SU(4) symmetry structure of the nonleptonic weak interaction

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Starting with the most general SU(4) weak Hamiltonian and assuming the nonexoticity of intermediate states and s-u channel symmetry, we obtain that the parity-violating (parity-conserving) Hamiltonian belongs to $\underline{15}_A \oplus \underline{45} \oplus \underline{45}^*$ ($\underline{15}_S \oplus \underline{20}'' \oplus \underline{84}$).

With the advent of charm and the other new flavors as indicated by recent experiments, the structure of basic weak currents has been extended. The situation is further complicated by the lack of a simple understanding of the nonleptonic decays,[1,2] especially of the $\Delta I = \frac{1}{2}$ enhancement, making its generalization unclear. In the theory of strong interactions (quantum chromodynamics) an enhancement at short distances does occur,[1,3] but numerical estimates are typically too small to account for the observation. Attempts have also been made to introduce a right-handed piece[4] in the weak current which indeed seems to help in explaining weak processes, though there is not yet any experimental evidence for its existence. Two types of right-handed currents, namely $(\bar{c}d)_R$ and $(\bar{c}s)_R$, have been considered.[5,6] Neither of these explains both the nonleptonic and the weak radiative decays simultaneously, but a linear combination of the two, i.e., $\bar{c}\gamma_\alpha(1-\gamma_5)(-d\sin\theta + s\cos\theta)$, seems to improve the situation.[7] Restricting to the charm scheme, the general current×current weak interaction belongs to the SU(4) representations present in the direct product

$$\underline{15} \otimes \underline{15} = \underline{1}_S \oplus \underline{15}_A \oplus \underline{20}_S'' \oplus \underline{45}_A \oplus \underline{45}_A^* \oplus \underline{84}_S . \qquad (1)$$

Owing to the presence of a right-handed current, the weak Hamiltonian would contain an H_w^{LR} piece arising from the left×right current-current interaction in addition to the conventional Glashow-Iliopoulos-Maiani (GIM) piece H_w^{LL} originating from pure left-handed currents. H_w^{LL} belongs to the symmetric representations[8] $\underline{20}'' \oplus \underline{84}$ for the parity-violating (PV) as well as the parity-conserving (PC) mode, while the H_w^{LR} part transforms[7] like

$$\underline{15}_A \oplus \underline{45}_A \oplus \underline{45}_A^* \text{ for the PV mode}$$

and (2)

$$\underline{15}_S \oplus \underline{20}_S'' \oplus \underline{84}_S \text{ for the PC mode}.$$

In general, therefore, all the representations present in the product (1) can contribute to the weak interactions. The most general Hamiltonian thus does not have any predictive power unless some additional assumptions are made.

In this paper, we explore the SU(4)-symmetry structure of the nonleptonic weak Hamiltonian by studying the hadronic weak decays in a dynamical consideration.[9,10] We start with the most general interaction and expand the decay amplitudes in terms of the reduced amplitudes corresponding to the intermediate states in all the s, t, and u channels. For instance, the hyperon decay process $B \to B' + P$ we write as

$$S + B \to B' + P .$$

The spurion S has the same structure as the Hamiltonian and is such that the strong quantum numbers in this reaction are conserved. The transition amplitudes are then expressed in terms of the following SU(4) eigenamplitudes (reduced matrix elements):

$$\langle B' \| P \| m \rangle \langle m \| S \| B \rangle, \text{ for } s \text{ channel},$$

$$\langle P \| \bar{S} \| m \rangle \langle m \| \overline{B'} \| B \rangle, \text{ for } t \text{ channel},$$

and

$$\langle B' \| \bar{S} \| m \rangle \langle m \| \bar{P} \| B \rangle \text{ for } u \text{ channel}.$$

We obtain constraints on these reduced matrix elements by assuming

(i) CP invariance,

(ii) that the eigenamplitudes corresponding to the physically observed (nonexotic) intermediate states contribute dominantly to the transitions, and

(iii) symmetry[10] of the weak Hamiltonian in s and u channels.

We have earlier shown[9] in SU(3) that with the general $(V-A)\times(V-A)$ weak Hamiltonian ($\underline{8} \oplus \underline{27}$), these assumptions give the following results:

For the PV mode, we get (i) $\Delta I = \frac{1}{2}$ rule, (ii) Lee-Sugawara sum rule, (iii) $\Sigma_+^+ = 0$, and (iv) the s and u channels do not contribute, and so the PV decays arise through the t channel.[11,12]

For the PC mode, our analysis led in the s and u channels to (i) $\Delta I = \frac{1}{2}$ rule, (ii) Lee-Sugawara(LS) sum rule, and (iii) a new sum rule

$$\sqrt{2}\,\Sigma_+^+ - \Sigma_0^+ = \sqrt{3}\,\Lambda_-^0\,,$$

$$(14.91 \pm 0.82) = (17.41 \pm 0.44)$$

which is well satisfied experimentally. Discrepancies in these relations can be explained by the t-channel contributions. The experimental validity of these relations indicates, however, that the t-channel contributions[13] are small and the PC decays arise predominantly through s and u channels.[11]

In order to obtain constraints on the SU(4) symmetry structure of the weak Hamiltonian, we now extend our considerations to SU(4). We start with the most general weak interaction, and expand the transition amplitudes in terms of the SU(4)-reduced matrix elements corresponding to the various possible intermediate states in the s, t, and u channels. We notice that with the assumption of the nonexoticity of the intermediate states and the s-u channel symmetry of the weak interaction, the contribution from the GIM weak Hamiltonian ($\underline{20}'' \oplus \underline{84}$) to the PV decays of the baryons, as well as of the mesons, vanishes in all the s, t, and u channels. So the PV decays occur only through H_w^{LR}, i.e., the $(V-A)\times(V+A)$ part of the weak interaction. For the PC decays where the representations $\underline{15}$, $\underline{20}''$, and $\underline{84}$ can appear in general, our analysis puts important constraints on the reduced matrix elements corresponding to these representations.

$B(\frac{1}{2}^+) \to B(\frac{1}{2}^+) + P(0^-)$. For the PV decays, the s-u channel symmetry together with the CP invariance makes the contribution of the GIM weak Hamiltonian ($\underline{20}'' \oplus \underline{84}$) zero for the s and u channels, whereas the nonexoticity of the intermediate states forbids the GIM Hamiltonian to contribute in the t channel. Therefore the PV weak decays, in our analysis, occur through the $(V-A)\times(V+A)$ part of the Hamiltonian, which leads to $\underline{15}$ dominance for the ordinary (uncharmed) decays. The Cabibbo-enhanced $\Delta C = \Delta S$ decays of charmed baryons get contributions from the $\underline{45} \oplus \underline{45}^*$ piece of the weak Hamiltonian. For PC decays, our considerations forbid the $\underline{84}$ Hamiltonian to contribute in the s and u channels. So we expect PC decays to arise predominantly through the $\underline{15}$ and $\underline{20}''$ representations. This, of course, amounts to the octet dominance at the SU(3) sublevel for the ordinary decays. For the Cabibbo-enhanced ($\Delta C = \Delta S$) mode only $\underline{20}''$ contributes.

Decays of $\frac{3}{2}^+$ isobars. In SU(4) analysis $\underline{20}''$ makes no contribution to the PV $D(\frac{3}{2}^+) \to D(\frac{3}{2}^+) + P(0^-)$ decays, and $\underline{84}$ vanishes, according to our assumptions, in all the channels. These decays also then arise from the H_w^{LR} part. This leads to the $\Delta I = \frac{1}{2}$ rule for $\Omega^- \to \Xi^*\pi$ decays in the PV mode. In our study, the $\Delta I = \frac{3}{2}$ piece is allowed to contribute to $\Omega^- \to \Xi\pi$ decays and to the PC mode of $\Omega^- \to \Xi^*\pi$. For the $\Omega^- \to \Lambda K^-$ mode we get a null asymmetry parameter.[9] Both of these features are found to be supported in a recent CERN experiment[14] which indicates a more than 20% violation of the $\Delta I = \frac{1}{2}$ rule and the value 0.06 ± 0.14 for the asymmetry parameter α ($\Omega^- \to \Lambda K^-$).

Pseudoscalar-meson decays. It is well known[15] that the $\underline{20}''$ Hamiltonian does not contribute to the PV, $P \to \underline{2P}$ decays. In our considerations the same reduced matrix element $\langle P \| P \| m \rangle \langle m \| S \| P \rangle$ appears in s, t, and u channels. Nonexoticity of the intermediate states leads to the vanishing of the $\underline{84}$ part. Therefore, here again, the decays occur through the H_W^{LR} part of the Hamiltonian, the $\underline{15}$ representation of which allows $K \to 2\pi$ decays. Cabibbo enhanced $\Delta C = \Delta S$ decays arise through the $\underline{45} \oplus \underline{45}^*$ piece. Assuming the nonexoticity of the intermediate states, we can express all the decay amplitudes in terms of a single parameter. The following relations are then obtained:

$$\begin{aligned} &\langle \overline{K}^0\pi^0 \,|\, D^0 \rangle = \langle \pi^0\pi^+ \,|\, F^+ \rangle = 0\,, \\ &\langle K^-\pi^+ \,|\, D^0 \rangle = \langle \overline{K}^0\pi^+ \,|\, D^+ \rangle = -\langle K^+\overline{K}^0 \,|\, F^+ \rangle \\ &\qquad = -\sqrt{6}/4 \langle \pi^+\eta \,|\, F^+ \rangle = -\sqrt{3}/2 \langle \pi^+\eta' \,|\, F^+ \rangle\,. \end{aligned} \tag{3}$$

Although the present data on these decays are rather limited, they do seem to favor these results. Experimentally,[16] the branching ratios are

$$\begin{aligned} &B(D^+ \to \overline{K}^0 + \pi^+) = 1.5 \pm 0.6\,, \\ &B(D^0 \to K^- + \pi^+) = 2.2 \pm 0.6\,, \\ &B(D^0 \to \overline{K}^0 + \pi^0) < 6\,. \end{aligned} \tag{4}$$

Branching ratios for $D^0 \to K^- + \pi^+$ and $D^+ \to \overline{K}^0 + \pi^+$ are nearly equal, as expected from (3) in contrast to the GIM scheme, where one gets[8]

$$\langle K^-\pi^+ \,|\, D^0 \rangle = \sqrt{2}\langle \overline{K}^0\pi^0 \,|\, D^0 \rangle$$

and (5)

$$\langle \overline{K}^0\pi^+ \,|\, D^+ \rangle = 0\,.$$

Further, $D^0 \to \overline{K}^0 + \pi^0$ and $F^+ \to \pi^+ + \pi^0$ have not yet been seen, and thus are in accord with our expectation.

The parity-conserving decays $P \to 3P$ may acquire contributions from the $\underline{15}$, $\underline{20}''$, and $\underline{84}$ parts of the weak Hamiltonian.

Finally, we conclude that for all of the processes considered, the PV decays seem to occur through the $\underline{15}_A \oplus \underline{45} \oplus 45^*$ components of the weak Hamiltonian H_w^{LR} arising from the $(V-A)\times(V+A)$ part of the interaction, and the PC decays get contributions from the $\underline{15}_S \oplus \underline{20}'' \oplus \underline{84}$ parts of the weak Hamiltonian coming from the $(V-A)\times(V-A)$

and/or $(V-A)\times(V+A)$ interaction. We may note here that the representation $(\underline{15},\underline{15})$ of the chiral SU(4) ×SU(4) has precisely this splitting for the odd-parity and even-parity terms. This may indicate the $(\underline{15},\underline{15})$ nature of the weak Hamiltonian H_w^{LR} in chiral SU(4)×SU(4), along with a possible term belonging to $(\underline{4},\underline{4}^*)+(\underline{4}^*,\underline{4})$. The ordinary PV and PC decays seem to arise from the H_w^{15} and $H_w^{20''}$ parts of the weak Hamiltonian, respectively. This picture is consistent with the observation made in the current-algebra technique, where different d/f ratios are required[17] to fit the experimental data for PV and PC decays. Owing to the different transformation properties of PC and PV decays, the two modes get decoupled.[18] For this reason, the above structure of the weak interaction appears to be free from the criticism of Golowich and Holstein.[2]

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[10]The s-u channel symmetry of the weak Hamiltonian has been earlier arrived at by Y. Kohara and K. Nishijima [Prog. Theor. Phys. 47, 648 (1972)] using dispersion relations, etc.

[11]These observations are in accordance with the results obtained in current algebra where the PC-decay amplitudes get contributions from the pole terms and the PV from the equal-time commutator.

[12]This result is in accord with the findings of S. Nussinov and J. L. Rosner [Phys. Rev. Lett. 23, 1266 (1969)] based on duality arguments that for s-wave baryonic decays, the low-energy pole contributions will be relatively small and that the Regge contribution will dominate. However, duality arguments run into difficulaties in the case of charmed-particle decays. See. J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B100, 313 (1975).

[13]In fact, recent experiment on $\Xi^0\to\Lambda\pi^0$ has indicated a $\Delta I=\frac{1}{2}$ violation [G. Bunce *et al.*, Phys. Rev. D 18, 633 (1978)]. If t-channel contributions are included, discrepancies of the sum rules are related as

$$(\tfrac{3}{2})^{1/2}\Delta\Sigma=-(\Delta\Lambda+2\Delta\Xi)=\Delta\,(\mathrm{LS}).$$

[14]J. Gaillard, results of CERN experiment presented at the 6th Trieste Conference on Elementary Particles, 1978 (unpublished):

$$\frac{\Gamma(\Omega^-\to\Xi^{-\,\circ}\pi^-)}{\Gamma(\Omega^-\to\Xi^-\pi^\circ)}=2.9\pm0.3.$$

$\Delta I=\frac{1}{2}$ predicts this ratio to be 2.

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