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Rescattering in the nucleus for $\pi^- d$ interactions at 15 GeV/c

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We present the $\pi^- d$ charged multiplicity distributions at 15 GeV/c and examine the probability that the products of a $\pi^- n$ collision in a deuterium nucleus rescatter on the proton. Averaged over all topologies, this probability is 0.13 ± 0.02. The rescatter probability as a function of the number of charged particles produced in the $\pi^- n$ interaction exhibits an increase at high multiplicity.

I. INTRODUCTION

It is well known that high-energy interactions with nuclei may, through rescattering within the nucleus, provide information on the development of hadronic matter during and shortly after an interaction. For example, the magnitude of the ratio of the mean charged multiplicity for interactions on nuclei to that for interactions on nucleons, $\langle n_{ch}\rangle_{nucleus}/\langle n_{ch}\rangle_{nucleon}$, is cited as evidence against a simple cascade picture for high-energy hadron-nucleus collisions.[1,2] While there exists a large body of theoretical literature on the nascent properties of hadronic matter, experimental information is still rather sparse.[2]

The potential for using multiplicity distributions in deuteron-target experiments in this connection has become apparent in recent years.[3,4] In this paper we obtain the charged-particle multiplicity distribution for $\pi^- d$ interactions at 15 GeV/c, and use it to investigate the probability that the product of a $\pi^- n$ interaction (in the deuteron) will rescatter on the proton. Most of the above experiments[3,4] only make measurements for charged multiplicities $\geqslant 3$, and either neglect or make estimates for the contribution of lower multiplicities. In this work, we include measurements of these lower charged multiplicities ($n_{ch} = 0$, 1, and 2).

II. EXPERIMENTAL DETAILS

This report covers a special scan of a 91 000-frame subsample from a 950 000-frame exposure of the 82-in. SLAC-LBL deuterium bubble chamber to a 15-GeV/c rf-separated π^- beam at SLAC.[5,6] In this special scan we have recorded 40 367 interactions of all event topologies. For each event, the scanner recorded the numbers of positive and negative outgoing tracks, classifying the positives further according to whether or not they stopped (without decaying) in the bubble-chamber liquid (i.e., were protons or deuterons of momentum $\lesssim$400–900 MeV/c). For the stopping positives the projected range and the angle with respect to the beam were recorded. In addition, angle and distance information was recorded for secondary interactions (of both charged and neutral products), and for neutral-strange-particle decays and $\gamma \to e^+e^-$ conversions. In order to determine scan efficiencies, a portion of the sample (~13%) was scanned a second time with subsequent comparison scanning by a physicist to resolve discrepancies.[6]

Although we are concerned primarily in this report with scan data, measurements of $\gamma \to e^+e^-$ conversions and neutral-strange-particle decays have been used, especially in the evaluation of the one- and two-prong inelastic cross sections.[7] Also, measurements made of three-, four-, five-, and six-prong events for the entire exposure[5] have been used to determine proton momentum spectra.

III. CORRECTIONS TO THE MULTIPLICITY DISTRIBUTION

The numbers of events found by the scanners are given for each charge multiplicity in Table I. To this scanned distribution we make a number of "standard" corrections, which do not directly depend on the nature of the target. We also make a "spectator correction," to account for those events which may be classified, in the impulse approximation, as $\pi^- n$ interactions with proton "spectators," but where the proton nevertheless leaves a visible track due to its Fermi momentum at the time of collision. (A "spectator" proton in the impulse approximation is a proton which does not participate in an interaction between its companion neutron in the deuteron and the beam particle.)

TABLE I. 15-GeV/c $\pi^- d$ charged-multiplicity distribution.

n_{ch}	Raw events	Inelastic events after "standard" corrections	Inelastic events after spectator corrections	Inelastic cross sections (mb)
0	356	438 ± 37	426 ± 36	0.461 ± 0.041
1	1 855	$1\,245 \pm 206$	$1\,798 \pm 225$	1.94 ± 0.25
2	8 742	$5\,353 \pm 581$	$4\,677 \pm 572$	5.06 ± 0.63
3	4 523	$4\,656 \pm 71$	$6\,479 \pm 95$	7.00 ± 0.20
4	10 913	$11\,208 \pm 115$	$9\,126 \pm 122$	9.87 ± 0.28
5	3 313	$3\,305 \pm 60$	$4\,734 \pm 83$	5.12 ± 0.15
6	6 431	$6\,411 \pm 90$	$4\,844 \pm 93$	5.24 ± 0.16
7	1 332	$1\,281 \pm 38$	$1\,751 \pm 51$	1.89 ± 0.07
8	2 125	$2\,040 \pm 50$	$1\,527 \pm 51$	1.65 ± 0.07
9	255	227 ± 16	302 ± 21	0.327 ± 0.024
10	432	401 ± 22	317 ± 22	0.343 ± 0.025
11	26	21 ± 5.1	24 ± 5.9	0.026 ± 0.006
12	55	45 ± 8.2	41 ± 8.3	0.044 ± 0.009
13	5	$4.4^{+3.5}_{-2.3}$	$4.4^{+3.5}_{-2.3}$	$0.005^{+0.004}_{-0.002}$
14	3	$2.8^{+3.0}_{-1.7}$	$2.7^{+2.9}_{-1.7}$	$0.003^{+0.003}_{-0.002}$
15	0	$0^{+1.2}_{-0}$	$0^{+1.2}_{-0}$	$0^{+0.002}_{-0}$
16	1	$1^{+2.3}_{-0.8}$	$1^{+2.2}_{-0.8}$	$0.001^{+0.002}_{-0.001}$
Total	40 367	$36\,639 \pm 645$	$36\,053 \pm 653$	38.98 ± 1.17
Even		$25\,900 \pm 603$	$20\,961 \pm 618$	22.67 ± 0.86
Odd		$10\,739 \pm 230$	$15\,092 \pm 264$	16.31 ± 0.49

A. "Standard" corrections

In the "standard" correction category, we take into account corrections for (i) scanning efficiencies (86%, 70%, 87%, and 98.6% for 0, 1, 2 and ≥ 3 prongs, respectively), (ii) unresolved close secondary interactions (very minor, ~0.4% of the events), (iii) unresolved close γ conversions and neutral-strange-particle decays (also minor, <0.2% of the events), (iv) Dalitz pairs (1024 ± 82 Dalitz pairs in the efficiency-corrected sample). However, very-low-momentum-transfer one- and two-prong events are essentially invisible to the scanner, and, hence, are not adequately corrected for by the scan efficiencies in (i) above. Since we really want the inelastic[7] cross sections we may circumvent this difficulty, and obtain directly the inelastic contributions to the one- and two-prong samples, by examining events with observed γ conversions and/or neutral-strange-particle decays and correcting for the $\pi^- d \to \pi^- \pi^+ nn$ channel (see Appendix A). We thus obtain very nearly the entire inelastic contribution. The multiplicity distribution with these standard corrections applied is given in Table I.

B. "Spectator" correction

The "spectator" correction, for the shift from odd-prong to even-prong events due to the presence of $\pi^- n$ interactions with a visible proton spectator, is estimated using events with a visible backward (in the laboratory) proton track. For each multiplicity, the total number of visible-spectator events is obtained by multiplying the number of visible-backward-proton events by the expected ratio of total visible to backward visible spectators, based on the deuteron wave function[8] and the flux factor.[9] The result is then added to the odd-prong sample, and subtracted from the even. We are using here the fact that even-prong events with a visible backward proton are nearly always $\pi^- n$ interactions, because the recoil proton from a $\pi^- p$ interaction never goes backward except in rare cases due to initial Fermi motion (see below).

A number of points should be made concerning the calculation of the expected ratio of total visible to backward visible spectator protons: (i) We must account for the fact that the cross section is dependent upon s (s = center-of-mass energy squared of the beam plus target neutron system), particularly at high multiplicities, and, hence upon the spectator proton direction and momentum. In the calculation we use the empirical scaling form for topological cross sections of Czyzewski and Rybicki.[10] (ii) We integrate over a Gaussian beam-momentum distribution (of mean 14.92 GeV/c and standard deviation 0.06 GeV/c), although this turns out to have negligible effect.

(iii) The spectator proton is assumed to be on the mass shell, while the target neutron is taken off shell to conserve energy. (iv) The effective minimum spectator-proton momentum (p_{min}) which is visible in the bubble chamber is estimated from the data as follows. Figure 1 shows a histogram of the events with a backward proton, as a function of the proton's projected range. Superimposed on the histogram is a curve giving the expected spectator distribution in projected range according to the Hulthén wave function,[6,8] normalized to the data in the region from 0.4 to 4 cm. The curve and data are very similar except in the first bin (0 to 0.2 cm), where there is a large loss of invisible protons. Comparison with the expected distribution in projected range indicates that ~68% of the spectators are invisible, corresponding to an effective p_{min} of ~95 MeV/c. This result is in agreement with estimates based on measured momentum distributions in the following fitted channels (by comparing with the Hulthén distribution prediction, and also by comparing the ratios of fitted events with and without visible spectators): $\pi^- d \to pp\pi^-\pi^-$, $\pi^- d \to pp\pi^-\pi^-\pi^0$, and $\pi^- d \to pp\pi^-\pi^-\pi^+\pi^-$. Thus, in our calculation we use $p_{min} = 95 \pm 5$ MeV/c.

The results of the calculation are given in Table II. We multiply the number of visible backward protons at each multiplicity by the appropriate factor from Table II to obtain the total spectator proton contribution to the even-prong events. This result is then subtracted from the even-prong sample and added to the next lower odd-prong sample. The final multiplicity distribution[11] is given in Table I, and plotted in Fig. 2. The corresponding partial cross sections are also given in Table I, using our measured microbarn equivalent of 925 ± 23 events/mb.[6] Various moments of the even and odd prong distributions are given in Table III.

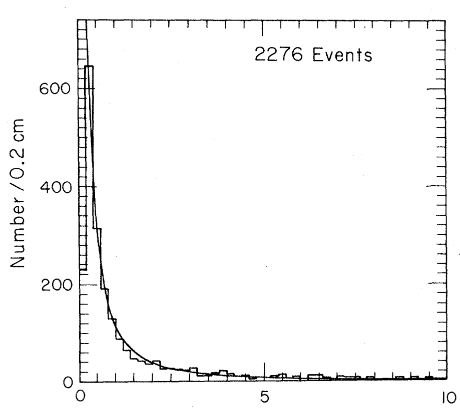

FIG. 1. Distribution of projected (parallel to the film plane) range for visible backward protons. The curve is the spectator-proton distribution prediction based on the Hulthén wave function normalized to the data between 0.4 and 4.0 cm.

C. Coherent events and invisible or backward proton recoils

It is now possible to interpret the odd-prong events (now, "odd" and "even" refer to the samples as corrected for spectators) as $\pi^- n$ interactions (with proton spectator), and the even as $\pi^- p$ events (with neutron spectator) or double-scatter events where both nucleons participated (incoherently). Note that double-scatter events contribute to the even-prong sample because the proton is always involved in the interaction, implying an even number of outgoing charged particles. There are, however, two potential complications.

First, coherent interactions may not be uniquely classified in terms of "$\pi^- n$" or "$\pi^- p$." The deuteron form factor limits significant coherent processes to low multiplicity; the dominant inelastic coherent reactions are expected to be $\pi^- d \to \pi^-\pi^0\pi^0 d$ and $\pi^-\pi^-\pi^+ d$. The cross section for the latter has been measured in this experiment[12] to be 0.694 ± 0.073 mb, and the cross section for the former should be approximately the same. It has also been determined in this experiment[12] that the $\pi^-\pi^-\pi^+ d$ sample contributes 308 ± 49 μb to the three-prong events and 386 ± 44 μb to the four-prong events. Thus, the inelastic coherent events are approximately equally distributed between the even- and odd-prong samples, and will not produce a large bias in the even-odd comparisons we shall make.[6] These effects will be discussed further where relevant.

The second concern is that we may have some $\pi^- p$ contamination in the odd-prong sample due to events where the momentum transfer to the recoil proton is too small to produce a visible proton track. Experiments in hydrogen demonstrate that this is rare, and primarily confined to the elastic events.[13] Since the minimum squared momentum transfer (t_{min}) required to produce a system of mass m against a recoil proton rises ap-

TABLE II. Ratio of total to backward visible proton spectators (calculated).

n_{ch}	(Total visible)/(Backward visible)
2	2.003 ± 0.001
4	2.095 ± 0.003
6	2.233 ± 0.009
8	2.421 ± 0.017
10	2.670 ± 0.029
12	3.001 ± 0.047

TABLE III. Moments of the 15-GeV/c $\pi^- d$ multiplicity distribution. n=charged multiplicity. D=dispersion$=(\langle n^2\rangle - \langle n\rangle^2)^{1/2}$. $f_2 = \langle n(n-1)\rangle - \langle n\rangle^2$.

Moment	Even-prong events	Odd-prong events
$\langle n\rangle$	4.334 ± 0.066	3.989 ± 0.047
$\langle n^2\rangle$	22.67 ± 0.53	19.37 ± 0.32
$\langle n^3\rangle$	135.9 ± 3.8	108.1 ± 2.1
$\langle n(n-1)\rangle$	18.34 ± 0.47	15.38 ± 0.27
$\langle n(n-1)(n-2)\rangle$	76.6 ± 2.4	57.9 ± 1.3
D	1.97 ± 0.11	1.860 ± 0.064
$\langle n\rangle/D$	2.199 ± 0.047	2.144 ± 0.050
f_2	−0.45 ± 0.12	−0.53 ± 0.13

proximately as m^4 (for $m_\pi^2 \ll m^2 \ll s$), the only potentially significant invisible recoil inelastic process is low-mass beam diffractive dissociation (e.g., $\pi \to A_1$, with $m \simeq 1$ GeV), where at most a few percent of the interactions will have an invisible recoil proton. Thus, the contamination will be small, and almost entirely confined to the one- and three-prong samples.

It is also possible that the low-t $\pi^- p$ interactions could contaminate the visible backward proton spectator sample, due to an initial backward Fermi momentum of the target proton. Monte Carlo comparisons of this contribution with the invisible recoil contamination indicate that this is the lesser of the two sources of contamination.

We conclude that the total $\pi^- p$ contamination in the final odd-prong sample may be as high as ~1%, but probably substantially less, for the one- and three-prong events and negligible for higher multiplicities. This result, which is expected to be approximately energy-independent, is consistent with the estimate made for pd collisions at 100 GeV/c (by Lys *et al.*[4]) using 100-GeV/c pp data by smearing the target-proton momentum according to a Hulthén distribution.

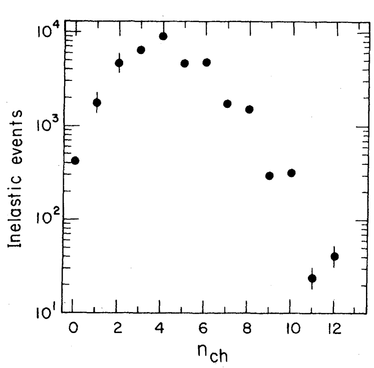

FIG. 2. Charged-multiplicity distribution for inelastic events after all corrections, including the spectator correction.

IV. RESCATTER PROBABILITIES

A. Overall rescatter probability

The first quantity we wish to investigate is the overall probability for a rescatter to occur in the deuteron following an inelastic interaction. Using the facts that rescatter events contribute to the even-prong sample, and that any events in the odd-prong sample are $\pi^- n$ interactions where a double-scatter has not occurred, this probability is readily evaluated from the difference between the numbers of events in the odd- and even-prong samples (after correcting for visible proton spectators):

$$P = \frac{N_e - \rho N_o}{N_e + N_o} = 0.130 \pm 0.020\,. \tag{1}$$

Here, P is the overall rescatter probability and N_e and N_o are the numbers of events in the even- and odd-prong samples, respectively (including zero-, one-, and two-prong events). The factor ρ, given by

$$\rho = \sigma_{\rm inel}(\pi^- p)/\sigma_{\rm inel}(\pi^- n) = 1.078 \pm 0.022\,,$$

compensates for the fact that the total inelastic $\pi^- p$ and $\pi^- n$ cross sections are not equal, and hence, that N_e would not equal N_o even in the absence of rescattering. To evaluate ρ, we assume that $\sigma_{\rm inel}(\pi^- n) = \sigma_{\rm inel}(\pi^+ p)$ (charge symmetry), and use the cross sections in Ref. 14.

We have, in this estimate, neglected the contamination of N_e from events where the first interaction is elastic and an inelastic rescatter occurs (N_e and N_o do not include the corresponding unrescattered, or elastically rescattered, events.) We obtain an upper estimate on the effect of this contamination by assuming that the inelastic rescatter probability following an initial elastic interaction is less than the *total* (elastic plus inelastic) probability following an initial inelastic interaction. This assumption would yield a maximum correction to P of −0.017. As discussed above, there may also be a shift from the even- to the odd-prong sample due to invisible or backward proton recoils from low-t $\pi^- p$ interactions. Note that the effect of the even-prong contamination by inelastic rescatters from an elastic first interaction is opposite in sign to the effect of the odd-prong contamination by low-t $\pi^- p$ interactions. Thus, these two contributions, which may be of the same order, will tend to cancel in their effect on our measurement of the rescatter probability. A shift of either sign could also be produced by an imbalance in the division of coherent events between even and odd multiplicities. If our bias were such that there was a shift of 0.25 mb (1% of the π^-N *total* cross section) from the even- to the odd-prong sample, the correction to P would be 0.014, while a shift in the opposite direction of 0.25 mb would imply a correction of -0.013. Our best estimate remains $P = 0.13 \pm 0.02$, although the systematic effects could be nearly as large as the error quoted.

This value for P is larger than that expected for a single beamlike particle emerging from the first interaction[15] by somewhat more than a factor of 2. Thus, on the average, the hadronic matter produced in the first interaction looks, by the time it reaches the second nucleon, "bigger" than a beam pion. On the other hand, since the average multiplicity (including neutrals) of a π^-N interaction at this energy is ~6, the first interaction product appears smaller, at the second-nucleon distance, than it does at an asymptotic distance (where the individual particle cross sections simply add).

Our measured value is compatible with a simple instantaneous-production (cascade) model calculation (see Appendix B). This instantaneous-production model calculation assumes that the particles produced in an interaction on one of the nucleons in the deuteron follow trajectories given by the extrapolation back in time of the asymptotic trajectories that they would have in a collision on a free nucleon. If one or more of these trajectories intersects the cross section presented by the second nucleon of the deuteron, with a Hulthén spatial distribution, then a rescatter is taken to occur. Particle production is assumed to be uniform in rapidity and exponential in transverse momentum, with a Poisson multiplicity distribution.

Noninstantaneous-production models, such as the energy flux cascade model of Gottfried,[1] generally assume that the rescattering hadronic matter remains beamlike in size [i.e., that the mean number of scatters is given by the parameter $\bar{\nu}$ (Ref. 16)]. The inconsistency of this with our measurement may result from the neglect (in some such models) of the rescattering of the target-recoil product.

Comparison with the results of $\pi^{\pm}d$ experiments at higher energies[3,4] indicates that the rescatter probability is consistent with being independent of the beam momentum. Note that we have included measurements of the zero-, one-, and two-prong cross sections in our analysis, while the higher-energy experiments with which we are comparing had not. However, if we similarly use estimates for these cross sections, instead of our measurements, the resulting rescatter probability (0.13–0.15) is still consistent with the higher-energy data. Since the π^-p average multiplicity doubles between 15 and 200 GeV/c, this energy independence is incompatible with naive cascade models which predict that the rescatter probability should be proportional to the number of outgoing particles from the first interaction. This alone cannot be taken as evidence against all "simple" cascade models; it is not difficult to develop such a model with just this behavior by including the overlap in particle trajectories and the angular distribution of particles in the target frame (Appendix B).

B. Multiplicity dependence of the rescatter probability

We now investigate further the early development of the product of a hadronic interaction by measuring the probability that the product of an initial interaction on the neutron will rescatter on the proton as a function of the final multiplicity that the π^-n interaction would have had in the absence of a second interaction. This is possible because we can measure the probability that the product of the π^-n interaction did *not* rescatter, since our corrected odd-prong sample constitutes precisely these events. Thus, the rescatter probability, P_n, as a function of the charged multiplicity (n) of the π^-n interaction is given by (Dziunikowska *et al.*[3])

$$P_n = 1 - \sigma_n(``\pi^-n")/\sigma_n(\pi^-n)\,. \qquad (2)$$

Here, $\sigma_n(\pi^-n)$ is the cross section to produce n charged particles on a free neutron, and $\sigma_n(``\pi^-n")$ is the measured n (odd) prong cross section in this experiment, i.e., the cross section for a π^-n interaction in deuterium followed by no rescatter.

The main difficulty in evaluating Eq. (2) is that the π^-n charged multiplicity distribution, $\sigma_n(\pi^-n)$, has not been directly measured. However, we are able to use the detailed π^+p cross sections at the nearby energy of 16 GeV,[17,18] plus the assumption of charge symmetry,[19] to estimate the 16-GeV/c π^-n multiplicity distribution, which we then scale to 15 GeV/c.[6,10] The results are tabulated in Table IV. The quoted errors include an estimate of the errors introduced by scaling uncertainties.[6]

Finally, we renormalize the $\sigma_n(``\pi^-n")$ so that

$$\sum_n \sigma_n(``\pi^-n") = (1-P)\sum_n \sigma_n(\pi^-n)\,. \qquad (3)$$

That is, we require that the odd-prong data, relative to the free-neutron-target cross section, give the correct overall rescatter probability measured above. This is necessary to correct for screening, where an initial π^-n interaction is prevented when the proton is "in the way." The resulting values for P_n are given in Table V and plotted as a function of the initial π^-n interaction

TABLE IV. Estimated $\pi^- n$ multiplicity distribution from $\pi^+ p$ data.

n_{ch}	$\sigma_n(\pi^- n)$ (mb) 16 GeV/c	15 GeV/c [a]
1 (total)	6.26 ±0.21	6.61 ±0.38
(inelastic)	2.05 ±0.21	2.18 ±0.22
3	8.11 ±0.17	8.31 ±0.23
5	6.62 ±0.15	6.45 ±0.20
7	2.38 ±0.07	2.18 ±0.08
9	0.570±0.021	0.487±0.024
11	0.072±0.006	0.057±0.006

[a] Scaled from the 16-GeV/c estimates according to the Czyzewski and Rybicki (Ref. 10) scaling function.

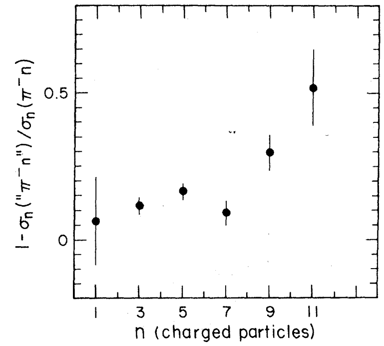

FIG. 3. Rescatter probability for an initial $\pi^- n$ interaction as a function of the charged multiplicity of the $\pi^- n$ interaction, for $\pi^- n$ at 15 GeV/c (this experiment).

asymptotic charged multiplicity in Fig. 3.

We find a substantial rise in P_n at the highest multiplicities ($n=9, 11$). The first four points ($n \leq 7$) are consistent with constant P_n; however, the confidence level for P_n to be constant for $n \leq 11$ is only 0.2%. The overall rescatter probability cancels out the χ^2 expression, so that the normalization according to Eq. (3) has no effect here. Also, the maximum sizes of the possible even-odd biases discussed earlier are too small to affect this result.

We conclude that, at 15 GeV, the product of a $\pi^- n$ interaction has time to develop some part of its final-state characteristics (i.e., at least the difference between low and high charged multiplicity) by the time it reaches the second nucleon in the deuteron. This effect is a signature for "cascading," since the more particles you have, in the cascade picture, the greater the chance for a rescatter (assuming that the angular distribution of emitted particles does not strongly narrow with increasing multiplicity). However, we expect that the fastest rise of P_n with n should be at low n in a cascade picture, because, for large n, the particles overlap and do not add as much rescatter probability for each additional particle as at low multiplicity. The observation of a significant rise only at the high multiplicities indicates that there may be more than simple cascading occurring. The issue might be partly resolved if, relative to high multiplicities, the low multiplicities have a larger diffractive component or are generally more peripheral.

TABLE V. Multiplicity dependence of rescatter probability for 15-GeV/c $\pi^- d$ interactions.

n_{ch}	P_n
1	0.066±0.152
3	0.116±0.028
5	0.167±0.030
7	0.090±0.043
9	0.295±0.060
11	0.521±0.130

Two other deuteron-target experiments have reported measurements of P_n. Both are $\pi^- d$, one at 21 GeV/c (Ansorge *et al.*[3]), the other at 205 GeV/c (Dziunikowska *et al.*[3]), and both see no significant rise in P_n with n. However, the 21-GeV/c data as reported do not include the fact that the forward-to-backward visible spectator correction should depend on multiplicity. In addition, the values for the free-neutron-target cross sections, $\sigma(\pi^- n)$, were estimated using $\pi^+ p$ data and statistical isospin assumptions (insufficiently detailed $\pi^+ p$ data are available near 21 GeV/c to make a charge-symmetry estimate such as we make at 15 GeV/c). Because of uncertainties in the importance of these issues, we hesitate to conclude that a transition has occurred between 15 and 21 GeV, or that there is a significant conflict between the two data sets. It does appear, however, that a change has occurred between 15 GeV and 205 GeV, with any indication of cascading becoming negligible at 205 GeV.

V. THE EVEN-PRONG DISTRIBUTION

Another distribution of value in double-scattering models, used by Dziunikowska *et al.*,[3] is the comparison of the measured even-prong multiplicity distribution, σ_n("$\pi^- p$"), including double-scatter events, with the corresponding topological cross sections on a free proton target, $\sigma_n(\pi^- p)$. The σ_n("$\pi^- p$") distribution includes effects of both the overall cross section excess from rescattered $\pi^- n$ events, and shifts among the multiplicities due to new-particle production in the double scatter. The ratio $\alpha_n = \sigma_n(\text{"}\pi^- p\text{"})/\sigma_n(\pi^- p)$ for our 15-GeV/c data is given in Table VI and shown in Fig. 4. Note that the σ_n("πp") are our even-prong cross sections as measured, with no renormalization involved. The $\sigma_n(\pi^- p)$ used are the 16-GeV/c

TABLE VI. The σ_n("$\pi^- p$")/$\sigma_n(\pi^- p)$ distribution at 15 GeV/c.

n_{ch}	σ_n("$\pi^- p$")/$\sigma_n(\pi^- p)$	BLRW model prediction [a]	CTM prediction [b]
0	0.92 ±0.41		0.86
2(inel)	0.89 ±0.12		0.92
4	1.091±0.049	1.12±0.04	0.97
6	1.139±0.055	1.16±0.04	1.07
8	1.364±0.098	1.20±0.07	1.27
10	2.02 ±0.26	1.29±0.14	1.68
12	2.9 ±1.7	1.38±0.73	2.50
14	4 ±5	1.63±1.63	4.26

[a] Reference 21.
[b] CTM = coherent tube model (Ref. 24).

values,[20] scaled slightly to 15 GeV/c.[10] The observed α_n distribution exhibits a tendency to increase with charged multiplicity, as expected if double scattering tends to increase the multiplicity of products in an interaction.

We can compare the α_n distribution with the prediction of a picture of hadronic interactions which assumes the absence of singular short-range forces. Here, we shall consider the prediction due to Baker *et al.* (BLRW).[21] It has been shown[21] that for high-energy hadron-deuteron interactions (≥100 GeV) the α_n distributions predicted by this picture agree well with the available data. In Table VI and Fig. 4 we give the prediction for 15 GeV obtained according to the prescription of BLRW.[21,22] Figure 4 suggests that the data exhibit a faster rise of α_n than the prediction. This would be consistent with the indication of cascading that we observe at 15 GeV/c.

The coherent tube model (CTM) also makes a prediction for the α_n distribution, which previously has been compared to data at higher energies.[24]

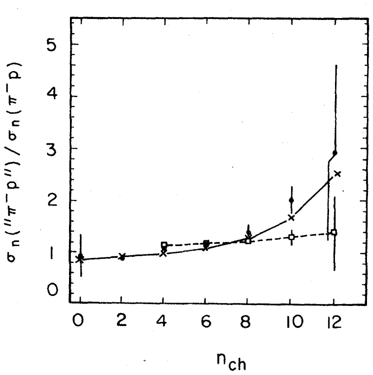

FIG. 4. The ratio of the (inelastic) even-prong "$\pi^- p$" cross section (this experiment) to the $\pi^- p$ cross section (Ref. 20) (scaled from 16 to 15 GeV/c) as a function of charged multiplicity. The "×" points are the prediction of the coherent tube model (Ref. 24) and the squares are the prediction of the BLRW model (Ref. 21).

We have estimated the prediction of this model for 15-GeV/c interactions,[25] given in Table VI and Fig. 4. The CTM prediction for α_n and the data are compatible, considering the possibility for systematic error in the estimation of the CTM predicted values.[25] However, because the observed P_n distribution is not constant (as the CTM would imply), the CTM fails to consistently describe double scattering at 15 GeV/c.

VI. CONCLUSIONS

We have measured the $\pi^- d$ charged-particle multiplicity distribution at 15 GeV/c, and have used the results to investigate double scattering inside the deuteron. The average probability for a rescatter to occur is $P = 0.13 \pm 0.02$ (statistical error only, systematic uncertainty nearly as large). As a function of the number of charged particles produced in an initial $\pi^- n$ interaction in the deuteron, the rescatter probability exhibits a rise at high multiplicities. This may be interpreted as evidence for cascading at 15 GeV/c. However, this distribution is constant at low multiplicities followed by a fast increase, which is inconsistent with a simple cascade picture which predicts a rapid increase at low multiplicities followed by a leveling off.

We have also compared the σ_n("$\pi^- p$"), for n even, including both single-scatter proton-target interactions and double scatters, with the corresponding $\sigma_n(\pi^- p)$ on free protons. Comparison of the ratio of the two with the prediction of the BLRW model suggests an incompatibility which may be due to cascading in 15-GeV/c $\pi^- d$ interactions.

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APPENDIX A: ESTIMATING THE ONE- AND TWO-PRONG INELASTIC SAMPLES

It is difficult to measure the one- and two-prong inelastic[7] cross sections by simply counting events because of the invisibility of very low-momentum-transfer events, and because of the contamination from elastic interactions. However, we may avoid this difficulty by using the fact that nearly all inelastic one- and two-prong events involve π^0's and/or neutral strange particles. By using the sample of events where a $\gamma \to e^+e^-$ conversion and/or a neutral-strange-particle decay is observed in the bubble chamber, we immediately eliminate any confusion with elastic events. This selection also minimizes the low-momentum-transfer problem because the low-momentum-transfer interactions are dominantly elastic, which we now exclude, and because the presence of a $\gamma \to e^+e^-$ conversion and/or a neutral-strange-particle decay provides an obvious flag for the existence of an event. The main inelastic channels missed by this technique are expected to be $\pi^-d \to \pi^-\pi^-\pi^+nn$, $\pi^-d \to K^-K^-K^+nn$, and $\pi^-d \to \Sigma^-K^+n$. The cross section for the first of these has been measured in the charge-symmetric channel (at 15 GeV/c) $\pi^+d \to \pi^+\pi^+\pi^-pp$ as 0.376 ± 0.025 mb.[26] This turns out to be smaller than the statistical error in our two-prong inelastic measurement (to which it applies); however, we correct for it anyway. The latter two channels are assumed to yield negligible contributions.

To determine the inelastic cross sections from the inclusive γ and neutral-strange-particle cross sections, we must determine the mean number of such particles in an interaction (selecting interactions that have at least one). This information may be obtained by studying the observed distribution of the number of γ conversions or strange-particle decays per event. The remainder of this Appendix describes the formalism that we have used.[6]

Let R_n be the probability for an event to have n particles of the desired type, given that it has at least one, and define Q_m as the probability to observe m particles (given there exists at least one). If ϵ is the efficiency for observing a particle, the moments of the R_n probability distribution are related to the Q_m by

$$\langle n(n-1)\cdots(n-k+1)\rangle = \frac{1}{\epsilon^k}\sum_{m=k}^{M}\frac{m!}{(m-k)!}Q_m, \qquad \text{(A1)}$$

where M is a number such that $R_n = 0$ for all $n > M$. The connection with the data is made in the ratios of Q's, which are just equal to the ratios of the corresponding numbers of events:

$$Q_i/Q_j = N_i/N_j, \qquad \text{(A2)}$$

where N_i is the number of events which are observed to have i particles of the desired type.

At this point, we turn to the specific cases at hand, starting with the γ's. Motivated by the facts that most γ's come from π^0 decays, and that our charged multiplicity distribution is very nearly Poisson, we assume that R_n is of the form

$$R_n = \frac{e^{-\mu}}{1-e^{-\mu}}\frac{\mu^n}{n!}. \qquad \text{(A3)}$$

With this probability distribution, (A1) and (A2) yield the result

$$\mu = \frac{1}{\epsilon}\frac{\sum_{n=k+1}^{\infty}\frac{n!}{(n-k-1)!}N_n}{\sum_{n=k}^{\infty}\frac{n!}{(n-k)!}N_n}. \qquad \text{(A4)}$$

The number of events with at least one γ (whether or not the γ is observed) $\mathfrak{N}_{\geq 1}$ is then estimated by[27]

$$\mathfrak{N}_{\geq 1} = (1-e^{-\mu})\left(\sum_{n=1}^{\infty} nN_n\right)\frac{\sum_{n=k}^{\infty}\frac{n!}{(n-k)!}N_n}{\sum_{n=k+1}^{\infty}\frac{n!}{(n-k-1)!}N_n}. \qquad \text{(A5)}$$

We note that the detection efficiency, ϵ, drops out of this equation except in the small $e^{-\mu}$ term.

Turning now to the neutral-strange-particle case, we start with the assumption that only single associated production is significant, i.e., we neglect the possibility that that number of strange particles (charged plus neutral) produced is different from two, in a strange-particle event.[6] Thus, we may set $M = 2$ in (A1), and a little algebra yields the number of events with at least one K^0 or Λ^0 (whether or not it is observed):

$$\mathfrak{N}_{\geq 1} = N_1 + N_2(2 - 1/\epsilon). \qquad \text{(A6)}$$

The symbols in (A6) have the corresponding meanings as for the γ case. Note that ϵ includes the contribution to the observational inefficiency from neutral decay modes, and K_L^0's.

APPENDIX B: AN INSTANTANEOUS-PRODUCTION MODEL FOR SCATTERING IN THE DEUTERON

The instantaneous production model (or "cascade" model) assumes that in a hadron-nucleon interaction, it is a good approximation to suppose that the asymptotic final state is achieved in a distance which is short compared with the internucleon spacing within a nucleus. Thus, a hadron-nucleus interaction is analyzed as a sequence of quasifree hadron-nucleon interactions. In this Appendix, we describe such a model for scattering in the deuteron. This model is intended to provide us with a useful intuitive foundation with which to interpret the data and the predictions of other models, some of which have much firmer theoretical foundations. Our model takes into account the fact that the products of a hadron-nucleon collision have strongly correlated trajectories (because of the Lorentz transformation to the laboratory frame, and because of the limited transverse momentum), and also avoids multiple counting in tallying double-scatter events. These points, which have sometimes been ignored in statements about cascade hypothesis predictions, are responsible for the fact that the overall rescatter probability is predicted by our model to be nearly independent of energy. However, because of the semiclassical nature of this model, we are forced to make some ad hoc assumptions, which we shall note in our discussion.

The quantity of interest to us here is P_N, the probability that a rescatter on the second nucleon of the deuteron occurs given an N-product (including neutrals) interaction on the first nucleon, all at a specified beam energy. The overall rescatter probability is obtained from this simply by

$$P=\sum_N P_N \sigma_N/\sigma_{\mathrm{TOT}}\,, \tag{B1}$$

where σ_N and σ_{TOT} refer to the N-particle and total inelastic cross sections on a single nucleon.

The instantaneous-production model makes the implicit assumption that, as far as rescattering properties are concerned, the outgoing state may be represented by an incoherent superposition of free-particle states, even though these "particles" may be within the range of the strong force of each other. In our model, motivated by the assumed instantaneous nature of the production, we shall take the product of a hadron-nucleon interaction to be an incoherent superposition of free-particle trajectories with origin in a common point.

An impulse approximation is used, so that no flux factor effects due to Fermi momenta are considered. Nucleon 1 is taken to be centered at the origin, while nucleon 2 has its center at coordinate $\vec{x}$. A beam + nucleon 1 interaction producing N outgoing particles occurs at $\vec{x}_i$, defined as the position of the center of the beam particle at the time of the interaction. This makes sense because the interaction is assumed to take place in a very short time, compared to other time scales for the event. P_N is then given by

$$\begin{aligned}P_N = \int_{\mathrm{all}\,\vec{x}}\int_{\mathrm{all}\,\vec{x}_i} &\times (\text{probability that a beam particle with impact parameter}\\ &\quad\text{between } \vec{b}_i \text{ and } \vec{b}_i+d\vec{b}_i \text{ will interact with nucleon 1},\\ &\quad\text{between } z_i \text{ and } z_i+dz_i \text{ along the beam direction given that nucleon 2 is at } \vec{x})\\ &\times (\text{probability that interaction at } \vec{x}_i \text{ will result in } N \text{ products})\\ &\times [\text{probability that nucleon 2 is centered between } \vec{x} \text{ and } \vec{x}+d^3(\vec{x})]\\ &\times (\text{probability that} \geq 1 \text{ products from the initial } N\text{-product}\\ &\quad\text{interaction at } \vec{x}_i \text{ will rescatter on nucleon 2 at } \vec{x})\,.\end{aligned} \tag{B2}$$

Since we take an N-product interaction on nucleon 1 as given to occur, the product of the first three factors above must integrate to unit total probability. The third factor is simply the absolute square of the deuteron wave function times the volume element, $|\psi(\vec{x})|^2 d^3(\vec{x})$, which we take to be spherically symmetric: $|\psi(\vec{x})|^2=|\psi(r)|^2$. We also assume that, on the average, it is sufficient to take the first interaction as occurring at the origin. Then the first factor is proportional to $\delta^{(3)}(\vec{x}_i)$, and, neglecting screening (for now) of the origin by nucleon 2, our formula reduces to

$$\begin{aligned}P_N = \int_{(\infty)} d^3(\vec{x})\,|\psi(r)|^2 \times (&\text{probability that} \geq 1 \text{ products from the initial } N\text{-product interaction}\\ &\text{at the origin will rescatter on the second nucleon if it is at } \vec{x})\,.\end{aligned} \tag{B3}$$

The remaining factor in parentheses we call $p_N(\vec{x})$. With due regard to trajectory correlations and care to avoid multiple counting, estimates for $p_N(\vec{x})$ are obtained as follows: Nucleon 2 at $\vec{x}$ is assumed to opaquely fill a solid angle $\vec{\Omega}_2(\vec{x})$ of magnitude $\Omega_2(\vec{x})$ and centered about the direction $\hat{x}$. Thus, a product of the first interaction whose trajectory intersects $\vec{\Omega}_2(\vec{x})$ rescatters, and it does not rescatter if its trajectory does not intersect $\vec{\Omega}_2(\vec{x})$. If $Q_{Ni}(\vec{\Omega}_2(\vec{x}))$ is the probability that particle

i intersects $\vec{\Omega}_2(\vec{x})$, we may now write

$$p_N(\vec{x}) = 1 - \prod_{i=1}^{N} [1 - Q_{Ni}(\vec{\Omega}_2(\vec{x}))] . \tag{B4}$$

Let us suppose (for now) that there is no very "special" class of particles on the average and set

$$Q_{Ni}(\vec{\Omega}_2(\vec{x})) = Q_N(\vec{\Omega}_2(\vec{x})) \text{ for all } i . \tag{B5}$$

Then

$$Q_N(\vec{\Omega}_2(\vec{x})) = \int_{\vec{\Omega}_2(\vec{x})} d^2(\hat{\Omega}')q_N(\hat{\Omega}') , \tag{B6}$$

where

$$q_N(\hat{\Omega}) = \frac{1}{\sigma_N} \frac{d\sigma_N}{d\hat{\Omega}} \tag{B7}$$

is the (normalized) inclusive differential cross section given N particles. Consolidating, we have

$$P_N = \int_{(\infty)} d^3(x) |\psi(r)|^2 \times \sum_{i=1}^{N} \binom{N}{i} (-1)^{i-1} \left(\int_{\vec{\Omega}_2(\vec{x})} d^2(\hat{\Omega}')q_N(\hat{\Omega}') \right)^i . \tag{B8}$$

For $\psi(r)$, we use the Hulthén wave function with the addition of a hard core:

$$\psi(r) = \begin{cases} A(R_{\min})e^{-\alpha r}(1 - e^{-\mu r})/r & \text{for } r \geq R_{\min} \\ 0 \quad \text{for } r < R_{\min} , \end{cases} \tag{B9}$$

where $R_{\min}$ is the radius of the core and A is a normalization factor. The addition of the hard core allows one to avoid the problem of nucleon 2 overlapping the origin.

For $\vec{\Omega}_2(\vec{x})$, we try two different forms reflecting different target shapes:

(1) disk target,

$$\vec{\Omega}_2(\vec{x}) = 2\pi\{1 - 1/[1 + (b/r)^2]^{1/2}\}\hat{x} ; \tag{B10a}$$

(2) sphere target,

$$\vec{\Omega}_2(\vec{x}) = 2\pi\{1 - [1 - (b/r)^2]^{1/2}\}\hat{x} . \tag{B10b}$$

The "disk-target" form takes nucleon 2 to present a cross section with the geometry of a disk of radius b, oriented with its axis along $\hat{x}$ and at distance r from the origin. The "sphere-target" form supposes this cross section to have the geometry of a sphere of radius b centered at $\vec{x}$. Note that the same radius, b, is used for both the disk and the sphere, so that both would present the same cross section to an incident plane wave.

The $q_N(\hat{\Omega}')$ term is important because it contains the trajectory correlation information, and hence, really determines the energy and N dependence in our model (we are assuming that particle-nucleon cross sections are independent of energy). In our calculations, we assume particles to be produced with a uniform distribution in rapidity and an exponential falloff in transverse momentum. This choice is motivated by the desire to examine the scaling properties of the rescattering as a function of energy. However, at 15 GeV/c it may be argued that the actual rapidity distribution is better described by a Gaussian (Bosetti *et al.*, 1973[18]). Assuming a Gaussian distribution at 15 GeV/c has approximately a 20% effect on the results, and flattens the P_N distribution somewhat, but does not alter our conclusions in this paper, so we shall use only the scaling form.

The actual model calculations are performed using the Monte Carlo method. The α and μ parameters of the Hulthén wave function are fixed at the typical values:

$$\alpha = 45.72 \text{ MeV} = 0.232 \text{ fm}^{-1},$$

$$\mu = 5\alpha = 1.16 \text{ fm}^{-1} .$$

$R_{\min}$ is assigned a value according to

$$R_{\min} = \sqrt{\sigma/(10\pi)} , \tag{B11}$$

where σ is a rough nucleon-nucleon total cross section in mb, and $R_{\min}$ is in fermis. With $\sigma = 39$ mb, $R_{\min} = 1.11$ fm, the value we have used for most of the calculations. $R_{\min}$ values of 0.892 fm (corresponding to the π-nucleon cross section of 25 mb) and 0.796 fm have also been tried to determine the effect of varying this parameter. Nucleon 2 is not allowed to have a position such that the beam particle would strike it before reaching nucleon 1 at the origin [a minor consideration, which was neglected in writing Eq. (B3)]. We take the b parameter (the radius of the 2 nucleon disk or sphere) to be fixed at 0.892 fm $[= \sqrt{\sigma_{\pi N}/(10\pi)}]$ for mesons and 1.11 fm $[= \sqrt{\sigma/(10\pi)} = R_{\min}]$ for baryons.

The total rescatter probability, P, is calculated assuming a Poisson multiplicity distribution for the "extra" particles produced in the first interaction (that is, the distribution is Poisson in $N - 2$). The mean of the Poisson distribution is taken to be

$$\langle N - 2 \rangle = \tfrac{3}{2}(a_1 + a_2 \ln s) - 2 , \tag{B12}$$

where $a_1 = -1$, $a_2 = 1.5$, and s is the square of the c.m. energy (in GeV) of the beam pion + target nucleon system. It is assumed that there are at least two outgoing particles from the first interaction. Thus, only those P_N for $N \geq 2$ are used in calculating P. A Poisson distribution in N, for $N \geq 2$ (normalized to 1 for $N \geq 2$), has also been tried, and gives results consistent with the above

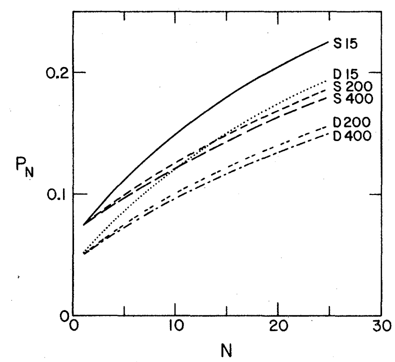

FIG. 5. The instantaneous-production-model prediction for the P_N distribution as a function of the total multiplicity (including neutrals) from the first interaction. The curves are labeled according to the target shape (S = sphere, D = disk), and the beam momentum in GeV/c.

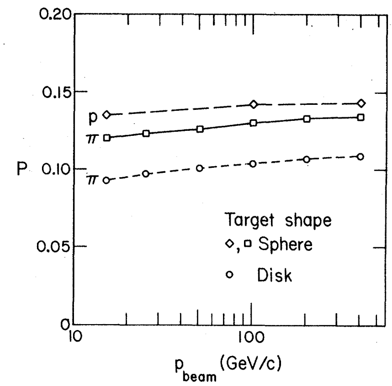

FIG. 6. The instantaneous-production-model prediction for the overall rescatter probability as a function of beam momentum. The curves are labeled p or π according to whether the beam particle is a proton or a pion.

method within the Monte Carlo errors.

Most of the results given here are for a pion beam, and assume one baryon (the target recoil) and $N-1$ identical mesons emerging from the first interaction. Calculations have also been made for a proton beam, giving two baryons from the first interactions. Production of baryon-antibaryon pairs is neglected.

The energy and N dependence of P_N, both for the sphere and the disk-target shapes, is shown in Fig. 5. Note that N is the total number of outgoing (hadronic) particles from the first interaction, not just the charged prongs. Thus, there will be some smearing over N when one sees only charged particles, and the average N corresponding to a given charged multiplicity (N_{ch}) may be something like $\frac{3}{2}N_{ch}$. Also, N is at least 2, so that one does not, in practice, have available the $N=1$ case. Thus, for comparison with experiment, the $N=1$ point is irrelevant, and, if only the charged-multiplicity dependence is available, the smearing will result in a somewhat flatter distribution (because a given charged multiplicity corresponds to contributions from several values of the total multiplicity) than in Fig. 5.

Figure 6 gives the total rescatter probability as a function of beam momentum for various parametrizations of our model. The estimates for P are not very sensitive to the slope assumed in the e^{-ap_T} falloff. For the 15-GeV/c, π beam with a spherical target shape, a slope of 4 GeV^{-1} gives $P=0.123$, and $a=8$ GeV^{-1} gives $P=0.117$, both 0.003 away from the 6-GeV^{-1} result. As P is only ~ twice as large as the rescatter probability expected for a single particle, and the remaining contribution is from particles which have small longitudinal momentum, the absence of substantial sensitivity to the precise slope of the p_T falloff should not be surprising.

Figure 6 shows a rather flat energy dependence between 15 and 400 GeV. The energy dependence of P may be simply understood in terms of two opposing factors. The increase in average multiplicity with energy tends to produce a corresponding rise in P, while the increasing forward collimation with energy tends to decrease P. At lower energies, the first factor wins and P rises with energy, but as the energy increases the two factors cancel and P becomes constant. That the asymptotic dependence should be flat is a simple consequence of the uniformity in y of the model (and of the assumption that the cross section of the outgoing particles is independent of energy). Since the average multiplicity grows like $\ln s$, and the total rapidity gap, Y, also grows as $\ln s$, the number of particles in a given y interval is independent of energy:

$$dN/dy \sim \text{constant} = \tfrac{3}{2}a_2 . \tag{B13}$$

Furthermore, as we increase the number of particles with increasing energy, the "added" particles are in the added rapidity region, namely at the highest value of y. Thus, the additional particles are in the very forward direction, where there are already plenty of other particles, and, hence, contribute nothing more to the rescatter probability. Consequently, the high-energy rescatter probability is energy independent. However, one should be aware that, especially at the lower energies, the assumptions that $\langle N\rangle \propto \ln s$ and dN/dy ~ constant are only approximations. These forms are reasonably good at Fermilab and CERN ISR energies, but are by no means established as asymptotic.

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[11]A small correction (586 ± 102 weighted events) has also been made to account for H_2 and HD contamination in the bubble-chamber liquid, distributed according to the expected $\pi^- p$ multiplicity distribution.

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[15]The rescatter probability for a single beamlike particle may be estimated for deuterium by $\bar{\nu}-1$ (see Ref. 16), with the result 0.049 ± 0.034.

[16]$\bar{\nu}$ is defined to be the mean number of inelastic collisions that a beam particle would undergo in a nucleus, assuming it remains beamlike after each collision, and given that it interacts at least once. Using our measured inelastic cross section for $\pi^- d$ at 15 GeV/c, and the equation (Ref. 2) $\bar{\nu} = 2\sigma_{\text{inel}}(\pi^- N)/\sigma_{\text{inel}}(\pi^- d)$, we obtain (Refs. 6, 14) $\bar{\nu} = 1.049 \pm 0.034$.

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[27]Note that in Eqs. (A1) and (A4), we use an average detection efficiency, ϵ. The spread in γ detection efficiencies allows possible correlations in multiple-γ (detected) events, resulting in a different effective detection efficiency than our overall average. The amount of bias introduced may be estimated from our data, and we find that there is no bias with a sensitivity of ~10% (Ref. 6). We add (quadratically) a 10% error to the statistical errors to allow for this uncertainty.