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Coherent detector for low-energy neutrinos

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The design of a coherent neutrino detector, based on total reflection of low-energy neutrinos from a plane surface, is reanalyzed using the standard model of neutrino interactions. An alternative design, using refraction in a thin-ruled surface, is shown to have the same sensitivity and simpler construction.

I. INTRODUCTION

Cross sections for neutrino interactions with other particles depend quadratically on the neutrino energy E_ν, and so the problem of detecting neutrinos apparently becomes increasingly difficult as E_ν decreases. At present, there is no workable method for detecting neutrinos in the energy range below about 1 MeV, despite the large fluxes available from fission reactors and the sun. It is particularly frustrating that the "cosmic neutrinos" produced during the early stage of our universe,[1] seem beyond the range of observation in terrestrial experiments.[2]

A provocative idea has been presented by Opher,[3] suggesting that cosmic neutrinos with $E_\nu \lesssim 10^{-2}$ eV could be detected by measuring the force exerted on a flat collector due to total reflection of neutrinos at its surface. I will reexamine the derivation given in this paper in order to draw further attention to the ideas involved, to update them somewhat using more recent models of neutrino scattering, and to extend them to a "coherent detector" of different design. This reconsideration of Opher's detector leaves me in basic agreement with his assertion that the measurement of low-energy neutrinos can be substantially improved by the use of large coherent detectors. Interesting experiments may be possible in measuring the total neutrino flux from reactors, from the sun, and from the "big bang."

II. THE INDEX OF REFRACTION FOR NEUTRINOS

The essential idea underlying Opher's work is the use of "coherent" neutrino processes in matter, for which the amplitudes from different atoms in the detector add together, rather than "incoherent" processes for which the rates add. In this way, as the energy E_ν is reduced the decrease in the amplitude can be compensated by an increase in the number of atoms which coherently combine. This important property of neutrino amplitudes has already been discussed by Freedman,[4] who emphasized that high-energy neutrinos would elastically scatter from nuclei with a cross section proportional to A^2, if the momentum transfer was below about $2m_\pi c \simeq 300$ MeV/c. The interaction can be detected only through measurement of the recoil momentum transferred to the target nucleus, which is a relatively poor signature for these events. As the neutrino energy is decreased and the momentum transfer further reduced, the coherent summation of scattering amplitudes is extended, first over the entire atom, then over many atoms in the solid, with a corresponding increase in the number of particles involved. At sufficiently low energies, there is appreciable scattering from the entire target; it is this momentum transfer which Opher proposes to detect.

The calculation of the index of refraction is a classic example of the use of coherence. The discussion is usually based on the modification of a plane-wave incident on a thin foil.[5] The derivation shows explicitly that the transmitted wave is formed by a superposition of the amplitudes from atoms lying within a lateral distance of order $\lambdabar = E_\nu^{-1}$. It follows that the number of atoms involved is proportional to E_ν^{-2}, and since the scattering amplitude is proportional to E_ν, the scattered wave grows like E_ν^{-1} as the energy decreases:

$$n - 1 \propto E_\nu^{-1}. \tag{1}$$

This increase suggests that at sufficiently low energy, neutrinos can be detected through coherent processes such as reflection and refraction, better than through incoherent reactions.

The precise relation involved in Eq. (1) is[6]

$$n = 1 + 2\pi N \lambdabar^2 f(0), \tag{2}$$

where N is the number density of atoms, $\lambdabar$ the neutrino wavelength, and $f(0)$ the forward scattering amplitude per atom. This relation is valid for homogeneous, isotropic media and for scattering amplitudes sufficiently smaller than $n - 1 \ll 1$. Notice that the sign in Eq. (2) implies that a weak attraction of neutrinos to matter, which leads to a negative potential energy and a positive scattering amplitude, gives a refractive index greater than unity.[7] Since the sign of $n-1$ is of interest, as well as the magnitude, we will carefully retain it in what follows.

When Opher's paper was written it was known that neutrinos could elastically scatter from electrons via charged currents, but there was only speculation about elastic scattering via neutral currents. In the intervening years, neutral currents have been discovered and the coupling strengths for neutrinos at high energies have been extensively studied. The combined results strongly confirm the "standard model" of Weinberg and Salam,[8] which gives an unambiguous prediction for the low-energy scattering amplitudes from electrons and nucleons. It is therefore of interest to calculate $f(0)$ in this model since it provides a good basis for predicting the index of refraction.

The standard model is normally expressed in terms of an effective Hamiltonian density for the interaction between a neutrino and another fermion ψ,[9]

$$\mathcal{H}_{\rm eff} = -2^{-3/2}G[\bar{\nu}\gamma_\lambda(1+\gamma^5)\nu] \times\{\bar{\psi}\gamma_\nu[c_L(1+\gamma_5)+c_R(1-\gamma_5)]\psi\}\ , \tag{3}$$

where the coupling constants c_L, c_R are simply related to the weak-hypercharge and weak-isospin quantum numbers of each fermion ψ. The model is defined in terms of quarks and leptons, with coupling constants given in Table I. We can apply Eq. (3) to another target such as a nucleon or an atom, by summing the coupling constants of the constituents. For example, since the proton is composed of two u quarks and one d quark, it has coupling constants

$$c_L = -\frac{2.25}{2} + 2\sin^2\theta_W, \quad c_R = \frac{0.25}{2} + 2\sin^2\theta_W. \tag{4}$$

This effective interaction includes contributions from both charged and neutral currents. The coupling constants for ν_μ are the same as for ν_e, excepting c_L for the electron which becomes $1-2\sin^2\theta_W$; the difference is due to the absence of a charged current in ν_μ-e scattering.

The calculation of the forward scattering amplitude for a neutrino from an atom requires evaluation of the matrix element[10]

$$f(0) = -(2\pi)^{-1} m_\nu V \sum_j \int d^3x \langle p|\mathcal{H}_j|p\rangle\ , \tag{5}$$

where the sum extends over the constituents of the atom: Z electrons, $(A+Z)$ u quarks, and $(2A-Z)$ d quarks. Here m_ν is the fictitious mass of the neutrino and V the normalization volume; both of these parameters will be canceled by corresponding factors in the plane-wave neutrino state $|p\rangle$. Taking the atom at rest in the laboratory frame, and assuming the constituent spins are unpolarized,[11] we obtain for ν_e

$$f(0) = -\frac{GE_\nu}{2\pi\sqrt{2}} \sum_j (c_L + c_R)_j = +\frac{GE_\nu}{2\pi\sqrt{2}}(3Z-A)\,. \tag{6}$$

The refractive index of a neutrino in matter is greater than unity, corresponding to an attractive interaction. For an antineutrino $\bar{\nu}_e$, the overall sign in Eq. (6) changes. For ν_μ ($\bar{\nu}_\mu$) the forward scattering amplitude is

$$f(0) = \mp\frac{GE_\nu}{2\pi\sqrt{2}}(A-Z)\,. \tag{7}$$

We conclude that the index of refraction for ν_e ($\bar{\nu}_e$) is

$$n = 1 \pm \frac{GN(3Z-A)}{\sqrt{2}E_\nu} \tag{8}$$

and for ν_μ ($\bar{\nu}_\mu$) is

$$n = 1 \mp \frac{GN(A-Z)}{\sqrt{2}E_\nu}\,. \tag{9}$$

These results are independent of the one undetermined parameter θ_W in the standard model. The factors $N(3Z-A)$ and $N(A-Z)$ lead to a characteristic dependence of the refractive index on the atomic weight. For e neutrinos n increases with A up to the region of Fe but does not grow further for heavier elements; for μ neutrinos n increases monotonically and is largest for Au, Pb. The signs imply that $\bar{\nu}_e$ and ν_μ should exhibit total *external* reflection and $\nu_e, \bar{\nu}_\mu$ should exhibit total *internal* reflection. The sign and magnitude of n enable one to recognize these four different neutrinos, at least in principle.

III. OPHER'S DETECTOR REVISITED

Remarkable as it may seem, the foregoing derivation implies that at sufficiently low energies, neutrinos can be totally reflected at angles which are not unreasonably small. For μ neutrinos of energy $E_\nu = 10^{-2}$ eV, the refractive index in gold is, from Eq. (9),

$$1-n = 4.5\times 10^{-11}$$

TABLE I. Coupling constants for ν_e with electrons and u, d quarks.

	e	u	d
c_L	$-1-2\sin^2\theta_W$	$-\frac{1}{2}\times 2.25+\frac{4}{3}\sin^2\theta_W$	$+\frac{1}{2}\times 2.25-\frac{2}{3}\sin^2\theta_W$
c_R	$-2\sin^2\theta_W$	$\frac{1}{2}\times 0.25+\frac{4}{3}\sin^2\theta_W$	$-\frac{1}{2}\times 0.25-\frac{2}{3}\sin^2\theta_W$

and the critical angle is

$$\theta_c=[2(1-n)]^{1/2}=0.95\times 10^{-5}\ \text{rad}\cong 2\ \text{arcsec}.$$

Opher has designed a neutrino detector based on the recoil energy from total reflection off a thin metal foil. If the surface area is made large and the mass kept small, the recoil energy can become comparable to the thermal energy. The detector would respond to the total flux of neutrinos in a narrow band of energies and angles.

Consider, as a prototype of such detectors, a square target of side length a and thickness b, exposed to a flux density j (neutrinos/sr cm^2 sec). Assuming for the moment that the neutrino energy spectrum is monochromatic, then there will be total reflection of the neutrinos within a solid angle $2\pi\theta_c$, defined by the critical angle for that energy. The collision rate is $ja^2 2\pi\theta_c$, and since the average momentum transfer per collision is $p_\nu\theta_c$, the force on the target is

$$\frac{dp}{dt}=2\pi j a^2 p_\nu\,\theta_c{}^2=4\pi j a^2 p_\nu(1-n)\,. \tag{10}$$

The force can be reexpressed in more fundamental terms using our results for the refractive index. For μ neutrinos we obtain

$$\frac{dp}{dt}=\sqrt{8}\;\pi G N(A-Z) j a^2\,, \tag{11}$$

which is proportional to the density and area of the detector, but independent of the neutrino energy.

A lower limit to the detectable flux density is set by the requirement that the recoil energy transferred to the target in time τ, be greater than the thermal energy per mode:

$$p^2/2M \geq kT\,, \tag{12}$$

where M is the target mass. This leads to the condition

$$j\tau \geq \frac{1}{2\pi G N_A (1-Z/A)a}\left(\frac{bkT}{\rho}\right)^{1/2}, \tag{13a}$$

or in numerical form (cgs units)

$$j\tau \geq 6.6\times 10^{26}\,\frac{1}{a(1-Z/A)}\left(\frac{bT}{\rho}\right)^{1/2}, \tag{13b}$$

where ρ is the target mass density. This result shows explicitly the dependence on the parameters of the detector and that a thin foil of large area is required, either in one large sheet or a large number of smaller sheets.

We have put off until this point any discussion of the actual form of the detector, which can be chosen in a number of different ways. A single plane target would *not* experience a net force in an isotropic flux, since reflections from the top and bottom surfaces would compensate each other. Since a very narrow band of angles is detected, the flux need only be isotropic within this band. Clearly the symmetry between top and bottom surfaces must be broken by the shape of the detector. Opher suggested a reflecting shield surrounding the central target, inclined at an angle θ_0 of the same order as θ_c. A single element like this would still not experience a force in an isotropic flux, since the reflector simply substitutes equal numbers of neutrinos striking the top from a slightly different incident direction; the neutrinos striking the bottom are unaffected and would still exactly compensate. However, a stack of many such elements would suffice, if placed close enough together that the adjoining reflectors serve as a "neutrino guide", deflecting particles from the bottom of one reflector onto the top of the next detector. A simpler solution is to place a horizontal reflector in such a position that it blocks neutrinos incident on one surface but not the other, or two equivalent detecting surfaces which shield each other, in such a way that one receives an excess flux on the top surface, the other on the bottom. There is no reason to doubt that a suitable configuration could be devised which differentiates between collisions from above and from below.

At this point, it is interesting to consider the relative merits of detecting coherent versus incoherent effects in matter. We can do so by comparing the rate of momentum transfer through incoherent scattering from individual atoms, with the rate of momentum transfer to the entire target given in Eq. (11). The former requires the calculation of the momentum-transfer cross section

$$\sigma_y \equiv E_\nu \int \frac{d\sigma}{dy}\,y\,dy\,, \tag{14}$$

where y is the fractional energy loss of the neutrino. We find for μ neutrinos,

$$\sigma_y=\frac{G^2E_\nu{}^4}{\pi M}\,(A-Z)^2\,, \tag{15}$$

where M is the atomic mass. Multiplying this by the neutrino flux and the number of target atoms, we obtain

$$\left(\frac{dp}{dt}\right)_{\text{incoherent}}=\frac{4G^2E_\nu{}^4}{M}\,(A-Z)^2 jNa^2 b\,. \tag{16}$$

The ratio of Eq. (16) to Eq. (11) is

$$\frac{\sqrt{2}}{\pi}\,GE_\nu{}^4(A-Z)b/M\,. \tag{17}$$

This ratio still depends on the thickness of the target b because the incoherent processes occur throughout the volume, while the coherent processes take place at the surface. We see that at sufficiently low energies, coherent effects are dominant over incoherent; the transition energy is approximately given by

$$E_\nu \leq \left(\frac{M_p}{Gb}\right)^{1/4}=\frac{6.3\ \text{MeV}}{[b\,(\text{cm})]^{1/4}}\,. \tag{18}$$

It is important to recognize that a detector of this type is inherently a narrow-band device, responding to the total number of neutrinos arriving in a narrow range of energies and angles. The use of total reflection geometry selects a narrow angular range; the reflecting device necessary to differentiate between collisions on the top and bottom restricts the energy roughly to those neutrinos having the predetermined critical angle. The use of total reflection also implies that the detector will not respond to fully degenerated neutrinos, since coherent scattering involves a finite momentum transfer but zero energy transfer, leading to a final state which is already occupied. Within these restrictions, such devices should be able to detect a large flux of very-low-energy neutrinos.

IV. REFRACTING DETECTOR

Opher's detector relies on the total reflection of a small fraction of the incident neutrinos. The momentum transfer involves the small angle θ_c twice: once in the scattering angle and once in the solid angle. As a result, the force on the detector is proportional to $\theta_c^2 = 2(1-n)$, and therefore to the weak scattering amplitude. Consider instead the momentum transfer due to refraction of neutrinos: Here the solid angle is of order unity and the scattering angle is of order $(n-1)$. This simple argument suggests that a refracting detector should have roughly the same sensitivity as the reflecting detector. We will show next that there is a detailed correspondence between the two detectors, and that an equation similar to Eq. (11) gives the net force exerted by refraction of neutrinos.

Consider a refracting metal prism in the shape of a triangular cylinder with refractive index n. A neutrino passing through the prism is deflected by an angle θ given by

$$\theta = (n-1)\tan\alpha\left(\frac{1+\tan^2\beta}{1+\tan\alpha\tan\beta}\right), \tag{19}$$

where α is the angle between the two faces and β is the angle of incidence. Unlike the totally reflecting detector, the angles α and β are not sharply restricted. Therefore, the refracting detector will respond to a broad spectrum of neutrino angles and energies. If the incident neutrino flux consists of a poorly collimated beam, then it is necessary to average the momentum transfer over the flux, giving a result of the form

$$\langle \Delta p\rangle = \gamma p_\nu (n-1), \tag{20}$$

where γ is a numerical coefficient of order unity.

Since we are interested in detecting cosmic neutrinos, we must also consider the response to an isotropic flux. It is clear that for a highly symmetrical prism, the coefficient γ can be zero in an isotropic flux. For example, the threefold symmetry of an equilateral triangular prism and the fourfold symmetry of a square prism can be shown to lead to $\gamma = 0$. On the other hand, a prism with lower symmetry can provide directions along which the momentum transfer does not vanish, even for an isotropic distribution of neutrino directions. A right isosceles triangle is a case in point; symmetry under exchanging the equal sides shows that the average momentum transfer must lie along the bisector of the right angle. A straightforward but tedious evaluation of the angular average shows that Eq. (20) does pertain, with $\gamma \cong 1.10$.

A detector made from one large refracting prism would be substantially less sensitive than the reflecting detectors we have described. The collision rate would scale like the surface area (L^2) and the total mass like the volume (L^3), where L is a typical dimension of the prism. The integration time would therefore contain a factor $L^{-1/2}$, in place of the much smaller factor $a^{-1}\sqrt{b}$ in Eq. (13), where a, b are lateral size and thickness, respectively. Since reflection and refraction are surface phenomena, greater sensitivity is obtained by reducing the thickness; the minimum thickness is the neutrino wavelength. A large refracting detector could be made like a Fresnel lens, with thin sheets ruled to provide a large number of fine prisms. For such a detector, in the shape of a square of side a and thickness b, the collision rate is $4\pi j a^2$, since the full solid angle is effective. The average momentum transfer per collision is $\gamma p_\nu(n-1)$ and so the force is

$$\frac{dp}{dt} = 4\pi\gamma j a^2 p_\nu (n-1), \tag{21}$$

which is the same as Eq. (10) except for the factor γ. The remainder of the discussion of the sensitivity is the same as before. We conclude that both refracting and reflecting detectors have the same sensitivity.

Comparing the two approaches, there are significant advantages to the use of refraction. This device accepts a broad band of neutrino directions, even including an isotropic flux. Consequently, the angular tolerances on the shape and orientation of the refracting surfaces are much larger than on the reflecting detector. Furthermore, there is a broad energy acceptance; the momentum transfer is proportional to the total number of neutrinos, integrated over all energies.

V. TORQUE VERSUS FORCE

Since neutrinos are two-component chiral particles, with spin oriented along the momentum, there is necessarily some angular momentum $\Delta\vec{J}$ transferred along with the linear momentum $\Delta\vec{p}$. This means that we are presented with the alternative of detecting the torque on the detector instead of the force.

In each individual collision, there is a fixed ratio of $\Delta\vec{J}$ to $\Delta\vec{p}$. Because neither $|\vec{J}|$ nor $|\vec{p}|$ change, the two vector diagrams for conservation of $\vec{J}$ and $\vec{p}$ are similar, and imply

$$\Delta\vec{J} = \mp \tfrac{1}{2}\lambda\!\!\!^{-}\Delta\vec{p}\,, \tag{22}$$

where $\lambda\!\!\!^{-}$ is the neutrino wavelength and the sign pertains to neutrinos (antineutrinos). It follows that the mean torque and force exerted on the detector have the same ratio

$$\left\langle \frac{d\vec{J}}{dt} \right\rangle = \mp \frac{1}{2} \left\langle \lambda\!\!\!^{-} \frac{d\vec{p}}{dt} \right\rangle . \tag{23}$$

We can compare the sensitivity of detecting torque and force by imposing, in place of Eq. (12), the condition

$$J^2/2I \geq kT\,, \tag{24}$$

where $I = M\rho^2$ is the moment of intertia and ρ the radius of gyration of the detector. The integration times for measuring torque and force are proportional:

$$t_1 = 2\frac{\rho}{\lambda\!\!\!^{-}}\, t_2\,, \tag{25}$$

where t_1 refers to the torque-sensing device and t_2 to the force measuring device. The sensitivities will be comparable only if the radius of gyration is comparable to $\lambda\!\!\!^{-}$. For the refracting detector, that simply means that the thin surface is ruled so that $\Delta\vec{p}$ and $\Delta\vec{J}$ lie *parallel* to the surface, not perpendicular to it.

There is a major advantage in the torque-measuring device: it would respond to a flux with equal mixture of neutrinos and antineutrinos. The opposite sense of their spins is compensated by a change in sign of their scattering amplitudes and angles of refraction, so that neutrinos and antineutrinos with the same incident momentum will have opposite momentum transfers $\Delta\vec{p}$, but the same angular momentum transfer $\Delta\vec{J}$. This could be an important consideration in the search for cosmic neutrinos.

VI. CONCLUSIONS

Our principal conclusion is that detectors using coherent processes have advantages over those using incoherent processes, for the detection of neutrinos in the energy range below several MeV. This idea has been stated earlier by Stodolsky and by Opher, and is strengthened by our result. Such detectors respond to the integrated flux of neutrinos, independent of their energy, and would provide an independent confirmation of calculations of the spectrum of neutrinos from fission reactors, from the sun, and from other sources. The use of coherent detectors makes the problem of measuring the flux of low-energy neutrinos analogous to the problem of detecting gravity waves. Recent progress in reducing the noise per mode in a large gravity wave detector to the thermal level provides hope that coherent neutrino detectors may become a practical reality.

We have considered here only the simplest coherent process, elastic scattering, involving recoil of the entire detector. This requires only a theory for the index of refraction. There is an obvious extension to other processes, involving the coherent excitation of low-lying collective modes such as acoustic phonons. We hope to investigate these next.

At a practical level, it seems clear the the refracting detector described in Sec. V is much easier to build than the reflecting detector. Because of the very high angular resolution of total reflection, the reflecting detector must be made of optically flat surfaces, aligned to 10^{-5} rad. On the contrary, the refracting surfaces need only be approximately flat and much more crudely aligned. It is an interesting challenge whether such a detector is capable of detecting the relatively well known flux of neutrinos from a fission reactor ($j \sim 10^{13}\bar{\nu}_e/\mathrm{cm}^2\,\mathrm{sec}$). An even more exciting and difficult challenge would be to search in this way for solar neutrinos ($j \sim 10^{11}\nu_e/\mathrm{cm}^2\,\mathrm{sec}$) and for cosmic neutrinos.

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[11]We have assumed either that the atom has zero net spin for electrons and quarks separately, or that these spins have zero ensemble average. If these conditions are relaxed, the refractive index would exhibit a parity nonconserving dependence on the neutrino direction; this has been discussed by Stodolsky in Ref. 2.