PHYSICAL REVIEW D VOLUME 21, NUMBER 3 1 FEBRUARY 1980

Spectra and strong decays of $c\bar{c}$ and $b\bar{b}$ states

E. van Beveren, C. Dullemond,* and G. Rupp
Institute for Theoretical Physics, University of Nijmegen, Nijmegen, The Netherlands
(Received 12 July 1979; revised manuscript received 14 January 1980)

An outline is given for the description of heavy-quark–antiquark systems by means of a set of coupled Schrödinger equations in which permanently confined two-fermion channels are coupled to free two-boson channels. Only two-particle decays satisfying the Okubo-Zweig-Iizuka rule are taken into account. Within this context the equations make a simple determination of the spectra and decay parameters possible. A comparison with the data is presented.

I. INTRODUCTION

In this paper we present a simple potential model for the simultaneous study of the spectra and the fast two-particle decays of heavy-quark-antiquark systems. It is understood that all "fast decays" satisfy the Okubo-Zweig-Iizuka (OZI) rule.[1]

The basic idea is that under certain circumstances one can write down equations which describe physical situations to a fair degree of accuracy and which are exactly solvable despite the fact that they involve the interplay of several free and bound channels.[2] For convenience let us restrict our attention to charmonium.

In the first place, of course, if charmonium were to be considered as a system of two heavy quarks which move nonrelativistically with respect to each other and which are bound by harmonic forces, then the spectrum would be just the isotropic-harmonic-oscillator spectrum and the wave functions would be elementary functions.

Suppose that the forces between the quarks were as simple as that; then obviously modifications are still necessary to account for the large width of the resonant states which occur as soon as the threshold for $D\bar{D}$ production is passed. We want to point out that this can be done within the context of nonrelativistic Schrödinger equations and that exact solvability can be retained. It should be clear that when unapproximated solutions of a potential problem of this kind can be found, many technical difficulties resulting from large coupling strengths, the proximity of thresholds and resonances, and the necessity of carrying out high-order approximations do not appear.

In order to obtain the proper conditions for such a favorable situation one important assumption must be made: When a resonant charmonium state decays in accordance with the OZI rule, then two-particle final states like $D\bar{D}$, $D\bar{D}^*+\bar{D}D^*$, or $F\bar{F}$ will occur, while the appearing charmed bosons are to be considered as nothing else than the original charmed quarks which for the occasion are "dressed" with light quarks of the u, d, or s variety, together with their counterparts $\bar{u}$, $\bar{d}$, or $\bar{s}$. These act as spectators and have no dynamical influence whatsoever. By thus ignoring an internal structure of the bosons the 2 particle $\rightarrow$4 particle problem is reduced to a 2 particle $\rightarrow$2 particle problem which can be treated in the usual nonrelativistic way, despite the fact that the two interacting particles can change their nature fundamentally.

Let $\vec{r}$ be the relative coordinates of the c and $\bar{c}$ quarks. We shall assume that at the moment a $u\bar{u}$, $d\bar{d}$, or $s\bar{s}$ pair is created, the newly formed bosons occupy the positions of the c and $\bar{c}$ quarks and thus have $\vec{r}$ as relative coordinates. The reverse is considered to be true at the moment of annihilation of a $u\bar{u}$, $d\bar{d}$, or $s\bar{s}$ pair. It is not strictly necessary to make that assumption, but it is not unreasonable and helps in the construction of manageable equations.

We shall also make the assumption that the c and $\bar{c}$ quarks do not feel each other's presence any more as soon as they are dressed. That means that the newly formed bosons do not mutually interact; they behave as free particles, no matter how near they are to each other. Also, this assumption is not strictly necessary.

The quark pairs are created out of the vacuum and have the quantum numbers of the vacuum. The fact that they borrow neither angular momentum nor parity from the neighboring charmed quarks and that they will be assumed to be created in an exactly $SU_F(3)$-symmetric way put severe limitations on the form of the transition "force". We will allow breaking of $SU_F(3)$ symmetry only in a kinematic sense, namely when it concerns the thresholds of the $D\bar{D}$ and $F\bar{F}$ channels, etc.

Although the above-mentioned conditions lead already to reasonably manageable equations, one final assumption will be made which makes solution by approximation methods manifestly unnecessary. When the transition potential consists of a δ shell or even a combination of δ shells the only criterion for exact solvability is that the problem be completely solvable *in the absence of any transition*. But we know that the harmonic-oscillator problem as well as the free-particle problem are exactly solvable.[3] The remaining questions are partly fundamental and partly phenomenological: How reasonable are harmonic oscillators and δ shells in the study of charmonium? Certainly, combinations of δ shells can be used to approximate any potential.

If the transition potential is strong enough, it not only causes bound states to become resonances, but it also causes a mass shift. In particular, those bound states which are lying below all thresholds and thus do not become resonances are affected and may shift in a way which make it less obvious that they belong to a harmonic-oscillator spectrum.

In Sec. II the harmonic oscillator is discussed. The model is presented in Sec. III, followed by an account for the specific form of the decay potential in Sec. IV. Comparison with the data is made in Secs. V and VI, where also leptonic decays are considered.

II. WHY A HARMONIC OSCILLATOR?

Let us first inspect the spectra: The $J^{PC}=1^{--}$ charmonium states are sufficiently equidistant in order to be interpreted as radial and angular excitations of a system of two heavy quarks which are bound by harmonic forces. We estimate for the energy-level separation of the harmonic oscillator a value of

$$2\omega = 0.35 \text{ GeV} \tag{2.1}$$

and for the position of the ground state

$$M = 2m_c + \tfrac{3}{2}\omega = 3.38 \text{ GeV}.$$

The comparison of the spectra is depicted in Fig. 1.

The $\psi(3100)$ bound state does not fit in this scheme. We will show, however, that this may be attributed to the considerable influence of the open charm channels.

It is attractive to treat the $b\bar{b}$ states in a similar way. Comparison of corresponding states in $c\bar{c}$ and $b\bar{b}$ states shows that the mass splittings in both spectra are very nearly equal (see Fig. 2). From this important observation we conclude that these level splittings are independent of the heavy-quark masses. This leads us to the introduction of a universal (i.e., independent of the flavor mass) oscillator frequency ω. It emphasizes that *a priori* a harmonic-oscillator potential cannot be ruled out on phenomenological grounds.

Moreover, the harmonic-oscillator potential for heavy quarks might very well be fundamental

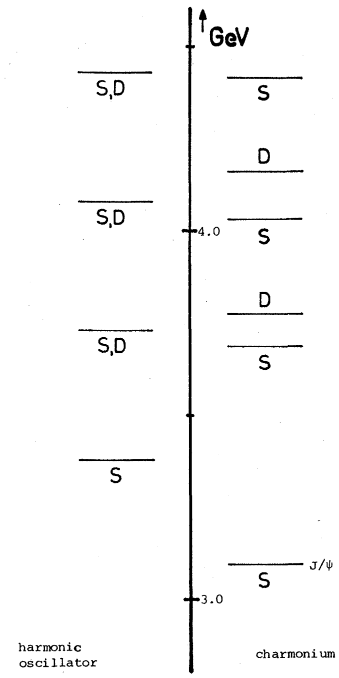


FIG. 1. The spectra of a harmonic oscillator ($2\omega = 0.35$ GeV, ground state at 3.38 GeV) and of charmonium (Ref. 4).

because various nonperturbative theories do predict harmonic forces between heavy quarks at short distances. For instance in nonperturbative quantum chromodynamics (QCD) the vacuum structure resulting from instanton solutions leads to a quark-antiquark potential of the form $V(r) \sim r^2$ (Ref. 5). A similar potential turns up in other nonperturbative gauge theories.[6]

Of course we noticed the success in describing

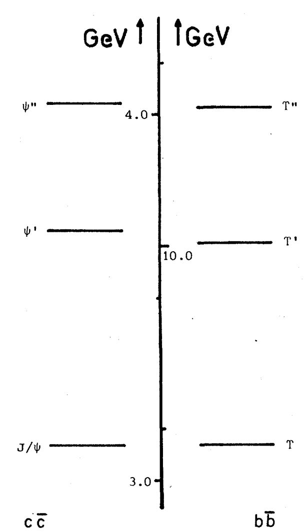


FIG. 2. Corresponding $c\bar{c}$ and $b\bar{b}$ states (Ref. 4).

charmonium of semiphenomenological potentials based on perturbative calculations in QCD,[7] and many of the ideas put forward there are adopted in the present model. Nevertheless, it is our opinion that it is worthwhile to study the possibility of harmonic forces in heavy-quark physics.

III. THE MODEL

Consider a system of n permanently confined channels in interaction with m free channels. In order to make the model simultaneously applicable to ψ, $\Upsilon, \ldots$, we introduce the invariants ω and

$$\rho_0 = \sqrt{\mu\omega}R\,, \tag{3.1}$$

where $\mu = \frac{1}{2}m_q$ is the reduced heavy quark mass and where R is the position of the δ-shell coupling of confined and free channels. Furthermore, we define the following quantities: a dimensionless distance

$$\rho = \sqrt{\mu\omega}r\,; \tag{3.2}$$

a reduced mass matrix

$$M = \begin{pmatrix} M_c & 0 \\ 0 & M_f \end{pmatrix}, \tag{3.3}$$

where M_c equals μ times I_n (the $n\times n$ unit matrix) and where $(M_f)_{ij} = m_i\delta_{ij}$, the diagonal elements m_j $(j=1,\ldots,m)$ being the reduced boson masses in the free channels; an orbital angular momentum matrix

$$L = \begin{pmatrix} L_c & 0 \\ 0 & L_f \end{pmatrix}, \tag{3.4}$$

with L_c and L_f diagonal $n\times n$ and $m\times m$ matrices, respectively, which contain the orbital angular momenta of the $n+m$ channels.

$$V = \begin{pmatrix} V_c & V_{\text{int}} \\ V_{\text{int}}^T & V_f \end{pmatrix} \tag{3.5}$$

represents the potential matrix as it appears in the set of coupled Schrödinger equations to be defined below. In (3.5) V_c is the potential in the confined channels

$$V_c = (\tfrac{1}{2}\omega\rho^2 + C)I_n\,. \tag{3.6}$$

V_f is the diagonal threshold matrix. $(V_f)_{jj} = D_j$ is the sum of the rest masses of the bosons in the jth channel.

The specific form of the off-diagonal term in (3.5) will be discussed in the next section. We have chosen, with (3.1) the form

$$V_{\text{int}} = g\frac{\omega}{\rho_0}\delta(\rho-\rho_0)\overline{V}_{\text{int}}\,, \tag{3.7}$$

where $\overline{V}_{\text{int}}$ is a real $n\times m$ matrix, independent of ρ.

Let $\phi(\rho)$ be a radial $(n+m)$-component wave function; then the set of radial Schrödinger equations to be solved is, with (3.2)–(3.5),

$$\left\{\tfrac{1}{2}\mu\omega M^{-1}\left[-\frac{d^2}{d\rho^2} + \frac{L(L+1)}{\rho^2}\right] + V(\rho)\right\}\phi(\rho) = E\phi(\rho)\,. \tag{3.8}$$

There are $2(n+m)$ independent solutions. The physical solutions must satisfy $n+m$ boundary conditions at the origin and an additional n boundary conditions at infinity for those components which belong to the permanently confined channels. No other boundary conditions are to be imposed as long as the energy is above all thresholds. In that case there are m independent physical solutions which can be found in a straightforward way and lead to an $m\times m$ unitary and symmetric S matrix.

Let us define the following diagonal matrices: a dimensionless $m\times m$ momentum matrix

$$Q = [2M_f(E - V_f)/\mu\omega]^{1/2} \tag{3.9}$$

with positive elements Q_{jj}, an $m\times m$ velocity matrix

$$v = \sqrt{\mu\omega}M_f^{-1}Q\,, \tag{3.10}$$

an $n\times n$ radial quantum number matrix s with (3.4) and (3.8) given by

$$E = \omega(2s + L_c + \tfrac{3}{2}) + C\,, \tag{3.11}$$

the $n\times n$ confluent hypergeometric function matrices

$$\Phi(-s, L_c + \tfrac{3}{2}; \rho_0^2) \text{ and } \Psi(-s, L_c+\tfrac{3}{2}; \rho_0^2)\,, \tag{3.12}$$

which are self-evident generalizations of the Φ and Ψ functions defined in Ref. 8, similar generalizations of the spherical Bessel, Neumann, and Hankel functions

$$J = Q\rho_0 j_{L_f}(Q\rho_0)\,,\quad N = Q\rho_0 n_{L_f}(Q\rho_0)\,,\quad H^{(1,2)} = J \pm iN\,, \tag{3.13}$$

and the $n\times n$ matrix

$$\mathfrak{a} = \tfrac{1}{2}\rho_0^{2L_c+1}e^{-\rho_0^2}\Gamma(-s)\frac{\Phi\Psi}{\Gamma(L_c+\frac{3}{2})}\,. \tag{3.14}$$

Then with (3.8)–(3.14) the expression for the S matrix becomes

$$S = -v^{1/2}\left(4g^2J\frac{M_f}{\mu}\overline{V}_{\text{int}}^T\mathfrak{a}\overline{V}_{\text{int}}H^{(1)} + iQ\rho_0\right)^{-1} \times\left(4g^2J\frac{M_f}{\mu}\overline{V}_{\text{int}}^T\mathfrak{a}\overline{V}_{\text{int}}H^{(2)} - iQ\rho_0\right)v^{-1/2}\,. \tag{3.15}$$

Although this is not manifestly so, the expression (3.15) is unitary and symmetric.[9]

Since $S - I$ contains the factor $\overline{V}_{\text{int}}^T\mathfrak{a}\overline{V}_{\text{int}}$, the rank of this matrix is smaller than or equal to min(m, n). This is an important observation because it means that the maximum number of nontrivial eigenphase shifts is equal to the number of permanently closed channels, if this happens to be smaller than the number of free channels to which they are coupled.

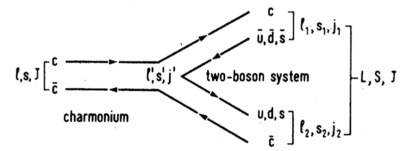
When E is smaller than some of the D_j, then the corresponding Q_{jj} as defined in (3.9) become purely imaginary. In order to satisfy additional boundary conditions at infinity for those components of the wave function which belong to the additional closed channels, the imaginary parts of these Q_{jj} should be chosen positive. If these values are substituted in (3.15), then the submatrix of S corresponding to the remaining open channels is again unitary and takes over the function of the S matrix.

For the study of bound and resonant states it is important to find the positions of the poles in S. This matrix is then to be considered as an analytic function of E. By taking E complex, the unitarity of S is lost, but not its symmetry.

IV. THE DECAY MECHANISM

In choosing for the decay potential the specific form (3.7), we are led by phenomenological and dimensional arguments. In the first place since the spectra of $c\bar{c}$ and $b\bar{b}$ states are very similar (see Fig. 2), it is necessary to construct an S matrix, the singularity structure of which is independent of μ for $\mu\to\infty$. It is easy to check that this is the case for the form (3.15). Furthermore, we prefer the quantities which compose $V_{\rm int}$ (3.7) to be invariants or dimensionless.

For the case of charmonium decay under the assumptions as stated before, the form of the $n\times m$ matrix $\overline{V}_{\rm int}$ can be determined from the following diagram[10]:



where

l = orbital angular momentum of $c\bar{c}$;

s = total spin of $c\bar{c}=0$ or 1;

l' = orbital angular momentum of $u\bar{u}$, $d\bar{d}$, or $s\bar{s}=1$;

s' = total spin of $u\bar{u}$, $d\bar{d}$, or $s\bar{s}=1$;

l_1 = orbital angular momentum of $c\bar{u}$, $c\bar{d}$, or $c\bar{s}=0$;

s_1 = total spin of $c\bar{u}$, $c\bar{d}$, or $c\bar{s}=0$ or 1;

l_2 = orbital angular momentum of $\bar{c}u$, $\bar{c}d$, or $\bar{c}d=0$;

s_2 = total spin of $\bar{c}u$, $\bar{c}d$, or $\bar{c}s=0$ or 1;

L = orbital angular momentum of final bosons;

S = total spin of final bosons;

J = total angular momentum;

j' = total angular momentum of $u\bar{u}$, $d\bar{d}$, or $s\bar{s}=0$;

j_1 = total angular momentum of $c\bar{u}$, $c\bar{d}$, or $c\bar{s}=s_1$;

j_2 = total angular momentum of $\bar{c}u$, $\bar{c}d$, or $\bar{c}s=s_2$.

The matrix elements of $\overline{V}_{\rm int}$ can now be computed and turn out to be the following:

$$\langle J, L, S, J_z, (s_1, s_2)|\overline{V}_{\rm int}|J, l, s, J_z\rangle = (-1)^{L-S+J+s_1-s_2+s} C^{J\ 0\ J}_{J_z 0 J_z} C^{l\,1\,L}_{0\,0\,0}\left[\frac{3(2l+1)}{4\pi(2L+1)}\right]^{1/2}\begin{pmatrix} l & s & J \\ 1 & 1 & 0 \\ L & S & J\end{pmatrix}\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & s \\ \frac{1}{2} & \frac{1}{2} & 1 \\ s_1 & s_2 & S\end{pmatrix} \tag{4.1}$$

independent of the channels $u\bar{u}$, $d\bar{d}$, or $s\bar{s}$. Here the C are Clebsch-Gordan coefficients and the expressions between square brackets are 9-j symbols as defined in Ref. 11.

At most, two confining channels are coupled to the boson channels, of which there are in general more than two. One therefore finds at most two nontrivial eigenphase shifts, the cotangents of which can be found by solving a quadratic equation.

Poles in the S matrix can be found by looking for zeros in the expression

$$\det(I_m - i\tan\Delta), \tag{4.2}$$

where the phase-shift matrix Δ is defined by

$$S=e^{2i\Delta}. \tag{4.3}$$

In the limit $g\to 0$ the pole positions approach the harmonic-oscillator bound states or else go to infinity. The "physical" poles can be traced by starting from the bound-state positions and slowly turning on the coupling constant. This tracing can be done by applying a linear or quadratic Newton method for finding the zeros in expression (4.2). It is only here that the use of a computer is strictly necessary.

The bound states which become resonances correspond to poles whose positions are moved into the unphysical energy plane in order to satisfy unitarity. However, the bound states which are lying below all thresholds can only move on the real axis and do in fact shift to lower energies. These shifts can be appreciable.

V. THE DATA

In the case of $c\bar{c}$ and $b\bar{b}$ $J^{PC}=1^{--}$ states the radial wave functions $\phi(\rho)$ consist of $2+10$ components. If we label the various channels by means of their quantum numbers $|L, S, (s_1, s_2)\rangle$, we have the two confined channels $|0, 1\rangle$ and $|2, 1\rangle$, the five nonstrange free channels $|1, 0, 0, 0\rangle$, $|1, 1, 0, 1\rangle$, $|1, 0, 1, 1\rangle$, $|1, 2, 1, 1\rangle$, and $|3, 2, 1, 1\rangle$, and a similar set of strange free channels.

The six different thresholds follow for the open $c\bar{c}$ channels from the data.[4] Owing to lack of data, for the open $b\bar{b}$ channels we have chosen a value of 10.5 GeV for the lowest threshold and the same separation between the various thresholds as for charmonium.

The parameter C in Eq. (3.6) is determined by the rest masses of the two heavy quarks, i.e.,

$$C=4\mu . \tag{5.1}$$

The other parameters ω, ρ_0, g^2, and μ are determined from an overall fit to the charmonium data.

We obtain for the universal frequency

$$\omega=0.178 \text{ GeV} . \tag{5.2}$$

This is very near to the value we already estimated in Sec. II. The invariant δ-shell radius

$$\rho_0=0.50 \tag{5.3}$$

is somewhat smaller than the average radius of the ground state of the unperturbed harmonic oscillator

$$(\langle\rho^2\rangle_{s=0})^{1/2} \simeq 0.7 .$$

The result for the coupling constant g in (3.7) is

$$\frac{g^2}{4\pi}=2.09 . \tag{5.4}$$

The mass of a charmed quark appears to be

$$m_c=2\mu_c=1.60 \text{ GeV} , \tag{5.5}$$

about $\frac{1}{2}$ the mass of J/ψ. For the Υ states the only parameter to be adjusted is the mass of the b quark, which comes out to be equal to

$$m_b=2\mu_b=4.76 \text{ GeV} . \tag{5.6}$$

In Table I the real parts of the computed pole positions are compared with the masses of the six established 1^{--} ψ resonances and the three known Υ resonances.

If we use the same procedure for the $t\bar{t}$ states, we predict a similar spectrum as for the other $q\bar{q}$ states only shifted to much higher energies.

VI. THE LEPTONIC DECAY WIDTHS

This section is devoted to the leptonic decay widths of the $\psi(3100)$ and the $\psi'(3685)$, which have become popular in discriminating between the various models for charmonium.[12] For instance, ordinary harmonic-oscillator models do not reproduce these widths. One can easily see this from the Van Royen–Weisskopf formula for the electromagnetic decay width of S waves, which reads[13]

$$\Gamma(\psi_n\to e^+e^-)=16\pi(\tfrac{2}{3}\alpha)^2\frac{|\psi_n(0)|^2}{M_n^2}, \tag{6.1}$$

where $\psi_n(0)$ and M_n are the wave function at the origin and the mass of the nth radial excitation, respectively, and where α is the electromagnetic fine-structure constant. In (6.1) the factor $\frac{2}{3}$ comes from the electric charge of a charmed quark; a factor 3 for its color degrees of freedom has also been taken into account. If we calculate the decay width for the ground state of the harmonic oscillator using (6.1), we obtain for the

TABLE I. Predicted and experimental masses of $J^{PC}=1^{--}$ $c\bar{c}$ and $b\bar{b}$ levels for the parameters (5.2)–(5.5). [We use the spectroscopic notation $n^{2S+1}L_J$, where n is the number of radial nodes plus one, S is the total $q\bar{q}$ spin (0 or 1), L is the orbital angular momentum of the $q\bar{q}$ system, and J is the total angular momentum of the state.]

	Level / Mass (GeV)	1^3S_1	2^3S_1	3^3S_1	4^3S_1	1^3D_1	2^3D_1
$c\bar{c}$	Predicted	3.08	3.66	4.04	4.42	3.80	4.14
	Expt. (Ref. 4)	3.095	3.684	4.03	4.414	3.772	4.16
$b\bar{b}$	Predicted	9.50	10.00	10.39	10.77	10.14	10.48
	Expt. (Ref. 4)	9.46	10.01	10.41	...	...	...

parameters (5.2)–(5.5) a value of 1.1 keV, which must be compared to 4.8 keV being the experimental width of the J/ψ.[4] For the first radial excitation we obtain similarly a value of 1.2 keV, whereas the experimental width of the $\psi'(3685)$ equals 2.1 keV.[4]

In their theoretical investigation on the behavior of the wave function at the origin,[12] Grosse and Martin show that convex potentials like the harmonic-oscillator potential yield $|\psi_1(0)| \geq |\psi_0(0)|$. Thus convex potentials are ruled out because the data require $|\psi_1(0)| < |\psi_0(0)|$.

The situation is, however, noticeably different if we couple open charm channels to the $c\bar{c}$ channel. We find, for instance, in the S-wave $c\bar{c}$ channel of the lowest bound state a quark distribution $\psi_0^\dagger(\vec{r})\psi_0(\vec{r})$ which is almost constant inside the δ shell and which drops very fast to zero outside the δ shell in contrast to the exponential falloff in the case of an ordinary harmonic-oscillator potential. This is, of course, a reflection of the breaking mechanism of the string. If at some length the string has the possibility to break, then there is less chance to find a larger quark separation. So it is no surprise that also the leptonic decay width is different in this case. If we neglect the contribution of all other channels, we obtain with (6.1)

$$\Gamma(\psi(3100) \to e^+e^-) = 3.2 \text{ keV}.$$

In fact, we can estimate that the contribution of the other channels is of little importance owing to the small quark content of these channels.

For the first radial excitation the situation is quite different. The breaking mechanism of the string has the effect that of the two regions in the quark distribution the region outside the node becomes more pronounced which in its turn causes a decrease of the wave function at the origin. The result is $|\psi_1(0)| < |\psi_0(0)|$ for the parameters (5.2)–(5.5). In the language of Grosse and Martin we might state that owing to the coupling to free channels the effective potential in the confined channels has become concave instead of convex for the bound states of charmonium.

The ratio of the leptonic decay widths is not in agreement with experiment. The reason for this failure might be that for charmonium the description of the string-breaking mechanism by means of a δ shell is a bit too far from reality, since the breaking point may be subject to some spreading. However, this certainly depends on the quark mass in such a way that for heavier masses the spreading is less. As a test we also compare the predicted leptonic decay widths of the Υ system with experiment. Our result is

$$\Gamma(\Upsilon(9.46) \to e^+e^-)/\Gamma(\Upsilon'(10.01) \to e^+e^-) = 3.9 .$$

This value is in agreement with experiment.[4]

VII. CONCLUSION

Even in its rather primitive form our model passes the test with experiment. In the near future we hope to present a more realistic form of the decay potential with several δ shells.

The above model also offers a starting point for the application of perturbation theory. For example, the influence of spin-spin and spin-orbit terms in the potential can be studied to any desired order.

ACKNOWLEDGMENT

We wish to thank Dr. T. A. Rijken for many helpful discussions.

*Temporary address until 13 August 1980: University of Rochester, Rochester, N. Y. 14627.

[1]S. Okubo, Phys. Lett. 5, 105 (1963); G. Zweig, CERN Report No. TH-401, 412, 1964 (unpublished); J. Iizuka, K. Okada, and O. Shito, Prog. Theor. Phys. 35, 1061 (1966).

[2]C. Dullemond and E. van Beveren, Ann. Phys. (N.Y.) 105, 318 (1977).

[3]A distinction must be made between the solvability of a Schrödinger equation for given energy independent of the boundary conditions and the solvability of an eigenvalue equation for the energy. The latter problem is intimately connected with the problem of finding the positions of poles in a scattering amplitude which also in our case cannot be written in the form of a closed expression involving elementary functions.

[4]Particle Data Group, Phys. Lett. 75B, 1 (1978); DASP Collaboration, R. Brandelik *et al.*, *ibid.* 76B, 361 (1978); K. Ueno *et al.*, Phys. Rev. Lett. 42, 486 (1979); G. Flügge, DESY Report No. DESY 79/26, 1979 (unpublished).

[5]C. G. Callan, R. Dashen, and D. J. Gross, Phys. Rev. D 17, 2717 (1978).

[6]A. Salam and J. Strathdee, Phys. Rev. D 18, 1323 (1978); E. van Beveren, C. Dullemond, and T. A. Rijken, Nijmegen Report No. THEF-NYM-79.11, 1979 (unpublished).

[7]E. Eichten *et al.*, Phys. Rev. D 17, 3090 (1978); V. A. Novikov *et al.*, Phys. Rep. 41C, 1 (1978).

[8]Bateman Manuscript Project, *Higher Transcendental Functions*, edited by A. Erdelyi *et al.* (McGraw-Hill, New York, 1953), Vol. 1, Chap. VI.

[9]Unitarity and symmetry can be proved in general for such a system, not only when δ-shell potentials are used. The proof involves the use of Wronskian relations. See, e.g., J. J. de Swart and C. Dullemond, Ann. Phys. (N.Y.) 16, 263 (1961).

[10]L. Micu, Nucl. Phys. B10, 521 (1969).
[11]M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957).
[12]H. Grosse and A. Martin, CERN Report No. TH-2674, 1979 (unpublished).
[13]R. Van Royen and V. F. Weisskopf, Nuovo Cimento 50A, 617 (1967).