$N \to \Delta(1232)$ electromagnetic transition form factor and pion-nucleon dynamics at moderate energies

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The dependence of the electromagnetic $N \to \Delta(1232)$ transition form factor $G_M^*(q^2)$ on q^2, the four-momentum transfer squared, has been calculated with the use of relativistic dispersion relations supplemented with some dynamical assumptions. In the first place, they regard the phase of the magnetic dipole amplitude of electroproduction of pions on nucleons in the p_{33} final state beyond the region of elastic unitarity. Namely, over the range from the lowest inelastic threshold up to 1780 MeV pion-nucleon c.m. energy, the phase in question has been identified with the real part of the respective phase shift of pion-nucleon scattering. Secondly, contributions to the dispersion integral from the higher energy region have been neglected. Finally, the polynomial ambiguity which appears in the problem has been fixed by requiring that the foregoing amplitude of electroproduction vanishes, independently of q^2, at the upper end of the integration interval as defined above. These assumptions which preserve unitarity were shown previously to lead to very good results when applied to the calculation of the multipole amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ of photopion production on nucleons in the $\Delta(1232)$ region. Now it is also shown that $G_M^*(q^2)$ calculated in that fashion follows remarkably well the data over the whole range $0 \le q^2 \le 2.5$ (GeV/c)^2 currently covered by quantitative experimental studies. Some speculation concerning a possible dynamical rooting of the foregoing assumptions is presented.

I. INTRODUCTION

The multipole analysis of electroproduction of pions on nucleons in the region of the first resonance $\Delta(1232)$ is at present still in a preliminary stage.[1] The existing data permit the extraction of information on the multipole amplitudes of the transition $N \to \Delta(1232)$ only under constraints relying usually on our past knowledge of photoproduction amplitudes. The assumption usually made for this purpose is that the amplitude $M_{1+}^{(3/2)}$ in the first resonance region is dominating so strongly that all terms not containing $M_{1+}^{(3/2)}$ can be neglected in multipole decomposition of the measurable quantities.

The values of $M_{1+}^{(3/2)}$ at resonance obtained in this fashion by several groups[2] are consistent, abundant enough, and covering a sufficiently large range of q^2, the squared four-momentum of the virtual photon,[3] to allow inference of the behavior of $G_M^*(q^2)$, the electromagnetic $N \to \Delta(1232)$ form factor of the magnetic dipole transition.

A conspicuous feature of the data is that the fall of $G_M^*(q^2)$ with q^2 is faster than that of the nucleon electromagnetic form factor approximated by the dipole formula $G_D(q^2) = [1 + q^2/0.71\ (\text{GeV}/c)^2]^{-2}$. This feature is nontrivial since e.g. the SU(6)-symmetry scheme predicts proportionality of the two form factors[4] following from the circumstance that N and Δ belong to the same 56-plet of SU(6).

There exist a number of more refined calculations[5-10] mostly making use of dispersion relations. From among them, the paper of Gutbrod and Simon[7] merits attention in view of the fine agreement between their prediction and the data of Galster *et al.*[2] and Köbberling *et al.*[2] over a wide range $0 \le q^2 \le 1.5$ (GeV/c)^2 and also because of some relevance their research has to the present calculation. In what concerns studies limited to small values of q^2, they are, unless expanded, not fully indicative of the possibilities of the dispersion formalism in predicting the shape of $G_M^*(q^2)$. Recent ambitious attempts to parametrize all electromagnetic $N \to N^*$ transition form factors[11] represent a different line of approach and need be mentioned here only incidentally.

The purpose of the present paper is to show that well established dispersion relations supplemented with some dynamical information are able to yield $G_M^*(q^2)$ which agrees extraordinarily well with the data over the whole range of q^2 now covered by experimental analysis, i.e., $0 \le q^2 \le 2.5$ (GeV/c)^2.

The dynamical information in question comprises in the first place the postulated identification of the multipole phase beyond the range of elastic unitarity with the real part of the p_{33} phase shift of pion-nucleon scattering, and in the second, far more important place, the postulated vanishing of multipole amplitudes of photoproduction and electroproduction of pions on nucleons in the p_{33} state at the energy where the multipole phase equals π. This is the way of fixing the Castillejo-Dalitz-Dyson (CDD) (polynomial) ambiguity[12] appearing in the final solution to the integral equation.

The relevance of this ambiguity to photoproduction and electroproduction calculations was emphasized a long time ago.[9,13,14] A big problem faced since then has always been how to find dynamical motivation for a chosen way of fixing the ambiguity. So far, not only the problem itself remains unsolved, but fixing devices themselves (e.g. a demand of reasonable threshold behavior[13,14]), which in general turn out to work in the case of the multipole $M_{1+}^{(3/2)}$ of photoproduction, fail as a rule when applied to the multipole $E_{1+}^{(3/2)}$ of photoproduction[9,13] or to $M_{1+}^{(3/2)}$ of electroproduction[14] even limited to small q^2.

It is also worth emphasizing that Gutbrod and Simon,[7] whose study represents a somewhat different line of approach, have nicely predicted $G_M^*(q^2)$ (as mentioned previously), but using the same "driving force" built up of box diagrams did not succeed in properly calculating the multipole $E_{1+}^{(3/2)}$ of photoproduction.

In contrast, the main point about the way of fixing the CDD ambiguity as proposed in the present study is that it is common to this and the related calculations. For this reason the conclusions drawn in this paper should be considered in association with those regarding the multipoles $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ of photoproduction[15,16] calculated earlier in exactly the same fashion.

II. THE INTEGRAL EQUATION AND THE MULTIPOLE PHASE

The electromagnetic $N \to \Delta(1232)$ transition form factor is customarily defined[2] through the expression

$$[G_M^*(q^2)]^2 = \frac{4p_\Delta m^2 \Gamma}{\alpha |\vec{q}_\Delta|^2} \left| M_{1+}^{(3/2)}(M, q^2) \right|^2, \tag{1}$$

where α denotes the electromagnetic fine-structure constant,

$$p_\Delta = \{[(M-m)^2 - \mu^2][(M+m)^2 - \mu^2]\}^{1/2}/2M,$$
$$|\vec{q}_\Delta|^2 = \{[(M-m)^2 + q^2][(M+m)^2 + q^2]\}^{1/2}/2M,$$

while m, μ, M, Γ, denote, respectively, the masses of N and π and the position and width of $\Delta(1232)$.

In order to calculate the magnetic dipole amplitude $M_{1+}^{(3/2)}$ of pion electroproduction we start from the well known integral equation

$$M_{\frac{3}{2}}^{(3)}(v,q^2)\exp[-i\varphi_{\frac{3}{2}}^{(3)}(v,q^2)]\cos\varphi_{\frac{3}{2}}^{(3)}(v,q^2) = B_{\frac{3}{2}}^{(3)}(v,q^2) + \frac{1}{\pi}\int_{v_0}^{\infty}\frac{dv'}{v'-v}\exp[-i\varphi_{\frac{3}{2}}^{(3)}(v',q^2)]\sin\varphi_{\frac{3}{2}}^{(3)}(v',q^2)M_{\frac{3}{2}}^{(3)}(v',q^2), \tag{2}$$

where v denotes a variable depending only on $\sqrt{s} = W$, the energy of the pion-nucleon system in its c.m. frame, v_0 being the threshold value. In the actual calculation $v = (s - m^2 - \mu^2)^2/(4m^2) - \mu^2$. The function $M_{\frac{3}{2}}^{(3)}(v,q^2)$ denotes the multipole amplitude $M_{1+}^{(3/2)}$ multiplied by a suitable kinematic threshold factor, while $\varphi_{\frac{3}{2}}^{(3)}(v,q^2)$ denotes the multipole phase. The function $B_{\frac{3}{2}}^{(3)}(v,q^2)$ represents the multipole projection of one-particle terms. These comprise the minimal gauge-invariant set corresponding to nucleon exchange in the s and u channels and pion exchange in the t channel. No attempt was made to include vector-meson exchange in the present calculation.

Specifically

$$B_{\frac{3}{2}}^{(3)}(v,q^2) = \frac{g\sqrt{4\pi\alpha}}{4mE_2(E_1+m)(W+m)}\left\{G_M^V(q^2)\left[m(E_1+m)(W-m)\frac{Q_1(z_1)}{p^2+|\vec{q}|^2} - \frac{Q_2(z_1)-Q_0(z_1)}{p|\vec{q}|} - \frac{m(W+m)}{E_2+m}\frac{Q_2(z_1)}{p|\vec{q}|}\right]\right.$$
$$\left. -2mF_\pi(q^2)\frac{Q_2(z_2)-Q_0(z_2)}{3p|\vec{q}|} + \frac{G_E^V(q^2)-G_M^V(q^2)}{1+q^2/(4m^2)}(W-m)\frac{Q_2(z_1)-Q_0(z_1)}{3p|\vec{q}|}\right\}. \tag{3}$$

In the foregoing equation g denotes the π-N coupling constant,

$$p = \{[(W-m)^2 - \mu^2][(W+m)^2 - \mu^2]\}^{1/2}/(2W), \quad |\vec{q}| = \{[(W-m)^2 + q^2][(W+m)^2 + q^2]\}^{1/2}/(2W),$$
$$E_1 = (q^2+m^2)^{1/2}, \quad E_2 = (p^2+m^2)^{1/2}, \quad z_1 = (2E_2 q_0 + q^2)/(2|\vec{q}|p), \quad z_2 = (2E_\pi q_0 + q^2)/(2|\vec{q}|p),$$

with

$$E = (p^2+\mu^2)^{1/2} \text{ and } q_0 = (|\vec{q}|^2 - q^2)^{1/2}.$$

The symbol

$$G_{E,M}^V(q^2) = \tfrac{1}{2}[G_{E,M}^p(q^2) - G_{E,M}^n(q^2)]$$

denotes the electric or magnetic isovector form factor of the nucleon. The functions $Q_l(z)$ are Legendre functions of the second kind.

The q^2 dependence of the proton electric form factor $G_E^p(q^2)$ was reproduced in the present calculation by the well-known dipole formula $G_E^p(q^2) = (1 + q^2/m_V^2)^{-2}$ with $m_V = 0.84$ GeV/c. For the proton and neutron magnetic form factors $G_M^{p,n}(q^2)$ the usual scaling formula $G_M^{p,n}(q^2) = \mu_{p,n} G_E^p(q^2)$ has been adopted, where $\mu_{p,n}$ denotes the anomalous magnetic moment of the proton or neutron.

The electric form factor of the neutron $G_E^n(q^2)$ is still poorly known. The fit of Bartoli *et al.*,[17]

$$G_E^n(q^2) = \mu_n q^2 (4m_n^2 + 5.6q^2)^{-2} G_E^p(q^2),$$

where m_n denotes the neutron mass, has been used in the present calculation. This choice, however, has no essential bearing on the results. The dependence of the pion form factor $F_\pi(q^2)$ on q^2 has been approximated by the single pole formula[18] $F_\pi(q^2) = (1+q^2/m_s^2)^{-1}$ where $m_s = 0.686$ GeV/c.

One should emphasize here the otherwise obvious, yet important, fact that the amplitude resulting from Eq. (2) satisfies the unitarity requirement.

Actually, the right-hand side of Eq. (2) represents only the so-called characteristic part of the complete integral equation for $M_{1+}^{(3/2)}$. It has always been widely accepted that nonsingular crossing terms representing the coupling of $M_{1+}^{(3/2)}$ to all the other multipoles can be safely neglected in that equation. In what concerns the nonsingular self-coupling term, it has been shown[15] that including it does not lead to essential changes in the final result in the case of photoproduction. There are good reasons to expect[19] that the same conclusion holds in the case of electroproduction. This allows for substantial simplification in the procedure of solving the integral equation for $M_2^{(3)}(v, q^2)$.

Although the foregoing reductions are partly of dynamical character it is through assumptions concerning the multipole phase $\varphi_2^{(3)}(v, q^2)$ that the lion's share of dynamics is brought into the integral equation. Since these assumptions have been repeatedly exposed and discussed in detail elsewhere[15] we give here only a brief account of their content:

(i) The elastic unitarity condition $\varphi_2^{(3)}(v, q^2) = \delta_{33}(v)$, where $\delta_{33}(v)$ denotes the phase shift of pion-nucleon scattering in the p_{33} state, is assumed to hold effectively up to $\sqrt{s} \cong 1500$ MeV.

(ii) Above the foregoing energy range the phase is assumed to be independent of q^2 as before. It is extrapolated smoothly in such a way that the expression $\exp[i\varphi_2^{(3)}(v)] \sin\varphi_2^{(3)}(v)$ would match the pion-nucleon scattering amplitude if inelasticity were neglected in the latter. The phase $\varphi_2^{(3)}(v)$ defined in this fashion reaches the value of π at v_c corresponding to $\sqrt{s} \cong 1780$ MeV.

(iii) The range of variation of $\varphi_2^{(3)}(v)$ as assumed above gives rise to a CDD ambiguity[12] in the solution of Eq. (2). The ambiguity is resolved by requiring that $M_2^{(3)}(v_c, q^2) = 0$. This point has turned out to be most crucial in the present calculation as well as in the calculations of $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ of photoproduction.[15,16] A comment on it will be given below.

(iv) The contributions to the integrand in (2) arising from the range $v > v_c$ are strongly damped. It is assumed that they can be safely omitted.

With the aid of (i)–(iv) the solution of Eq. (2) can be easily written in the form

$$\begin{aligned} M_2^{(3)}(v, q^2) = {} & B_2^{(3)}(v, q^2) \cos\varphi_2^{(3)}(v) \exp[i\varphi_2^{(3)}(v)] \\ & + \frac{1}{\pi} \exp[\rho(v) + i\varphi_2^{(3)}(v)] \\ & \times \mathrm{P}\int_{v_0}^{v_c} \frac{dv'}{v'-v} B_2^{(3)}(v', q^2) e^{-\rho(v')} \sin\varphi_2^{(3)}(v') \\ & + c_2^{(3)}(q^2) \exp[\rho(v) + i\varphi_2^{(3)}(v)], \end{aligned} \tag{4}$$

where

$$\rho(v) = \frac{1}{\pi} \mathrm{P}\int_{v_0}^{v_c} \frac{\varphi_2^{(3)}(v')}{v'-v} dv' - \ln(v_c - v). \tag{5}$$

The arbitrary quantity $c_2^{(3)}(q^2)$ reflects the presence of a CDD ambiguity. The values of $c_2^{(3)}(q^2)$ have to be fixed so as to satisfy requirement (iii).

It is now a matter of simple integration to calculate the values of $M_{1+}^{(3/2)}$ at resonance as function of q^2. The details of the procedure have been reported previously[15] and there is no need to repeat them here.

One remark seems nevertheless in order. The values of $c_2^{(3)}(q^2)$ have not been determined explicitly in the actual calculation. This is because Eq. (2) was solved on the interval $v_0 \leq v \leq v_c - \epsilon$ where $\epsilon > 0$ represents a small quantity. As $\varphi_2^{(3)}(v_c - \epsilon) < \pi$ the CDD ambiguity is absent in this case and the solution of Eq. (2) falls to zero at $v_c - \epsilon$.[20] By continuity arguments slight extrapolation of the values calculated at resonance for a few ϵ's gives the required $M_{1+}^{(3/2)}(M, q^2)$. Owing to this device, the burden of determining the limit of the integral in Eq. (4) can be lifted at the expense of ignoring the explicit values of $c_2^{(3)}(q^2)$ which, however, do not seem relevant to further discussion.

Then Eq. (1) yields the values of the transition form factor $G_M^*(q^2)$ immediately.

III. RESULTS AND DISCUSSION

The outcome of the calculation is shown in Fig. 1. The computed curve follows with remarkable accuracy all the data over the wide q^2 range currently covered by quantitative experimental study. As a result the manifest feature of the data, the decrease of $G_M^*(q^2)$ faster than of $G_D(q^2)$, is well reproduced. The correct absolute normalization of the curve follows from the outcome of calcula-

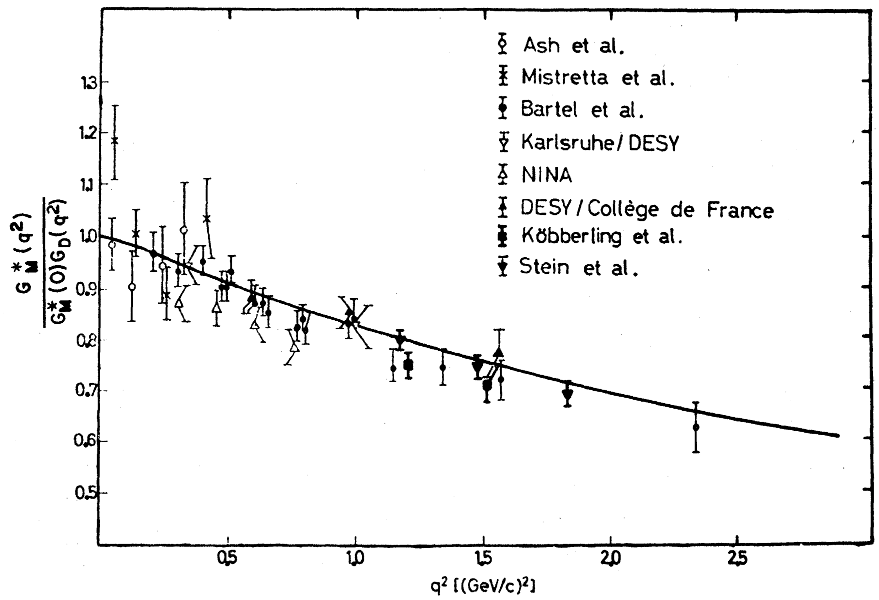

FIG. 1. The transition form factor $G^*_M(q^2)$ as compared to $G_D(q^2)$. The solid curve is the result of the present calculation. Details concerning the experimental points can be found in Ref. 2.

tions in the photoproduction case.

An attempt to approximate the calculated curve by the dipole formula $G^*_M(q^2)=G^*_M(0)(1+q^2/M_D{}^2)^{-2}$ over the range $0\le q^2\le 3.0$ $(\text{GeV}/c)^2$ yields the best fit, within a $\pm 11\%$ error band, for $M_D=0.765$ GeV/c. This is to be compared with $M_D=0.71$ GeV/c of the experimental fit of Köbberling *et al.*[2] No satisfactory fit could be found with the use of a single pole formula.

It should be emphasized that the whole calculation proceeded without fitting since there were *no free parameters* involved. Assumptions (i)–(iv) of the preceding section, whose partial arbitrariness should not be blurred, cannot be viewed as adjustable components of this calculation. They have proved very effective in the case of $M^{(3/2)}_{1+}$ of photoproduction treated previously, and arbitrariness in their choice, if any, should be referred only to that calculation. Retained unchanged in the present study, assumptions (i)–(iv) should henceforth be considered as a given dynamical ingredient. Incidentally the latter standpoint applies also to the calculation of $E^{(3/2)}_{1+}$ of photoproduction.[16] It seems therefore appropriate to look more closely at a possible significance of (i)–(iv).

Assumptions (i) and (iv) certainly should not raise serious doubts as they have become almost standard in this type of calculations. In what concerns (ii) there are good reasons to believe that the detailed behavior of $\varphi^{(3)}_2(v)$ in the range of higher energies has no appreciable bearing on the calculated values of $M^{(3/2)}_{1+}$ in the resonance region as long as $\varphi^{(3)}_2(v)$ approaches π sufficiently fast.[21] Certainly the extrapolation used in the present approach is arbitrary, but it does represent a controllable deviation from the actual behavior of the πN scattering phase shift in the p_{33} state[22] connected after all to the multipole amplitude $\varphi^{(3)}_2(v)$ and is consistent with the idea of using a totally elastic $\Delta(1232)\to\pi+N$ amplitude as input to dispersion integrals.

In contrast with the foregoing, the *ad hoc* character of assumption (iii) is clear. It would be difficult to give a convincing reason for imposing a CDD zero on the amplitudes in this calculation and those related to it. Yet their successful outcome strongly indicates that assumption (iii) is pivotal and very likely rooted in true dynamics.

Note in this connection that the extrapolation procedure as adopted by assumption (iii) indicates that some inelastic effects have been neglected in the calculation. Therefore the standard single-channel approach may not be strictly applicable to the present case. The appearance of a CDD zero in our single-channel amplitude could be then interpreted as a manifestation of a multichannel resonance as indicated by numerous studies.[23,24] The latter pertain, however, to amplitudes of strong processes, satisfying nonlinear N/D equations, while in the present case we deal with strong interactions in the final state of an electromagnetic process whose amplitude satisfies a linear integral equation. Multichannel equations of this kind were studied in some detail not long ago.[24] Unfortunately it turns out that the existing possibility of combining polynomial ambiguities from different channels gives rise to a large number of admissible solutions in the multichannel case. Therefore, arbitrariness in picking one of them is even greater than in a single-channel process, and there is again no indication how to fix the ambiguity in the latter case. This leaves us with little more than general statements such as, e.g., that the role of CDD ambiguities should be to represent the neglected contributions from distant singularities.[25]

It is tempting nevertheless to interpret the presence of a CDD zero in (4) as a reflection of inelastic effects setting in. No credible calculation can be presented in support of this opinion, but since casual coincidence between the calculated results and the data is improbable, some dynamical justification to the "CDD ansatz" is necessary. The given interpretation is just one possibility.

Among the earlier studies relevant to the electromagnetic $N \to \Delta(1232)$ transition form factor, the calculations of Walecka and Zucker[8] and of Gutbrod and Simon[7] are those which the present approach resembles most.

In Ref. 8, in addition to the minimal gauge-invariant set of one-particle exchange terms such as of Eq. (2), ω exchange has been included in the expression approximating the left-hand singularity contribution to the dispersion integral. No reference to CDD ambiguity in the transition amplitude has been made, and, moreover, its possible role in shaping the final result is obscured by the use of narrow-resonance approximation and by treating the coupling constant at the ωNN vertex as a free parameter.

The curve of Gutbrod and Simon agrees with the data of Köbberling *et al.*[2] perfectly, but it should be borne in mind that the use of box diagrams to approximate the contribution of the left-hand cut is motivated in Ref. 7 by some analogies rather than by more sound dynamical statements. Moreover, photoproduction calculations as reported in Ref. 7 do not lead to very satisfactory results, especially where the multipole amplitude $E_{1+}^{(3/2)}$ is concerned.

The above-quoted papers do not seem therefore to be of great help in understanding a possible dynamical meaning of assumption (iii).

A natural extension of the present study would be to calculate the electric and Coulombic $N \to \Delta(1232)$ transition form factors[11] expressible in terms of the amplitudes $E_{1+}^{(3/2)}(M,q^2)$ and $S_{1+}^{(3/2)}(M,q^2)$, respectively. At present only qualitative estimates of the ratios $|E_{1+}^{(3/2)}|/|M_{1+}^{(3/2)}|$ and $|S_{1+}^{(3/2)}|/|M_{1+}^{(3/2)}|$ at the resonance are available.[2] Owing to the experience with photoproduction amplitudes,[15,16] there is little doubt that the present approach can properly account for the foregoing ratios. A more precise calculation would require the solution of a system of coupled integral equations and consequently would be much more complex than the one presented here. Nevertheless with the advent of new data such a study may prove necessary.

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