}{}_{\alpha}\partial^\alpha[\Delta(x)G_c^\beta(x\,|\,0)] - 2g^+{}_\alpha g^\nu{}_\beta[\partial^\alpha\Delta(x)]G_c^\beta(x\,|\,0) - \epsilon^{+\nu}{}_{\alpha\beta}\partial^\alpha[\Delta(x)G_c^{5\beta}(x\,|\,0)]\}, \tag{3.4}$$

where

$$G_c^\beta(x\,|\,0) = d_{abc}A_c^\beta(x\,|\,0) + f_{abc}S_c^\beta(x\,|\,0), \quad G_c^{5\beta} = d_{abc}S_c^{5\beta}(x\,|\,0) - f_{abc}A_c^{5\beta}(x\,|\,0), \tag{3.5}$$

$$S_a^\mu(x\,|\,0) = \tfrac{1}{2}[\bar{q}(x)\gamma^\mu \tfrac{1}{2}\lambda_a q(0) + \bar{q}(0)\gamma^\mu \tfrac{1}{2}\lambda_a q(x)], \tag{3.6}$$

$$A_a^\mu(x\,|\,0) = \frac{1}{2i}[\bar{q}(x)\gamma^\mu \tfrac{1}{2}\lambda_a q(0) - \bar{q}(0)\gamma^\mu \tfrac{1}{2}\lambda_a q(x)], \tag{3.7}$$

$\Delta(x)$ or $\partial^+\Delta(x)$ reads $-\epsilon(x^-)\delta(\vec{x}^\perp)\delta(x^+)/4$ or $-\delta(x^-)\delta(\vec{x}^\perp)\delta(x^+)/2$, respectively, and $s^{\mu\nu\alpha\beta} = g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\alpha\beta}$. By taking $\mu=+$ and $\nu=+$ or i, Eq. (3.3) on the null plane becomes

$$\begin{aligned} C^{+\nu} = {} & p^+p^\nu \int_0^1 d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p\cdot x)[-\beta^2 h_1^{ab}(\lambda^2,\beta) + (\lambda^2 - \beta^2 m^2)h_2^{ab}(\lambda^2,\beta) + \beta^3 m^2 g_1^{ab}(\lambda^2,\beta)]f(x) \\ & + \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p\cdot x)[h_2^{ab}(\lambda^2,\beta) - \beta g_1^{ab}(\lambda^2,\beta)]p\cdot\partial(p^+\partial^\nu + p^\nu\partial^+)f(x) \\ & + i\int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p\cdot x)\beta[h_1^{ab}(\lambda^2,\beta) + m^2 h_2^{ab}(\lambda^2,\beta) - \beta m^2 g_1^{ab}(\lambda^2,\beta)](p^+\partial^\nu + p^\nu\partial^+)f(x) \\ & + \int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p\cdot x)[h_1^{ab}(\lambda^2,\beta) + (ip\cdot\partial - \beta m^2)g_1^{ab}(\lambda^2,\beta)]\,\partial^+\partial^\nu f(x), \end{aligned} \tag{3.8}$$

where

$$f(x) = -\tfrac{1}{4}i\epsilon(x^-)\delta(\vec{x}^\perp), \quad \partial^- f(x) = -\tfrac{1}{8}ix^-\epsilon(x^-)(\partial^j\partial^j - \lambda^2)\delta(\vec{x}^\perp). \tag{3.9}$$

If we require Eq. (3.8) to agree with Eq. (3.4), we obtain the conditions

$$\int_0^\infty d\lambda^2[h_2^{ab}(\lambda^2,\beta) - \beta g_1^{ab}(\lambda^2,\beta)] = 0, \tag{3.10}$$

$$\int_0^\infty d\lambda^2[-\beta^2 h_1^{ab}(\lambda^2,\beta) + \lambda^2 h_2^{ab}(\lambda^2,\beta)] = 0, \tag{3.11}$$

$$\int_0^\infty d\lambda^2[h_1^{ab}(\lambda^2,\beta) + (ip\cdot\partial - \beta m^2)g_1^{ab}(\lambda^2,\beta)] = 0, \tag{3.12}$$

$$G_c(p\cdot x, 0) = -i\int_0^\infty d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p\cdot x)h_1^{ab}(\lambda^2,\beta), \tag{3.13}$$

where

$$\langle p\,|G_c^\beta(x\,|\,0)|\,p\rangle_c = p^\beta G_c(p\cdot x, x^2) + x^\beta \bar{G}_c(p\cdot x, x^2), \tag{3.14}$$

and we use the letter S or A instead of G, whenever necessary, to specify whether it comes from the symmetric or the antisymmetric bilocal defined in Eqs. (3.6) and (3.7). Equations (3.10)–(3.13) should be understood to hold in the sense of Eq. (2.16) when they operate on the function which satifies $(\Box - \lambda^2)F(x) = 0$. Since the DGS representation of the current anticommutator in the case of a stable one-particle state is given by changing $i\Delta(x,\lambda^2)$ into $\Delta^{(1)}(x,\lambda^2)$, its null-plane restriction becomes Eq. (3.8) with

$$f(x) = -\frac{1}{2\pi}\ln|x^-|\,\delta(\vec{x}^\perp)$$

and

$$\partial^- f(x) = \frac{1}{4\pi}x^-\ln|x^-|\,(\lambda^2 - \partial^j\partial^j)\delta(\vec{x}^\perp).$$

Therefore, with use of Eqs. (3.10)–(3.13), we obtain after some algebra

$$\langle p\,|\{J_a^+(x), J_b^\nu(0)\}|\,p\rangle_c\delta(x^+) = \langle p\,|\{s^{+}{}_{\beta}{}^{\nu}{}_{\alpha}\partial^\alpha[\Delta^{(1)}(x)G_c^\beta(x\,|\,0)] - 2g^+{}_\alpha g^\nu{}_\beta[\partial^\alpha\Delta^{(1)}(x)]G_c^\beta(x\,|\,0)\}|\,p\rangle_c, \tag{3.15}$$

where $\Delta^{(1)}(x)$ or $\partial^+\Delta^{(1)}(x)$ reads $-(1/2\pi)\ln|x^-|\,\delta(\vec{x}^\perp)\delta(x^+)$ or $-(1/2\pi)\mathrm{P}(1/x^-)\delta(\vec{x}^\perp)\delta(x^+)$, respectively. The case $\mu=+$ and $\nu=-$ is discussed similarly and we obtain the condition

$$p\cdot x\,\frac{\partial\overline{G}_c(p\cdot x,0)}{\partial p\cdot x}=\tfrac{1}{2}\int_0^\infty d\lambda^2\int_{-1}^1 d\beta\exp(i\beta p\cdot x)\lambda^2[h_1^{ab}(\lambda^2,\beta)+(ip\cdot\partial-\beta m^2)g_1^{ab}(\lambda^2,\beta)]\,, \tag{3.16}$$

in addition to Eqs. (3.10)–(3.13). Under these conditions, we see that Eq. (3.15) holds even in the case $\nu=-$. Finally, we derive the sum rules from Eq. (3.15): In the case of $\mu=\nu=+$, we obtain

$$\int_0^\infty d\nu\, W_2^{(ab)}(\nu,-\vec{q}^{\perp 2})=d_{abc}\int_{-\infty}^\infty d\alpha\,\mathrm{P}\,\frac{1}{\alpha}A_c(\alpha,0)=\mathrm{P}\int_0^1\frac{d\omega}{\omega}F_2^{(ab)}(\omega)\,, \tag{3.17}$$

where

$$C_{ab}^{\mu\nu}(p,q)=\int d^4x\exp(iq\cdot x)\langle p\,|\{J_a^\mu(x),J_b^\nu(0)\}\,|p\rangle_c$$
$$=-\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)W_L^{ab}(\nu,q^2)+\left[p^\mu p^\nu-\frac{\nu}{q^2}(p^\mu q^\nu+p^\nu q^\mu)+\frac{\nu^2}{q^2}g^{\mu\nu}\right]W_2^{ab}(\nu,q^2)\,, \tag{3.18}$$

the symmetric part of W_2^{ab} is defined similarly as Eq. (2.25), and $F_2^{ab}(\omega)$ is the Bjorken scaling limit of νW_2^{ab}. In the case of $\mu=+$ and $\nu=i$, we obtain

$$\frac{1}{\vec{q}^{\perp 2}}\int_0^\infty d\nu\,\nu W_2^{[ab]}(\nu,-\vec{q}^{\perp 2})=-\tfrac{1}{2}f_{abc}\int_{-\infty}^\infty d\alpha\ln|\alpha|S_c(\alpha,0)=\tfrac{1}{2}\mathrm{P}\int_0^1\frac{d\omega}{\omega^2}F_2^{[ab]}(\omega)\,, \tag{3.19}$$

where the antisymmetric part of W_2^{ab} is defined as

$$W_2^{[ab]}(\nu,q^2)=\frac{1}{2i}[W_2^{ab}(\nu,q^2)-W_2^{ba}(\nu,q^2)]\,. \tag{3.20}$$

In the case of $\mu=+$ and $\nu=-$, we obtain

$$\int_0^\infty d\nu\, W_L^{(ab)}(\nu,-\vec{q}^{\perp 2})=0\,. \tag{3.21}$$

Note that Eq. (3.21) can be derived from Eq. (3.12) as follows: We represent $F(x)$ which satisfies $(\Box-\lambda^2)F(x)=0$ as

$$F(x)=\int d^4k\exp(-ik\cdot x)f(k)\delta(k^2-\lambda^2)\,, \tag{3.22}$$

where $f(k)$ is an arbitrary function. Then if we take the $\Lambda\to\infty$ limit, corresponding to Eq. (2.16), Eq. (3.12) becomes

$$\int_{-\infty}^\infty d\nu\int_0^\infty d\lambda^2\int_{-1}^1 d\beta[h_1^{ab}(\lambda^2,\beta)+\nu g_1^{ab}(\lambda^2,\beta)]f(q+\beta p)\delta((q+\beta p)^2-\lambda^2)=0\,, \tag{3.23}$$

where the q^- integration is changed into the ν integration, and h_1^{ab} and g_1^{ab} are assumed to satisfy the appropriate conditions discussed in Sec. II. Since Eq. (3.23) holds for the arbitrary function $f(q+\beta p)$, we obtain

$$\int_{-\infty}^\infty d\nu\,[h_1^{ab}(\lambda^2(\nu,\beta),\beta)+\nu g_1^{ab}(\lambda^2(\nu,\beta),\beta)]=0\,, \tag{3.24}$$

where $\lambda^2(\nu,\beta)=q^2(\nu)+2\beta\nu+\beta^2m^2$. Now if we take $f(k)=1/(2\pi)^3$, Eq. (3.23) is nothing but the expression

$$\int_{-\infty}^\infty d\nu\,\frac{1}{q^2}W_L^{ab}(\nu,q^2)=0\,. \tag{3.25}$$

Therefore, if we assume the interchange of setting $q^+=0$ and the ν integration, Eq. (3.25) becomes Eq. (3.21).

The generalization to the polarized target is straightforward and implicitly included in Ref. 4.

IV. THE RELATION BETWEEN THE SEA-QUARK DISTRIBUTION AND THE POMERON

There is essentially no difficulty in generalizing the discussions in Secs. II and III to the axial-vector current except the symmetry-breaking effects which might occur in the case of $\mu=+$ and $\nu=i$ or $-$. Now we consider the connected diagonal matrix element of the anticommutator of axial-vector currents taken between the spin-averaged proton state with momentum p, defined as[5]

$$C_{ab}^{\mu\nu}(p,q)=\int d^4x\exp(iq\cdot x)\langle p\,|\{J_a^{5\mu}(x),J_b^{5\nu}(0)\}\,|p\rangle_c$$
$$=p^\mu p^\nu W_1^{ab}+p^\mu q^\nu W_2^{ab}+p^\nu q^\mu W_3^{ab}+q^\mu q^\nu W_4^{ab}+g^{\mu\nu}W_5^{ab}\,. \tag{4.1}$$

By applying the same method in Secs. II and III, we obtain the sum rule in the case of $\mu=\nu=+$,

$$\int_0^\infty d\nu\, W_1^{(ab)}(\nu, -\vec{q}^{\perp 2}) = d_{abc} \mathrm{P} \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} A_c(\alpha, 0)$$

$$= \mathrm{P} \int_0^1 \frac{dx}{x} F_2^{(ab)}(x), \quad (4.2)$$

where $F_2^{ab}(x)$ is the same as the one defined in Sec. III and $x = -q^2/2\nu$. At $q^+ = \vec{q}^\perp = 0$, W_1^{ab} is related to the off-shell pion-nucleon total cross section through the PCAC relation as[8]

$$W_1^{ab} = \frac{4}{\nu} F_a{}^2 \sigma^{ab}, \quad (4.3)$$

where a is the complex conjugate of b, $F_{1\pm i2} = \sqrt{2} f_\pi$, $F_3 = f_\pi$, and f_π is the pion decay constant. By substituting Eq. (4.3) into Eq. (4.2), we obtain

$$g_A{}^2(0) + \frac{2f_\pi{}^2}{\pi} \int_0^\infty \frac{d\nu}{\nu} [\sigma^+(\nu) + \sigma^-(\nu)]$$

$$= \tfrac{1}{2} \mathrm{P} \int_0^1 \frac{dx}{x} [F_2^{\nu p}(x) + F_2^{\bar{\nu} p}(x)], \quad (4.4)$$

where ± denotes the charge of the pion and the Born term is separated out at the left-hand side. Expressing the right-hand side of Eq. (4.4) by the quark-distribution function in the proton, taking into account the fact that there are two valence u quarks and one valence d quark in the proton, we obtain

$$\mathrm{P} \int_0^1 \frac{dx}{x} [F_2^{\nu p}(x) + F_2^{\bar{\nu} p}(x)] = 6 + 8\mathrm{P} \int_0^1 dx\, \lambda(x), \quad (4.5)$$

where $\lambda(x)$ denotes the sea-quark distribution in the nucleon. According to the usual parametrization,[19] we take $\lambda(x)$ as

$$\lambda(x) = \frac{a(1-x)^n}{x}. \quad (4.6)$$

Now at the left-hand side of Eq. (4.4), we assume the smooth extrapolation of the off-shell pion-nucleon total cross section to the on-shell one and approximate it at high energy by the Regge parametrization as[20]

$$(\nu^2 - m_\pi{}^2 m_N{}^2)^{1/2} (\sigma^+ + \sigma^-)$$

$$= 8\pi (\beta_P \nu^{\alpha_P(0)} + \beta_{P'} \nu^{\alpha_{P'}(0)}). \quad (4.7)$$

If we substitute Eqs. (4.6) and (4.7) into Eq. (4.4), both sides diverge logarithmically as far as $\alpha_P(0) = 1$. Since the behavior $1/x$ of the sea-quark distribution near $x \approx 0$ is assumed to be due to the Pomeron exchange,[21] the sum rule (4.4) interrelates the sea-quark distribution with the Pomeron. In order to see the fact clearly we first assume $\alpha_P(0)$ as

$$\alpha_P(0) = 1 - \epsilon, \quad (4.8)$$

where ϵ is a small positive constant. Accordingly, we change the sea-quark distribution as

$$a x^{-1+\epsilon} (1-x)^n. \quad (4.9)$$

Then the sum rule (4.4) can be estimated and the divergent piece as $\epsilon \to 0$ appears as the simple pole $1/\epsilon$. Therefore, if the sum rule is meaningful, the coefficient of this simple pole should be equal on both sides. Thus we obtain the condition

$$a = 4 f_\pi{}^2 \beta_P. \quad (4.10)$$

Under the condition (4.10), the limit $\epsilon \to 0$ can be taken and we obtain the sum rule which determines the power n in the sea-quark distribution.[5] The condition (4.10) is the relation between the residue of the Pomeron and the coefficient a of the sea-quark distribution in the nucleon, and a is estimated as 0.15 with use of the experimental value of $f_\pi \approx 0.094$ (GeV) and $\beta_P \approx 4.2$. The value 0.15 is about the same as the one currently used.[19] The same analysis in the kaon-nucleon scattering gives

$$a = 4 f_K{}^2 \beta_P', \quad (4.11)$$

where the kaon decay constant f_K and β_P' are defined similarly as the f_π and the β_P in Eq. (4.7), respectively. From Eqs. (4.10) and (4.11) we obtain

$$\frac{f_K}{f_\pi} = \left(\frac{\beta_P}{\beta_P'}\right)^{1/2}. \quad (4.12)$$

If we use the experimental value of β_P and β_P',[20] we obtain

$$\frac{f_K}{f_\pi} \approx 1.1. \quad (4.13)$$

This value is in good agreement with the experimental value 1.3 obtained by the decay rate for the kaon and nearer to the symmetric value 1.

V. SUMMARY

We have studied the assumption necessary to restrict current anticommutators to the null plane. We find that this assumption is the same as that which restricts current commutators to the null plane. Moreover, when applied to sum rules the problem is reduced to the class II graph problem discussed previously. Here the DGS representation is powerful, but very complex. Effectively, sum rules can be derived by the ones derived through the canonical null-plane quantization. However, as we often emphasize, their applications should be restricted to the case where the DGS representation holds, such as the stable-one-particle matrix element. Though the many sum rules derived are concerned with the divergent quantities, the one method to treat these quantities is given, and we find the quantitative relation between the sea-quark distribution and the Pomeron, Eqs. (4.10) and (4.11), makes the sum rules meaningful. Further, the f_K/f_π ratio is related to the residues of the Pomeron in pion-nucleon and kaon-nucleon scattering, Eq. (4.12).

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$$\langle p^+, \vec{p}^\perp | \vec{p}'^\perp, p'^+ \rangle = (2\pi)^3 2p^+ \delta(\vec{p}^\perp - \vec{p}'^\perp)\delta(p^+ - p'^+).$$

[11]Mathematically, it is a difficult problem to eliminate the unphysical mode corresponding to $p^+ = 0$, and because of this mode, $\Delta^{(1)}(x, \lambda^2)$ on the null plane diverges or becomes ambiguous. Usually, such a divergent piece is regarded as zero with use of a suitable test function or something similar. The integral representation of $\ln|x^-|$ should be understood in this sense. Further, such $p^+ = 0$ modes do not contribute to the connected one-particle matrix element. I am grateful to Professor M. Ida for helpful discussions concerning these points. [M. Ida, Nuovo Cimento 40, 354 (1977) and references cited therein.]

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