PHYSICAL REVIEW D VOLUME 21, NUMBER 3 1 FEBRUARY 1980

Isospin-violating mass differences and mixing angles: The role of quark masses

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(Received 28 September 1979)

The phenomenology of quantum-chromodynamics-based constituent quark models is employed to discuss isospin-violating mass differences and mixing angles which can arise from an intrinsic mass difference between down and up quarks. Such effects appear not only directly from the quark mass difference $m_d - m_u$ but also indirectly via isospin violations induced by this mass difference in the strong interactions and tend to be much larger than electromagnetic effects.

I. INTRODUCTION

The idea that isospin violations may arise in part from a difference in the mass of the down and up quarks is very old. The reason for this is simply that even if isospin symmetry were exact in the absence of electromagnetism, the mass difference $m_d - m_u$ would still be renormalized by electromagnetic interactions and must consequently be treated as a (usually incalculable) isospin-violating parameter.

It now seems much more likely that isospin is an "accidental" symmetry in the sense that the short-distance ("current") quark masses m_d^{current} and m_u^{current} violently break isospin symmetry but that the dressed ("constituent") quark masses $m_d^{\text{constituent}}$ and $m_u^{\text{constituent}}$ are approximately equal since they are mainly determined by the mass scale $\Lambda \sim 0.5$ GeV at which the strong interactions hadronize, and only marginally by m^{current}. In this view approximate isospin symmetry is a consequence of the fact that $\Lambda \gg m_d^{\text{current}}, m_u^{\text{current}}$. This notion is appealing since it provides an explanation for what would otherwise simply be an accidental degeneracy in the fermion mass spectrum.

There is a very simple, though rough, intuitive picture of this effect.[1] A constituent quark will have a size determined by the strong-interaction mass scale Λ. Such a quark of charge e_q will carry an electromagnetic mass in its electric field

$$\Delta m_{\text{em}} \sim e_q{}^2 \alpha \Lambda \sim 1 \text{ MeV}, \tag{1}$$

while its flavor-independent chromoelectric field will give it a mass

$$\Delta m_{\text{strong}} \sim \alpha_s \Lambda \sim \Lambda\,. \tag{2}$$

Thus it is that the very small and asymmetric short-distance masses appear as approximately symmetric masses $m_d \simeq m_u \simeq 0.33$ GeV. Despite this intuitive connection between the short- and long-distance quark masses, however, the relationship between low-energy hadron physics described in terms of current and constituent quarks is unclear. The pion is the best example: In the current quark picture it is the (almost) Goldstone boson of spontaneously broken chiral symmetry while in the constituent picture it is the partner of the ρ meson from which it is split by color hyperfine interactions.

There have been a number of discussions in the current quark picture of the possibility of observing the effects of $(m_d - m_u)_{\text{current}} \neq 0$, and especially of the possibility of observing very large isospin-violating effects since

$$\left(\frac{m_d - m_u}{m_d + m_u}\right)_{\text{current}} \sim 1\,.$$

It is our intention here and in the sequel[2] to this work to discuss such effects in the constituent quark framework. Since $(m_d - m_u)_{\text{current}}$ evolves into $(m_d - m_u)_{\text{constituent}}$, a determination of the latter quantity can be used to fix the former, thereby shedding light on the structure of the fermion mass spectrum. On the other hand, one would at first sight think that isospin-violating effects in the constituent quark picture would be much smaller, of order

$$\left(\frac{m_d - m_u}{m_d + m_u}\right)_{\text{constituent}},$$

and therefore very difficult to observe. That this is not necessarily true may easily be seen by considering how the constituent quarks arrange themselves in isospin multiplets. Just as one gets pure $s\bar{s}$ mesons in those systems where annihilation forces (which cause $u\bar{u} \leftrightarrow d\bar{d} \leftrightarrow s\bar{s}$ mixing) are small with respect to $m_s - m_d$, so one would get pure $u\bar{u}$ and $d\bar{d}$ mesons, and maximal isospin violation, if the *annihilation amplitudes* are small with respect to $m_d - m_u$.

In this paper we will use the recent developments of constituent models along the lines suggested by quantum chromodynamics (QCD),[3] which have been very successful phenomenologically, to discuss isomultiplet mass differences and isoscalar-isovector mixing angles. We will be led in this way to a satisfactory understanding of the observed mass differences and consequently to a favorable sense of the reliability of our predicted mixing angles, the effects of which are discussed in the sequel.[2] The final section contains a summary and our conclusions, and includes a comparison with related results as well as estimates of the current quark masses based on our results.

II. ISOMULTIPLET MASS DIFFERENCES AND ISOSCALAR-ISOVECTOR MIXING ANGLES

A. Baryon isomultiplet mass differences

We begin by discussing baryon isomultiplet mass differences. The mass difference between two members of an isomultiplet, say n and p, is composed of a large number of effects:

(1) the difference in the constituent quark masses;
(2) the change induced by $m_d - m_u$ in the color hyperfine interactions which go like

$$\frac{S_i \cdot S_j}{m_i m_j}\delta^3(r_{ij})$$

between quarks i and j, both from the change in the masses and from the change in the expectation value of $\delta^3(r_{ij})$ from the resultant wave-function shifts;
(3) the change in the "zero-point" energy of the system;
(4) the Coulombic and hyperfine interactions of electromagnetism.

In practice it turns out here (and in most of the effects we consider) that the actual electromagnetic contributions are small so that the observed isospin violations are dominated by $m_d - m_u$ and strong-interaction shifts.

To make these calculations we rely on a recent analysis[4] of the ground-state baryons in QCD. When considering a baryon with unequal quark masses it is useful to use the analog of the uds basis introduced for discussing strange baryons.[3,5] For example, one has

$$n = (ddu)\chi^\lambda\psi_{00}\,, \tag{3}$$

$$p = (uud)\chi^\lambda\psi_{00}\,, \tag{4}$$

where (ddu) denotes that the u quark has been assigned the label 3 so that the Pauli principle is applied only to the two truly identical quarks; χ^λ is the spin-$\frac{1}{2}$ spin wave function of three quarks which is symmetric in quarks 1 and 2 [see Eq. (50)], and ψ_{00} is the asymmetric ground-state wave function which we approximate by the harmonic-oscillator solution

$$\psi_{00} = \frac{\alpha_\rho^{3/2}\alpha_\lambda^{3/2}}{\pi^{3/2}}\exp(-\tfrac{1}{2}\alpha_\rho^2\rho^2 - \tfrac{1}{2}\alpha_\lambda^2\lambda^2) \tag{5}$$

in which

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2)\,, \quad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)\,,$$

and where $\alpha_i \equiv (3Km_i)^{1/4}$ with $m_\rho = m_1 = m_2 \equiv m$ and $m_\lambda = 3mm'/(2m+m')$ where $m' \equiv m_3$.

The results of the calculation are shown in Table I, broken down into components (1), (2), (3), and (4). The first column arises simply from quark counting and is given in units of

$$\delta m \equiv m_d - m_u\,. \tag{6}$$

The second column requires explicit calculation of the shift in the color hyperfine interaction

$$H_{\text{hyp}} = \sum_{i<j}\frac{16\pi\alpha_s}{9m_i m_j}\vec{S}_i\cdot\vec{S}_j\delta^3(\vec{r}_{ij})\,. \tag{7}$$

This contribution has two parts. In column (a) we show the contribution from the shift in the first-order hyperfine interaction, including the effects of wave-function distortion, in terms of

$$\delta \equiv \frac{4\alpha_s\alpha^3}{3\sqrt{2\pi}\,m_d^{\,2}} \simeq \Delta - N\,, \tag{8}$$

where

$$\alpha \equiv (3Km_d)^{1/4}\,, \tag{9}$$

and in terms of

$$x_i \equiv \frac{m_u}{m_i}\,, \tag{10}$$

and w_s, w_{ss}, w_c, w_s^*, w_{ss}^*, and w_c^* which are calculable factors (1.10, 1.05, 1.05, 1.01, 1.05, and 0.98, respectively) which take into account the small wave-function shifts due to the presence of heavy quarks. In column (b) we show the small contribution from changes in the second-order hyperfine interactions; the results of these latter calculations do not lend themselves to symbolic expression so here we show only the numerical value of the shift in units of δm. Column 3 gives the zero-point energy shift, using the perturbation

$$\Delta K \equiv -\sum_i (1-x_i)\delta_{id}\frac{p_i^{\,2}}{2m_u}\xi_\alpha\,, \tag{11}$$

where ξ_s, ξ_{ss}, and ξ_c are calculable factors (1.04, 1.08, and 1.12, respectively) which take into account the dependence of the kinetic energy on the presence of heavy quarks. The results are given in terms of $K \equiv \langle p_3^{\,2}/2m_u\rangle = \alpha^2/2m_u$ and the ξ_α. Finally columns 4(a) and 4(b) give the calculated electric and magnetic shift, respectively, using

TABLE I. Baryon isomultiplet mass shifts.

	Quark mass	Strong hyperfine (a) First order	Strong hyperfine (b) Second order	Kinetic energy	Electromagnetism (a) Electric	Electromagnetism (b) Magnetic	Total theory (MeV)	Expt (MeV)
$p-n$	$-\delta m$	$+\frac{5}{24}\delta\frac{\delta m}{m}$	$+0.08\delta m$	$+K\frac{\delta m}{m}$	$+\frac{1}{3}\epsilon$	$-\frac{1}{12}\mu$	-1.3	-1.3
$\Sigma^+-\Sigma^0$	$-\delta m$	$-\frac{5}{48}(2x_s-1)w_s\delta\frac{\delta m}{m}$	$-0.06\delta m$	$+K\xi_s\frac{\delta m}{m}$	$+\frac{1}{3}\epsilon$	$-\left(\frac{1+x}{6}\right)\mu$	-3.5	-3.1 ± 0.1
$\Sigma^--\Sigma^0$	$+\delta m$	$+\frac{5}{48}(2x_s-1)w_s\delta\frac{\delta m}{m}$	$+0.06\delta m$	$-K\xi_s\frac{\delta m}{m}$	$+\frac{1}{3}\epsilon$	$+\left(\frac{2x-1}{12}\right)\mu$	$+4.5$	$+4.9\pm0.1$
$\Sigma^++\Sigma^--2\Sigma^0$	0	0	0	0	$+\epsilon$	$-\frac{1}{4}\mu$	$+1.6$	$+1.8\pm0.2$
$\Xi^--\Xi^0$	$+\delta m$	$+\frac{5}{12}x_s w_{ss}\delta\frac{\delta m}{m}$	$+0.18\delta m$	$-K\xi_{ss}\frac{\delta m}{m}$	$+\frac{2}{3}\epsilon$	$+\frac{x}{3}\mu$	$+6.3$	$+6.4\pm0.6$
$\Delta^{++}-\Delta^0$	$-2\delta m$	$+\frac{5}{12}\delta\frac{\delta m}{m}$	$-0.18\delta m$	$+2K\frac{\delta m}{m}$	$+\frac{5}{3}\epsilon$	$-\frac{5}{12}\mu$	-3.0	-2.6 ± 0.4
$\Delta^{++}-\Delta^-$	$-3\delta m$	$+\frac{5}{8}\delta\frac{\delta m}{m}$	$-0.27\delta m$	$+3K\frac{\delta m}{m}$	$+\epsilon$	$-\frac{1}{4}\mu$	-6.9	-7.9 ± 6.8
$\Delta^+-\Delta^0$	$-\delta m$	$+\frac{5}{24}\delta\frac{\delta m}{m}$	$-0.09\delta m$	$+K\frac{\delta m}{m}$	$+\frac{1}{3}\epsilon$	$-\frac{1}{12}\mu$	-2.3	
$\Sigma^{*+}-\Sigma^{*0}$	$-\delta m$	$+\frac{5}{48}(1+x_s)w_s^*\delta\frac{\delta m}{m}$	$-0.04\delta m$	$+K\xi_s\frac{\delta m}{m}$	$+\frac{1}{3}\epsilon$	$-\left(\frac{2-x}{12}\right)\mu$	-2.0	$+0.3\pm2.6$
$\Sigma^{*-}-\Sigma^{*0}$	$+\delta m$	$-\frac{5}{48}(1+x_s)w_s^*\delta\frac{\delta m}{m}$	$+0.04\delta m$	$-K\xi_s\frac{\delta m}{m}$	$+\frac{2}{3}\epsilon$	$-\left(\frac{1+x}{12}\right)\mu$	$+3.7$	$+5.4\pm2.6$
$\Sigma^{*+}-\Sigma^{*-}$	$-2\delta m$	$+\frac{5}{24}(1+x_s)w_s^*\delta\frac{\delta m}{m}$	$-0.09\delta m$	$+2K\xi_s\frac{\delta m}{m}$	$-\frac{1}{3}\epsilon$	$+\left(\frac{2x-1}{12}\right)\mu$	-5.8	-5.1 ± 0.7
$\Xi^{*-}-\Xi^{*0}$	$+\delta m$	$-\frac{5}{24}x_s w_{ss}^*\delta\frac{\delta m}{m}$	$+0.04\delta m$	$-K\xi_{ss}\frac{\delta m}{m}$	$+\frac{2}{3}\epsilon$	$-\frac{x}{6}\mu$	$+3.8$	$+3.2\pm0.6$
$\Sigma_c^{++}-\Sigma_c^+$	$-\delta m$	$+\frac{5}{48}(1-2x_c)w_c\delta\frac{\delta m}{m}$	$-0.03\delta m$	$+K\xi_c\frac{\delta m}{m}$	$+\frac{2}{3}\epsilon+\frac{2}{3}\epsilon'$	$-\frac{\mu}{6}+\frac{\mu'}{3}x_c$	-0.2	
$\Sigma_c^+-\Sigma_c^0$	$-\delta m$	$+\frac{5}{48}(1-2x_c)w_c\delta\frac{\delta m}{m}$	$-0.03\delta m$	$+K\xi_c\frac{\delta m}{m}$	$-\frac{1}{3}\epsilon+\frac{2}{3}\epsilon'$	$+\frac{\mu}{12}+\frac{\mu'}{3}x_c$	-1.8	
$\Sigma_c^{*++}-\Sigma_c^{*+}$	$-\delta m$	$+\frac{5}{48}(1+x_c)w_c^*\delta\frac{\delta m}{m}$	$-0.03\delta m$	$+K\xi_c\frac{\delta m}{m}$	$+\frac{2}{3}\epsilon+\frac{2}{3}\epsilon'$	$-\frac{\mu}{6}-\frac{\mu'}{6}x_c$	-0.1	
$\Sigma_c^{*+}-\Sigma_c^{*0}$	$-\delta m$	$+\frac{5}{48}(1+x_c)w_c^*\delta\frac{\delta m}{m}$	$-0.03\delta m$	$+K\xi_c\frac{\delta m}{m}$	$-\frac{1}{3}\epsilon+\frac{2}{3}\epsilon'$	$+\frac{\mu}{12}-\frac{\mu'}{6}x_c$	-1.7	

$$H_{\text{em}}=\alpha_{\text{em}}\sum_{i<j}\frac{e_ie_j}{e^2}\left[\frac{1}{r_{ij}}-\tfrac{8}{3}\pi\frac{\vec{S}_i\cdot\vec{S}_j}{m_im_j}\delta^3(r_{ij})\right] \tag{12}$$

as appropriate to a nonrelativistic S-wave state. The Coulombic effects are given in terms of

$$\epsilon\equiv\left\langle\frac{\alpha_{\text{em}}}{r_{12}}\right\rangle=\left(\frac{2}{\pi}\right)^{1/2}\alpha_{\text{em}}\alpha \tag{13}$$

and $\epsilon'=1.15\epsilon$ which allows for SU(4)-breaking effects, while the magnetic effects are expressed in terms of

$$\mu\equiv\left\langle\frac{8\pi\alpha_{\text{em}}}{3m_u^2}\delta^3(\vec{r}_{12})\right\rangle=\frac{4\alpha_{\text{em}}\alpha^3}{3\sqrt{2\pi}\,m_u^2} \tag{14}$$

and $\mu'=1.50\mu$ which is once again introduced to take into account SU(4) breaking. Apart from the quantity δm which we are studying here, all parameters are known from the previously mentioned study[4] of the ground-state baryons. They are $m_u\simeq m_d=0.33$ GeV, $m_s-m_u=0.22$ GeV, $m_c-m_d=1.40$ GeV, $\delta=260$ MeV, and $\alpha=0.32$ GeV.

A good fit to the observed baryon isomultiplet splittings can be seen to emerge when we assign the value

$$\delta m=m_d-m_u=+6\text{ MeV}. \tag{15}$$

We shall adopt this value for use henceforth.

B. Meson isomultiplet mass differences

In calculating the baryon isomultiplet mass differences we relied heavily on explicitly calculating the effect of each component interaction contributing to the mass of a given state. In the case of mesons we have in most cases the luxury of foregoing this reliance on theory. In an ideally mixed nonet (we deal with charmed mesons separately below) the experimental masses of the mesons,

$$|\bar{M}_1\rangle\equiv\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}), \tag{16}$$

$$|\bar{M}_{ns}\rangle\equiv\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}), \tag{17}$$

$$|\bar{M}_{1/2}\rangle\equiv d\bar{s}, \tag{18}$$

$$|\bar{M}_s\rangle\equiv s\bar{s}, \tag{19}$$

supply us with empirical information on the behavior of the mass of a meson versus the masses of its constituent quarks, and we choose simply to exploit this information to decrease the model dependence of our results. Of course in practice nonets are not ideal; however, we shall assume, following Refs. 6, 7, and 8, that the mixing matrix is of the form

$$\begin{pmatrix} m_{u\bar{u}}+A_{uu} & A_{ud} & A_{us}\\ A_{ud} & m_{d\bar{d}}+A_{dd} & A_{ds}\\ A_{us} & A_{ds} & m_{s\bar{s}}+A_{ss}\end{pmatrix}\begin{matrix}|u\bar{u}\rangle\\ |d\bar{d}\rangle,\\ |s\bar{s}\rangle\end{matrix} \tag{20}$$

where $A_{qq'}$ (which depends on the $q\bar{q}$ quantum numbers) is the $q\bar{q}\to q'\bar{q}'$ transition amplitude which proceeds through gluon annihilation channels. We expect $A_{qq'}$ to have a rather complicated dependence on q and q' since we must at least anticipate that

$$A_{qq'}\propto[\alpha_s(m_q,m_{q'})]^n\,\frac{\psi_{q\bar{q}}(0)\psi_{q\bar{q}}(0)}{m_qm_{q'}}, \tag{21}$$

where n is the number of annihilation gluons; we respond to this complication by adopting the simple ansatz that the mass dependence of α_s and $\psi(0)$ will approximately compensate so that we may, following Ref. 8, take

$$A_{qq'}\simeq A\,\frac{m_u^2}{m_qm_{q'}}=Ax_qx_{q'}. \tag{22}$$

Apart from the case of the pseudoscalars where A is large, our results are insensitive to this assumption. When A is small, a quick look at the mass matrix (20) allows the use of the experimental values of the masses and mixing angle in a nonet to deduce the three masses $m_{u\bar{u}}\simeq m_{d\bar{d}}$, $m_{d\bar{s}}$, and $m_{s\bar{s}}$. We then assume that [in the SU(3) sector] these masses are a smooth function of the average quark mass

$$\bar{m}\equiv\tfrac{1}{2}(m_q+m_{\bar{q}'}), \tag{23}$$

and we use the three "data points" to perform a quadratic fit to the annihilation-free meson masses $\mathfrak{M}(\bar{m})$ about $\bar{m}_K=\frac{1}{2}(m_d+m_s)$. Since $\bar{m}$ differs from twice the reduced mass (the relevant variable for the binding energy) or from $(m_qm_{\bar{q}'})^{1/2}$ (the relevant variable for the hyperfine interaction) by less than 10% over the range of $\bar{m}$, our conclusions are not very sensitive to the choice of $\bar{m}$ as our interpolation parameter. The resulting fits are

$$\mathfrak{M}_P(\bar{m})\simeq 0.49+2.3(\bar{m}-\bar{m}_K)-8.0(\bar{m}-\bar{m}_K)^2, \tag{24}$$

$$\mathfrak{M}_V(\bar{m})\simeq 0.90+1.1(\bar{m}-\bar{m}_K), \tag{25}$$

$$\mathfrak{M}_T(\bar{m})\simeq 1.43+0.9(\bar{m}-\bar{m}_K)-1.8(\bar{m}-\bar{m}_K)^2, \tag{26}$$

$$\mathfrak{M}_3(\bar{m})\simeq 1.78+0.8(\bar{m}-\bar{m}_K), \tag{27}$$

where we have adopted the notation $P\leftrightarrow 0^{-+}$, $V\leftrightarrow 1^{--}$, $T\leftrightarrow 2^{++}$, and $3\leftrightarrow 3^{--}$. Only $\mathfrak{M}_P$ requires further comment. In this case the mixing is very strong so that it is not possible to identify, for example, the $s\bar{s}$ meson. We proceed in this case to find A_P and $m_{s\bar{s}}=\mathfrak{M}_P(m_s)$ by demanding that the matrix (20) yield an approximately 50-50 mixture of $|\bar{M}_{ns}\rangle$ and $|\bar{M}_s\rangle$ in the η—to give a pseudoscalar mixing angle of $-10°$ (Ref. 7)—and then adjusting A to give values for m_η and $m_{\eta'}$ as close as possible to those observed. (We obtain in this way $m_\eta = 0.50$, $m_{\eta'} = 1.02$ for $A_p \simeq 0.31$ GeV, $m_{s\bar{s}} \simeq 0.65$ GeV.) All of the formulas (24) to (27) can also be reproduced approximately by methods similar to those used for the baryons; for example, the very large curvature of $\mathfrak{M}_P$ is due to the very large hyperfine interaction component in these states. In particular our results are quite similar to those one would obtain from the explicit models of Ref. 8.

We can now apply these results to calculate isomultiplet mass differences in the SU(3) sector. For example, we have

$$m_{d\bar{s}} - m_{u\bar{s}} \simeq \frac{1}{2}\frac{d\mathfrak{M}}{d\bar{m}}(\bar{m}_K)\delta m + (m_{d\bar{s}} - m_{u\bar{s}})_{\rm em} \,. \tag{28}$$

In this way we can generate most of the first column of Table II: the "strong" contribution to meson isomultiplet splittings (we include in this column a small contribution to the $I = 1$ splittings, which would otherwise be zero, from isovector-isoscalar mixing which we calculate in the next subsection). The next three columns are the electromagnetic contributions. The first two are from the interaction (12) and are based on assuming that the meson Gaussian shape factor β analogous to the baryon parameter α of Eq. (5) is given by[3]

$$\beta = 3^{-1/4}\alpha \tag{29}$$

as expected from the QCD flux-tube model—a relation that is consistent with the ratios of the measured π^+ and p electromagnetic radii. The third column lists other known (and unknown) electromagnetic contributions. Here the π^+-π^0 mass difference requires special treatment since with $m_\pi^{-1} \sim r_\pi$ we cannot expect our static nonrelativistic calculation of the electromagnetic shifts to work; in fact we know that a fully relativistic treatment[9] [which includes (12) (see Ref. 10) along with other contributions known to be important] gives $\pi^+ - \pi^0 \simeq 5$ MeV. Of course there may also be some residue of such effects in the kaon since m_K is not that much larger than r_K^{-1}, but we may assume that they are unimportant for more massive states.

For charmed mesons we follow an exactly analogous procedure based on the values[11] of the $D^+(c\bar{d})$, $F^+(c\bar{s})$, $D^{*+}(c\bar{d})$, and $F^{*+}(c\bar{d})$ masses, once again interpolating in the average mass $\bar{m}$. To calculate the electric and magnetic contributions we use the harmonic-oscillator model to scale our results from the SU(3) sector, but our results are not very sensitive to this extrapolation because of the suppressed magnetic contribution. The results are once again shown in Table II.

Possibly as a result of the problems mentioned above, these results for mesons are considerably less conclusive than those for baryons. Nevertheless, the resultant understanding may be deemed adequate and certainly we are presented with no compelling reason here to abandon the conclusion we drew from baryons that $\delta m \simeq 6$ MeV.

C. Isovector-isoscalar mixing

In addition to contributing to isomultiplet splittings, δm will also cause isovector-isoscalar mixing in (among others) the π-η-η', ρ-ω-ϕ, A_2-f-f', g-ω_3-ω_3', and Σ-Λ systems. This mixing may easily be calculated by treating the change in the relevant mass matrix under $m_d = m_u \to m_d = m_u + \delta m$ as a perturbation. The eigenvectors of the unperturbed mass matrix may then be approximated by the physical states with their ob-

TABLE II. Meson isomultiplet mass shifts in MeV.

	Strong	Electromagnetic (a) Electric	(b) Magnetic	(c) Other	Total	Expt
$\pi^+ - \pi^0$	+0.2	+1.0	+0.5	? [a]	? [a]	+4.6
$K^+ - K^0$	−6.9	+0.7	+0.2	~0 ?	−6.0	-4.0 ± 0.1
$\rho^+ - \rho^0$	0.0	+1.0	−0.2	−1.4 [b]	−0.6	-2.4 ± 2.1
$K^{*+} - K^{*0}$	−3.3	+0.7	−0.1	~0	−2.7	-4.1 ± 0.6
$A_2^+ - A_2^0$	−0.2	+0.7	0.0	~0	+0.5	
$K_T^{*+} - K_T^{*0}$	−2.7	+0.4	0.0	~0	−2.3	-5.2 ± 3.2
$D^+ - D^0$	+4.1	+1.7	+0.2	~0	+6.0	$+5.0 \pm 0.8$
$D^{a+} - D^{a0}$	+3.0	+1.7	−0.1	~0	+4.6	$+2.6 \pm 1.8$

[a] See text.

[b] From $\rho^0 \to \gamma \to \rho^0$.

served masses and widths so that we may do simple perturbation theory in δm. We begin this time with the mesons where we must calculate

$$m_{d\bar{d}} - m_{u\bar{u}} \simeq \frac{d\mathfrak{M}}{d\bar{m}}(0.33)\delta m + (m_{d\bar{d}} - m_{u\bar{u}})_{\text{em}} . \tag{30}$$

Exploiting the fact that $(m_{d\bar{d}} - m_{u\bar{u}})_{\text{em}}$ is related to the electromagnetic part of the isovector isomultiplet splitting we find

$$(m_{d\bar{d}} - m_{u\bar{u}})_P \simeq 4.1\delta m + \tfrac{2}{3}(\pi^+ - \pi^0)_{\text{em}} \simeq 27 \text{ MeV}, \tag{31}$$

$$(m_{d\bar{d}} - m_{u\bar{u}})_V \simeq 1.1\delta m + \tfrac{2}{3}(\rho^+ - \rho^0)_{\text{em}} \simeq 6.2 \text{ MeV}, \tag{32}$$

$$(m_{d\bar{d}} - m_{u\bar{u}})_T \simeq 1.3\delta m + \tfrac{2}{3}(A_2^+ - A_2^0)_{\text{em}} \simeq 8.2 \text{ MeV}, \tag{33}$$

$$(m_{d\bar{d}} - m_{u\bar{u}})_3 \simeq 0.8\delta m + \tfrac{2}{3}(g^+ - g^0)_{\text{em}} \simeq 5.2 \text{ MeV}. \tag{34}$$

Of course there is also a perturbation of the annihilation matrix by

$$\delta A = A\frac{\delta m}{m_u}\begin{bmatrix} 0 & -1 & 0 \\ -1 & -2 & -x_s \\ 0 & -x_s & 0 \end{bmatrix}, \tag{35}$$

though this effect is negligible except for the pseudoscalars; and in the case of the vectors there is an additional contribution to the mass matrix from one photon mixing which we have already included in (32) via column 2(c) of Table II. We can now reexpress the mixing problem in each case in terms of the physical masses and isovector-isoscalar mixing matrix elements,

$$\begin{pmatrix} m_1 - i\Gamma_1 2 & m_{10} & m_{10'} \\ m_{10} & m_0 - i\Gamma_0/2 & 0 \\ m_{10'} & 0 & m_{0'} - i\Gamma_{0'}/2 \end{pmatrix} \begin{matrix} |\bar{M}_1\rangle \\ |\bar{M}_0\rangle , \\ |\bar{M}_{0'}\rangle \end{matrix} \tag{36}$$

where

$$|\bar{M}_1\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \tag{37}$$

$$|\bar{M}_0\rangle = \alpha\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) - \beta s\bar{s} + \gamma X, \tag{38}$$

$$|\bar{M}_{0'}\rangle = \alpha'\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) + \beta' s\bar{s} + \gamma' X'. \tag{39}$$

Here we have adopted the results of the more complete mixing analysis of Ref. 8 for the physical states: The X's are radial excitations mixed by annihilation into the ground states, and states are approximately ideally mixed ($\alpha \simeq \beta' \simeq 1$, $\beta = \gamma = \alpha' = \gamma' \simeq 0$) except for the pseudoscalars where $\alpha_P \simeq 0.75$, $\beta_P \simeq 0.65$, $\alpha'_P \simeq 0.45$, $\beta'_P \simeq 0.62$ (a pseudoscalar mixing angle of $-10°$ corresponds to the "perfect" mixing eigenstates $1/\sqrt{2}\,|\bar{M}_{ns}\rangle \mp 1/\sqrt{2}\,|\bar{M}_s\rangle$,[7] i.e., to $\alpha_P = \beta_P = \alpha'_P = \beta'_P = 1/\sqrt{2}$). We then find

$$P: \; m_{10} = -7.6 \text{ MeV}, \quad m_{10'} = -2.2 \text{ MeV}, \tag{40}$$

$$V: \; m_{10} = -3.1 \text{ MeV}, \quad m_{10'} \simeq 0 \text{ MeV}, \tag{41}$$

$$T: \; m_{10} = -4.1 \text{ MeV}, \quad m_{10'} \simeq 0 \text{ MeV}, \tag{42}$$

$$3: \; m_{10} = -2.6 \text{ MeV}, \quad m_{10'} \simeq 0 \text{ MeV}. \tag{43}$$

Our results for isovector-isoscalar mixing can now be given in Table III in terms of mixing angles χ and χ' defined by

$$|M_1\rangle = |\bar{M}_1\rangle - \chi|\bar{M}_0\rangle - \chi'|\bar{M}_{0'}\rangle, \tag{44}$$

$$|M_0\rangle = |\bar{M}_0\rangle + \chi|\bar{M}_1\rangle, \tag{45}$$

$$|M_{0'}\rangle = |\bar{M}_{0'}\rangle + \chi'|\bar{M}_1\rangle . \tag{46}$$

Note that: (1) χ' is in each case much smaller than χ, and (2) in the 3^{--} mesons $(m_{\omega_3} - i\Gamma_{\omega_3}/2) - (m_g - i\Gamma_9/2) = [(-20 \pm 22) + i(10 \pm 16)]$ MeV, so χ_3, though not at present calculable, should be very large.

To complete this section we turn finally to the Σ-Λ system. The unperturbed eigenstates are simply

$$\bar{\Sigma}^0 \equiv \frac{1}{\sqrt{2}}(uds + dus)\chi^\lambda\psi_{00}, \tag{47}$$

$$\bar{\Lambda}^0 \equiv \frac{1}{\sqrt{2}}(uds - dus)\chi^\rho\psi_{00}, \tag{48}$$

where ψ_{00} is given in (5) and where

$$\chi^\rho = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \tag{49}$$

$$\chi^\lambda = -\frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow). \tag{50}$$

Σ and Λ are mixed only by the strong hyperfine interactions (via its $1/m_i m_j$ dependence) and by H_{em}:

$$\langle\bar{\Lambda}^0|H_{\text{hyp}}|\bar{\Sigma}^0\rangle = -\frac{1}{2\sqrt{3}}x_s\delta\frac{\delta m}{m_u} \simeq -0.8 \text{ MeV}, \tag{51}$$

TABLE III. Isovector-Isoscalar mixing angles in radians.

	χ	χ'
P	-0.02	-0.003
V	$0.04e^{i(100°)}$	~ 0
T	$0.07e^{i(45°)}$	~ 0
3	large	~ 0
$\Sigma - \Lambda$	$\chi_{\Sigma\Lambda} \simeq +0.01$	

$$\langle\overline{\Lambda}^0|H_{\rm em}|\overline{\Sigma}^0\rangle = -\frac{1}{4\sqrt{3}}x_s\mu \simeq -0.1\ \text{MeV}. \qquad (52)$$

The resulting value for $\chi_{\Sigma\Lambda}$ defined by

$$\Sigma^0 = \overline{\Sigma}^0 - \chi_{\Sigma\Lambda}\overline{\Lambda}^0, \qquad (53)$$

$$\Lambda^0 = \overline{\Lambda}^0 + \chi_{\Sigma\Lambda}\overline{\Sigma}^0 \qquad (54)$$

is also listed in Table III. These angles will be compared with those obtained by other means in the next section. They form the basis for the sequel[2] to this work in which it is shown that they are not only consistent with established data, but they also lead to the prediction of some very large isospin-breaking effects in special circumstances.

III. DISCUSSION

In the preceding section we have presented a description of isospin violations induced by $m_d - m_u$ in the constituent quark picture. These results and some of their consequences which are discussed in the sequel[2] to this work show that where data exist the picture is reasonably consistent; we have in addition found and discussed in the sequel several systems where predictions of large isospin violations may be amenable to experimental study. Pending such study, we believe we have presented a reasonable case for a quite large value for $\delta m = m_d - m_u \simeq 6$ MeV.

Aside from the intrinsic interest of studying these isospin violations, if the value we have deduced for $\delta m_{\rm constituent}$ is correct it may be possible to relate it to the even more interesting quantity $\delta m_{\rm current}$ relevant to the mass spectrum of fundamental fermions.[12] In accord with the qualitative discussion in the Introduction, we expect that

$$m_i(Q^2) = m_i^{\rm current}(Q^2) + \mathcal{U}(Q^2), \qquad (55)$$

where $\mathcal{U}$ is the flavor-independent part of m_i which we have associated with the chromoelectric field (in current quark language, such a term must arise from spontaneous breakdown of chiral symmetry). As $Q^2 \to \infty$, the soft contribution $\mathcal{U}(Q^2)$ should vanish more rapidly than the renormalized current quark masses $m_i^{\rm current}(Q^2)$. Moreover, the Q^2 evolution of $m_i(Q^2)$ should be independent of i, at least in leading order, so we expect that

$$\Delta \equiv \frac{m_d^{\rm current}(Q^2) - m_u^{\rm current}(Q^2)}{m_s^{\rm current}(Q^2) - m_u^{\rm current}(Q^2)} \qquad (56)$$

is independent of Q^2 and hence can be evaluated at $Q^2 = Q_h^2$ appropriate to our constituent quarks:

$$\Delta = \frac{\delta m}{m_s - m_u} \simeq \tfrac{6}{220}. \qquad (57)$$

On the other hand from current algebra

$$\frac{m_d^{\rm current} + m_u^{\rm current}}{m_s^{\rm current} + m_u^{\rm current}} \simeq \frac{m_\pi^{\,2}}{m_K^{\,2}}, \qquad (58)$$

so that (using $m_u^{\rm current} \ll m_s^{\rm current}$) we find

$$\frac{m_d^{\rm current} - m_u^{\rm current}}{m_d^{\rm current} + m_u^{\rm current}} \simeq \tfrac{1}{3}. \qquad (59)$$

Moreover, we can now calculate explicitly the current quark masses at Q_h^2 since $m_i(Q_h^2) \equiv m_i^{\rm constituent} = m_i^{\rm current}(Q_h^2) + \mathcal{U}(Q_h^2)$. We find the eminently reasonable values

$$m_u^{\rm current}(Q_h^2) \simeq 6\ \text{MeV}, \qquad (60)$$

$$m_d^{\rm current}(Q_h^2) \simeq 12\ \text{MeV}, \qquad (61)$$

$$m_s^{\rm current}(Q_h^2) \simeq 230\ \text{MeV}. \qquad (62)$$

Of course for large values of Q^2, these masses should all decrease together presumably like powers of $\ln Q^2$.[12]

These estimates are all reasonably consistent with standard discussions in the current-quark framework. Of course Eq. (58) is a current-quark equation; however, our Eq. (59) is also consistent with the current-quark equation

$$\frac{m_d^{\rm current} - m_u^{\rm current}}{m_d^{\rm current} + m_u^{\rm current}} = \frac{m_{K^0}^{\,2} - m_{K^+}^{\,2} + m_{\pi 4}^{\,2} - m_{\pi^0}^{\,2}}{m_{\pi^0}^{\,2}} \simeq 0.29. \qquad (63)$$

This is a nontrivial correspondence between the two pictures.

We now turn to a comparison of our results with some analogous calculations made in broken SU(3) (Refs. 13 and 14) and various quark models[15,16]:

(a) The effects of π^0-η_8 mixing have been considered[15] for a variety of processes, based on a mixing angle

$$\Theta_{\pi^0\eta_8}^{\rm current} = -\frac{\sqrt{3}}{4}\left(\frac{m_d - m_u}{m_s}\right)_{\rm current} \simeq -0.012, \qquad (64)$$

which is certainly comparable to our $\pi\eta$ mixing angle $\chi_p \simeq -0.02$, especially when corrected for η-η' mixing.

(b) By taking an F/D ratio from SU(3) breaking in the octet, the "tadpole" contributions of $m_d - m_u$ to isomultiplet splittings can be calculated.[14] The results are reasonably good. Of course in the approach presented here the F/D ratio emerges dynamically from the strong hyperfine interactions. The same effect leads to a Σ-Λ mixing angle[13]

$$\chi_{\Sigma\Lambda}^{\rm current} = \frac{\sqrt{3}}{4}\left(\frac{m_d - m_u}{m_s}\right) \simeq 0.01 \qquad (65)$$

in perfect agreement with our calculation.

(c) There have also been several calculations recently based on a constituent-quark picture similar to the one employed here.[16] While there is generally qualitative agreement with our calculations, the various results differ from ours significantly in a quantitative comparison.

Finally we would like to repeat our belief that verification of the picture adopted here—namely, that isospin is an accidental symmetry that is not reflected in a degeneracy of the quark mass spectrum—is of fundamental importance as it bears directly on a confirmation of present views on the fermion masses as well as on future efforts to understand the spectrum of those masses.

ACKNOWLEDGMENTS

This work has benefited enormously from discussions and critical comments from I. Cohen and H. J. Lipkin who have been engaged in very similar calculations. The production of this paper was initiated by conversations with H. R. Rubinstein, A. Schwimmer, and H. J. Lipkin who are coauthors in its logical (but not temporal) sequel; their contributions to this paper were important. Very helpful conversations with H. Harari and G. Karl during the course of this work are also gratefully acknowledged. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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