PHYSICAL REVIEW D VOLUME 21, NUMBER 3 1 FEBRUARY 1980

Interpretation of single-tagged events in $\gamma\gamma$ experiments

C. Carimalo, P. Kessler, and J. Parisi
Laboratoire de Physique Corpusculaire, Collège de France, Paris, France
(Received 27 August 1979)

In the framework of the helicity formalism, we show that single-tagged events in $\gamma\gamma$ experiments performed with electron storage rings involving finite-angle tagging systems may be analyzed, with a very good accuracy, as electroproduction on a free photon target, i. e., through a four- or six-term formula.

I. INTRODUCTION

High-energy electron-position storage rings will be used increasingly in the next few years for an experimental study of $\gamma\gamma$ collisions. However, a rather unfortunate feature of such machines as PETRA and PEP is that because of the bremsstrahlung background tagging of the forward scattered electrons will only be possible at finite angles ($\gtrsim 1°$). That shortcoming will not only result in a sharp reduction of counting rates, in particular for double-tagged events; it will also make the analysis of $\gamma\gamma$ processes considerably more difficult.

Indeed, as we have shown in a previous paper on the limits of validity of the double equivalent-photon approximation—hereafter called (I) (Ref. 1)—that approximation becomes invalid as soon as either of the Q values of the virtual photons becomes larger than some rather small fraction of the invariant mass M_X of the system X produced in the $\gamma\gamma$ collision.

In a second paper—hereafter called (II) (Ref. 2)—we studied a typical deep-inelastic configuration, where one of the electrons undergoes a large-angle elastic scattering with a quasireal muon (or a quark) originating from the other (small-angle scattered) electron's vertex. There we have shown that the factorization procedure, eventually leading to a determination of the structure functions (F_2, F_L) of the photon, may only be applied validly, in practice, as long as the small-angle electron is emitted at a few milliradians, not more.

This paper will be devoted to the general problem of analyzing single-tagged events in $\gamma\gamma$ experiments performed with finite-angle tagging systems. Single-tagging has its obvious disadvantages, a higher rate of accidentals and, in general, more problems with background than in an analysis based on double-tagging. Its most serious shortcoming, however, is that the determination of the most fundamental parameters (such as M_X, or the scaling parameter x_B in a deep-inelastic experiment) becomes uncertain, in general, insofar as one cannot be sure that all particles produced are seen in the central detector. It therefore appears that a satisfactory analysis—in the sense of precise, quantitative physics—of single-tagged events will only be possible, in practice, in the case of rather simple $\gamma\gamma$ processes, such as materialization into pairs (possibly including pairs of jets) or production of resonances followed by their decay into simple (two- or three-body) channels.

Keeping these restrictions in mind, it must be said, on the other hand, that—always considering finite-angle tagging systems—single-tagging has two advantages: (i) Counting rates are obviously much higher than for double-tagging. (ii) The analysis of single-tagged events is considerably simplified, as far as the untagged electron may be considered as "tagged by absence."

Tagging by absence has been extensively discussed by one of us.[3] Its principle is that, in a $\gamma\gamma$ event, if one of the scattered electrons is not seen above some minimal tagging angle $\theta_{\min}$ (assuming the efficiency of electron tagging counters to be almost 100%), one may practically be sure that this electron was emitted between 0° and $\theta_{\min}$. Now, assuming $\theta_{\min} \approx 1°$, such electrons may be considered as having generated essentially quasireal photons (i.e., Q extremely small), since in any case an overwhelming contribution is due to those electrons which were scattered at extremely small angles ($\approx m_e/E_0$, E_0 being the beam energy).

We may thus consider that, in single-tagged events, we have on one side a flux of quasireal photons, and on the other side a much broader γ spectrum extending, in general, from quasireal to highly virtual photons. If we stick to the quasireal end of the latter spectrum, we come back to the configuration treated in paper (I). On the other hand, in considering its highly virtual part, we return to the case of deep-inelastic electron-photon scattering; however, instead of the specific configuration of paper (II), we shall treat here the more general case where particles produced are all measured at large angle.

In Sec. II, starting from the general helicity treatment used in papers (I) and (II), we shall derive (in the case of two-body or quasi-two-body reactions) a four-term formula for single-tagged events. That formula will then be numerically checked for lepton pair production under realistic experimental conditions, inspired by those of the first $\gamma\gamma$ experiments performed at PETRA by the PLUTO collaboration.[4] Section III contains our conclusions.

II. DERIVATION OF THE FOUR-TERM FORMULA

We consider the kinematic configuration shown in Fig. 1, where, for instance, $X_1 \equiv l^+$, $X_2 = l^-$ (we shall treat those leptons as massless). The same notations as in papers (I) and (II) will be used. We shall call "left-hand" (LH) and "right-hand" (RH) the electrons (e_0, e) and (e_0', e'), respectively, and the corresponding virtual photons as well. In paper (II), we have written down a ten-term expression [formula (2.1) of (II)], established by starting from the general helicity formula as given by Carlson and Tung[5] for the case of two-body or quasi-two-body $\gamma\gamma$ reactions, and by assuming θ(or Q) small enough to justify the neglect of all terms involving longitudinal polarization of the left-hand photon. We now rewrite that formula in a slightly different way, introducing the new variable φ_1' $(=\varphi-\varphi_1)$ which is the relative azimuthal angle between X_1 and e' in the $\gamma\gamma$ c.m. frame. We get

$$\begin{aligned}\frac{32E_0^2Q^4Q'^4}{e^8}\frac{d\sigma}{d\boldsymbol{P}_{\mathrm{LI}}} = {}& 2L_{++}(C_{++,++}+C_{++,--})R_{++}+4L_{++}(\mathrm{Re}\,C_{++,+-})R_{+-}\cos2\varphi_1'+4L_{+-}(\mathrm{Re}C_{+-,++})R_{++}\cos2(\varphi-\varphi_1')\\ &+2L_{+-}C_{+-,+-}R_{+-}\cos2\varphi+2L_{+-}C_{+-,-+}R_{+-}\cos2(\varphi-2\varphi_1')\\ &+4L_{++}(\mathrm{Re}C_{++,+0}-\mathrm{Re}C_{++,0-})R_{+0}\cos\varphi_1'+4L_{+-}(\mathrm{Re}C_{+-,+0})R_{+0}\cos(2\varphi-\varphi_1')\\ &+4L_{+-}(\mathrm{Re}C_{+-,0+})R_{+0}\cos(2\varphi-3\varphi_1')+2L_{++}C_{++,00}R_{00}+2L_{+-}C_{+-,00}R_{00}\cos2(\varphi-\varphi_1')\end{aligned} \tag{2.1}$$

Here we are interested in the differential cross section defined in such a way that M_X is fixed (instead of E, the LH electron's final energy), and that the LH electron's solid angle of emission is integrated over, i.e.,

$$\sigma^* \equiv \frac{d\delta}{dM_X\,dE'\,d\Omega'\,d\Omega_1} = \int \left|\frac{dE}{dM_X}\right| K\,\frac{d\sigma}{dP_{\mathrm{LI}}}\,d\Omega\ . \tag{2.2}$$

Taking $\theta\to0$, one gets from formula (A6) of the Appendix of paper (I)

$$M_X^2 \simeq 4\left[(E_0-E)(E_0-E')-EE'\sin^2\frac{\theta'}{2}\right], \tag{2.3}$$

and therefore

$$\left|\frac{dE}{dM_X}\right| \simeq \frac{M_X}{2(E_0-\bar{E}')}\ ,\ \text{defining } \bar{E}'=E'\cos^2\frac{\theta'}{2}\ , \tag{2.4}$$

whereas the approximate expression of the kinematic factor K is derived from (A15) and (A16) of (I) as follows:

$$K \simeq \frac{EE'E_1}{16(2\pi)^8\{2E_0-E(1-\cos\psi)-E'[1+\cos\theta'\cos\psi-\sin\theta'\sin\psi\cos(\phi'-\phi_1)]\}} \tag{2.5}$$

where, according to (A10) and (A11) of (I) one defines

$$E_1 \simeq \frac{M_X^2}{2\{2E_0-E(1-\cos\psi)-E'[1+\cos\theta'\cos\psi-\sin\theta'\sin\psi\cos(\phi'-\phi_1)]\}}\ . \tag{2.6}$$

Rewriting (A1) of (I), we have

$$\begin{aligned}L_{++}&=(Q^2+4m_e^2)\sinh^2\alpha+2Q^2,\\ L_{+-}&=-(Q^2+4m_e^2)\sinh^2\alpha,\end{aligned} \tag{2.7}$$

whereas, according to (A2), one gets, neglecting m_e^2,

$$R_{++}\simeq Q'^2(\sinh^2\alpha'+2),\quad R_{+-}\simeq -Q'^2\sinh^2\alpha'\ , \tag{2.8}$$

to which we here add the expressions

$$R_{+0} \simeq -\frac{1}{\sqrt{2}} Q'^2 \sinh 2\alpha', \quad R_{00} \simeq 2Q'^2 \sinh^2\alpha' . \tag{2.9}$$

(A4) of (I) is rewritten as

$$Q^2 = E_0 E \theta^2 + Q_0{}^2, \quad Q_0{}^2 = m_e{}^2 \frac{(E_0 - E)^2}{E_0 E}, \tag{2.10}$$

whereas from (A5), (A7), (A8), and (A9) one obtains

$$Q'^2 \simeq 4E_0 E \sin^2\frac{\theta'}{2}, \tag{2.11}$$

$$\sinh^2\alpha \simeq \frac{4(Q^2 - Q_0{}^2)}{Q^2 + 4m_e{}^2,} \frac{E_0 E}{(E_0 - E)^2}, \tag{2.12}$$

$$\sinh^2\alpha' \simeq \frac{4E_0 \overline{E}'}{(E_0 - \overline{E}')^2}, \tag{2.13}$$

$$\varphi \simeq \phi' - \phi . \tag{2.14}$$

From (A12) we get

$$\cos\chi \simeq 1 - \frac{E_1(1 - \cos\psi)}{E_0 - \overline{E}'} . \tag{2.15}$$

From the expressions given by (A13) of (I) for $\sin\varphi_1$ and $\cos\varphi_1$, one easily derives (by LH-RH symmetry) those to be used for $\sin\varphi_1'$ and $\cos\varphi_1'$. Then, making $\theta \to 0$, one gets

$$\sin\varphi_1' \simeq \frac{B \sin(\phi' - \phi_1)}{[A^2 + B^2 + 2AB\cos(\phi' - \phi_1)]^{1/2}},$$
$$\cos\varphi_1' \simeq \frac{A + B\cos(\phi' - \phi_1)}{[A^2 + B^2 + 2AB\cos(\phi' - \phi_1)]^{1/2}} \tag{2.16}$$

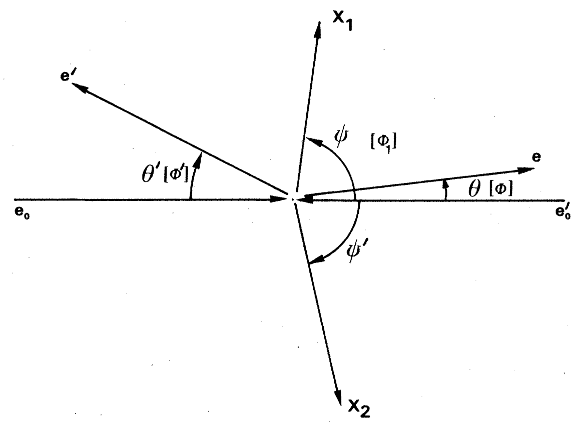

FIG. 1. Kinematic configuration for single-tagged events in the measurement of a process $e_0 e_0' \to ee'X_1X_2$, where e is the untagged electron and e' the tagged one. Orbital angles of outgoing particles are shown, and the corresponding azimuthal angles are indicated in brackets.

with

$$A = E' \sin\theta' \sin\tfrac{1}{2}\psi, \quad B = 2(E_0 - \overline{E})\cos\tfrac{1}{2}\psi .$$

Finally, the tensor elements $C_{m\bar{m},n\bar{n}}$ to be used become functions of M_X, Q', and χ only.

In the RH member of Eq. (2.2) the integrand then depends on ϕ, the laboratory azimuthal angle of the LH electron, only through the cos functions in the expression of $d\sigma/dP_{\rm LI}$ [see (2.1)], and, more precisely, only through those cos functions which contain a dependence on φ. Accounting for (2.15), one concludes that integration over ϕ between 0 and 2π will lead to vanishing of all helicity terms with φ dependence. We are thus left with the four-term formula

$$\frac{32E_0{}^2Q'^4}{e^8}\sigma^* \simeq \int Q^{-4}\left|\frac{dE}{dM}\right| K\, 2L_{++}[(C_{++,++} + C_{++,--})R_{++} + 2(\mathrm{Re}C_{++,+-})R_{+-}\cos 2\varphi_1'$$
$$+ 2(\mathrm{Re}C_{++,+0} - \mathrm{Re}C_{++,0-})R_{+0}\cos\varphi_1' + C_{++,00}R_{00}]\, d\Omega . \tag{2.17}$$

The $C_{m\bar{m},n\bar{n}}$ involved here are expressed (for pair production of massless leptons, making $Q \to 0$) by

$$C_{++,++} + C_{++,--} \simeq \frac{8(M_X{}^4 + Q'^4)}{(M_X{}^2 + Q'^2)^2}\frac{2 - \sin^2\chi}{\sin^2\chi}, \quad C_{++,+-} \simeq -\frac{8M_X{}^2Q'^2}{(M_X{}^2 + Q'^2)^2},$$
$$C_{++,+0} - C_{++,0-} \simeq \frac{8\sqrt{2}\, M_X Q'(M_X{}^2 - Q'^2)\cot\chi}{(M_X{}^2 + Q'^2)^2}, \quad C_{++,00} \simeq \frac{16M_X{}^2Q'^2}{(M_X{}^2 + Q'^2)^2} . \tag{2.18}$$

One may notice that after the trivial integration over θ (between 0 and $\theta_{\min}$) and ϕ (between 0 and 2π), formula (2.17) can be rewritten in the form

$$E_0\theta'\sigma^* = N(x)N'(x', \theta', \phi')\frac{d\Sigma^{\gamma\gamma}}{d\omega_1}\frac{d\omega_1}{d\Omega_1}\frac{dx}{dM_X}, \tag{2.19}$$

where ω_1 is the solid angle of particle X_1 in the $\gamma\gamma$ c.m. frame, and one defines $x=(E_0-E)/E_0$, $x'=(E_0-E')/E_0$.

Here $N(x)$ is the usual Williams-Weizsäcker spectrum for the quasireal photon given by

$$N(x)=\frac{2\alpha}{\pi}\frac{1}{x}\left[(1-x+\tfrac{1}{2}x^2)\ \ln\left(\frac{E_0}{m}\ \frac{1-x}{x}\ \theta_{\min}\right)-\tfrac{1}{2}(1-x)\right]. \tag{2.20}$$

$N'(x',\theta',\phi')$ is a "generalized Williams-Weizsäcker spectrum" for off-shell photons; it is expressed by

$$N'(x',\theta',\phi')=\frac{\alpha}{\pi^2}\ \frac{1}{\theta'}\ \frac{1-\bar{x}'+\frac{1}{2}\bar{x}'^2}{\bar{x}'}\ \frac{M_X{}^2}{M_X{}^2+Q'^2}\ , \tag{2.21}$$

with

$$\bar{x}'=(E_0-\bar{E}')/E_0.$$

In addition, one has

$$\frac{d\omega_1}{d\Omega_1}=\frac{M_X{}^2}{\{2E_0-E(1-\cos\psi)-E'[1+\cos\theta'\cos\psi-\sin\theta'\sin\psi\cos(\phi'-\phi_1)]\}^2} \tag{2.22}$$

$$\frac{dx}{dM_X}=\frac{M_X}{2E_0(E_0-\bar{E}')}\ . \tag{2.23}$$

Then, finally, the "generalized $\gamma\gamma$ cross section" $\Sigma^{\gamma\gamma}/d\omega_1$ is actually composed of four terms, as

$$\frac{d\Sigma^{\gamma\gamma}}{d\omega_1}=\frac{d\sigma_U^{\gamma\gamma}}{d\omega_1}+\eta\ \frac{d\sigma_L^{\gamma\gamma}}{d\omega_1}+\eta\ \frac{d\sigma_P^{\gamma\gamma}}{d\omega_1}\cos 2\varphi_1' +[2\eta(1+\eta)]^{1/2}\ \frac{d\sigma_I^{\gamma\gamma}}{d\omega_1}\cos\varphi_1'\ , \tag{2.24}$$

where the various $d\sigma^{\gamma\gamma}/d\omega_1$ are related by trivial proportionality factors to the corresponding $C_{m\bar{m},n\bar{n}}$ of formula (2.17), and η is the virtual photon's polarization parameter defined as

$$\eta=\frac{1-\bar{x}'}{1-\bar{x}'+\frac{1}{2}\bar{x}'^2}\ . \tag{2.25}$$

From (2.23), one notices the similarity with formulas used for electroproduction of one particle (e.g., a pion) from a nuclear target[6]; one may indeed consider that here one is electroproducing a particle pair on a quasifree photon target.

We shall add two remarks:

(i) If one does not limit oneself to the assumption of a two-body (or quasi-two-body) $\gamma\gamma$ reaction, the four-term formula is to be generalized into a six-term formula, i.e., $\sigma_P\cos 2\varphi_1'$ is to be replaced by $(\mathrm{Re}\sigma_P)\cos 2\varphi_1'-(\mathrm{Im}\sigma_P)\sin 2\varphi_1'$, and $\sigma_I\cos\varphi_1'$ is to be replaced by $(\mathrm{Re}\sigma_I)\cos\varphi_1'-(\mathrm{Im}\sigma_I)\sin\varphi_1'$.

(ii) The various virtual $\gamma\gamma$ cross sections in formula (2.24) are defined according to a prescription given by Hand[7] for virtual photoproduction cross sections, i.e., the kinematic factor for the incoming flux is defined as if the incoming virtual photon were real. If that prescription is changed, the factor $M_X{}^2/(M_X{}^2+Q'^2)$ may be taken off from (2.21) and transferred into a redefined expression of $d\Sigma^{\gamma\gamma}/d\omega_1$.

Let us now check formula (2.17) or (2.19) versus an exact computation, for realistic experimental conditions, inspired by those of the first $\gamma\gamma$ experiment recently performed at PETRA.[4] We here consider θ integrated over between 0 and $\theta_{\min}=23$ mrad and θ' and M_X taking a set of fixed values, as shown in Table I. As for the remaining independent variables (ψ, E', or x', and the relative azimuthal angle $\phi'-\phi_1$), we let them go through a wide variety of values, however, imposing the restriction $40°<\psi<140°$, as well as the same restriction on the nonindependent variable ψ'. The beam energy is set at (1) $E_0=8.5$ GeV, (2) $E_0=16$ GeV. For each couple of fixed values of θ' and M_X, we notice in Table I the range of the relative error found, i.e., of

$$\Delta=(\sigma^*_{\text{approx}}-\sigma^*_{\text{exact}})/\sigma^*_{\text{exact}}\ .$$

As Table I shows, the accuracy of the four-term formula is almost perfect, except for the lowest values of M_X where Δ may become relatively large when θ' is increased. A close analysis shows that those inaccuracies are due to nonnegligible errors resulting from the Lorentz transformation, i.e., errors involved in the expression of $\cos\chi$, as given by formula (2.15) for the specific configurations considered.

III. CONCLUSIONS

We have shown that single-tagged events in $\gamma\gamma$ experiments with finite-angle tagging systems can be analyzed with high accuracy (except for very low values of M_X/E_0) just like any electro-

TABLE I. Range of the relative error $\Delta = (\sigma^*_{\text{approx}} - \sigma^*_{\text{exact}})/\sigma^*_{\text{exact}}$ involved in the four-term approximation [formula (2.17) or (2.19)] for the process $e_0 e_0' \to ee'e^+e^-$, considering single-tagged events, for various fixed values of the tagging angle θ' and the invariant mass M_X produced. We let the other independent variables (ψ, E' or x', and the relative azimuthal angle $\phi' - \phi_1$) go through a wide variety of values, with, however, the restriction $40° < \psi, \psi' < 140°$. The untagged forward-scattered electron's angle θ was integrated over between 0 and 23 mrad. The beam energy was taken as (1) $E_0 = 8.5$ GeV, (2) $E_0 = 16$ GeV.

(1) $E_0 = 8.5$ GeV				
θ' (mrad) \ M_X (GeV)	0.3	0.5	1	2 and above
23	$-5\% < \Delta < +5\%$	$-6\% < \Delta < +3\%$	$-1\% < \Delta < +3\%$	$\lvert\Delta\rvert < 2\%$ everywhere
45	$-1\% < \Delta < +10\%$	$-5\% < \Delta < +3\%$	$-1\% < \Delta < +1\%$	
70	$-1\% < \Delta < +17\%$	$-2\% < \Delta < +3\%$	$-3\% < \Delta < +1\%$	
120	$-4\% < \Delta < +23\%$	$-1\% < \Delta < +11\%$	$-3\% < \Delta < +2\%$	
250	$-1\% < \Delta < +28\%$	$-1\% < \Delta < +18\%$	$-1\% < \Delta < +4\%$	

(2) $E_0 = 16$ GeV				
θ' (mrad) \ M_X (GeV)	0.5	1	2	3 and above
23	$-3\% < \Delta < +5\%$	$-5\% < \Delta < +3\%$	$-1\% < \Delta < +3\%$	$\lvert\Delta\rvert < 3\%$ everywhere
45	$-1\% < \Delta < +12\%$	$-4\% < \Delta < +2\%$	$-1\% < \Delta < +3\%$	
70	$-1\% < \Delta < +19\%$	$-2\% < \Delta < +3\%$	$-2\% < \Delta < +2\%$	
120	$-6\% < \Delta < +24\%$	$-1\% < \Delta < +9\%$	$-3\% < \Delta < +2\%$	
250	$0 < \Delta < +28\%$	$-2\% < \Delta < +15\%$	$-1\% < \Delta < +3\%$	

production processes, i.e., using a four- or six-term helicity formula. However, in order to separate the various terms (differential virtual $\gamma\gamma$ cross sections), one needs high statistics, since distributions with respect to various kinematic variables must be analyzed. With low statistics, obviously, a comparison of theory and experiment can only be performed on cross sections for $ee \to eeX$, integrated over most of the variables; in other words, only model-fitting shall be possible. At least, such a model-fitting will require only the computation of a small number of terms.

Ad for the simplest analysis, i.e., the one-term or Williams-Weizsäcker formula, it can be applied only to part of the data, located in a kinematic range which satisfies $Q' \ll M_X/2$ [as shown in paper (I)].

From papers (I), (II), and this paper, we draw the following general conclusions.

Back-factorization—the procedure allowing one to extract the information about $\gamma\gamma$ interactions directly from the brute experimental data—will be rather difficult to perform in $\gamma\gamma$ experiments involving finite-angle tagging systems. To a large extent, double-tagged events will not be usable for a simple analysis of that type. Single-tagged events are more promising, since they can be analyzed, as just shown, by using a small number or helicity terms; however, as already said in the Introduction, in many cases they will not allow a precise reconstitution of the $\gamma\gamma$ process involved.

In that sense, it may be foreseen that $\gamma\gamma$ physics, to be performed with finite-angle tagging systems, will remain semiquantitative to some extent (like neutrino physics), using such concepts as "visible energy," "visible mass," or "apparent scaling parameter."

It should, however, be said that—whereas a neutrino will never be directly measured, so that some uncertainty will always remain in experimental neutrino physics—electron tagging at 0° is not an impossibility, even with electron storage rings of very high energy. We are aware, of course, that it involves difficult and costly technical solutions which cannot be applied systematically to all machines of that type.

Therefore, we conclude that $\gamma\gamma$ physics—if one wants to make it an area of precise quantitative investigation—requires a machine of its own, with a specific technology allowing for electron tagging at 0°.

ACKNOWLEDGMENTS

The authors wish to thank Dr. A. Courau and Professor P. Waloschek for useful discussions, and Dr. Ch. Berger for communication of details on the PLUTO collaboration experiment, here mentioned, before publication.

[1]C. Carimalo, P. Kessler, and J. Parisi, Phys. Rev. D 20, 1057 (1979).
[2]C. Carimalo, P. Kessler, and J. Parisi, Phys. Rev. D 20, 2170 (1979).
[3]J. Parisi, thèse de Doctorat d'Etat, University of Paris, 1974 (unpublished).
[4]See G. Flügge, DESY Red Report No. 79/26, 1979 (unpublished).
[5]C. E. Carlson and W. K. Tung, Phys. Rev. D 4, 2873 (1971).
[6]C. W. Akerlof, W. W. Ash, K. Berkelman, and M. Tigner, Phys. Rev. Lett. 14, 1036 (1965).
[7]L. N. Hand, Phys. Rev. 129, 1834 (1963).