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Scalar-field algebraic chromodynamics

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I give the Lagrangian density and field equations for a generalized scalar-field chromodynamics, which has full local gauge invariance under general operator-valued gauge transformations.

In a recent letter[1] I gave the equations of motion and Lagrangian density for a generalized fermion chromodynamics, based on a U(n) gauge group, which has full local gauge invariance under general operator-valued gauge transformations. In this note I give the Lagrangian density and field equations for an analogous model in which the fermion fields are replaced by scalar fields.

As in Ref. 1, I introduce an n^2-plet potential operator b^a_ν, and the corresponding field strength $f^a_{\mu\nu}$, related by

$$F_{\mu\nu}=f^a_{\mu\nu}\tfrac{1}{2}\lambda^a=\frac{\partial B_\mu}{\partial x^\nu}-\frac{\partial B_\nu}{\partial x^\mu}-ig[B_\mu,B_\nu]\,,\qquad B_\nu=b^a_\nu\tfrac{1}{2}\lambda^a\,, \tag{1}$$

with λ^a the usual Hermitian basis matrices for U(n). Instead of an n-component complex spinor field ψ_A, I introduce now an n-component complex scalar field ϕ_A, and define a covariant derivative acting on it by

$$D_\nu\phi_A=\frac{\partial}{\partial x^\nu}\phi_A-ig\phi_C B_{\nu CA}\,,\qquad D_\nu\phi^\dagger_A=\frac{\partial}{\partial x^\nu}\phi^\dagger_A+igB_{\nu AC}\phi^\dagger_C\,, \tag{2}$$

or in a natural matrix notation

$$D_\nu\phi=\frac{\partial}{\partial x^\nu}\phi-ig\phi B_\nu\,,\qquad D_\nu\phi^\dagger=\frac{\partial}{\partial x^\nu}\phi^\dagger+igB_\nu\phi^\dagger\,. \tag{3}$$

Under the general operator-valued gauge transformations

$$\delta_g\phi=-i\phi U\,,\qquad \delta_g\phi^\dagger=iU\phi^\dagger\,,\qquad \delta_g B_\nu=-g^{-1}D_\nu U=-g^{-1}\left(\frac{\partial}{\partial x^\nu}U+ig[B_\nu,U]\right), \tag{4}$$

one finds by a simple calculation

$$\delta_g D_\nu\phi=-i(D_\nu\phi)U\,,\qquad \delta_g D_\nu\phi^\dagger=iUD_\nu\phi^\dagger\,. \tag{5}$$

Hence writing the Lagrangian density

$$\mathcal{L}=\mathrm{Tr}\{-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}[D_\mu\phi^\dagger D^\mu\phi]-\tfrac{1}{2}m_0^2[\phi^\dagger\phi]-\tfrac{1}{4}\lambda_0[\phi^\dagger\phi]^2\}\,, \tag{6}$$

with Tr a trace over all matrix and operator structure and with $[\phi^\dagger\phi]$ the matrix defined by $[\phi^\dagger\phi]_{AB}=\phi^\dagger_A\phi_B$, it is easily verified, using cyclic invariance of the trace, that

$$\delta_g\mathcal{L}=0\,. \tag{7}$$

From the action principle

$$\delta\int d^4x\,\mathcal{L}=0\,, \tag{8}$$

one gets the equations of motion

$$D_\mu F^{\nu\mu}=gJ^\nu=g\tfrac{1}{2}i[(D^\nu\phi^\dagger)\phi-\phi^\dagger(D^\nu\phi)]\,,$$
$$D_\mu D^\mu\phi+m_0^2\phi+\lambda_0\phi[\phi^\dagger\phi]=\frac{\partial}{\partial x^\mu}(D^\mu\phi)-ig(D^\mu\phi)B_\mu+m_0^2\phi+\lambda_0\phi[\phi^\dagger\phi]=0\,, \tag{9}$$

which by virtue of Eq. (7) are covariant under general operator-valued U(n) gauge transformations. This covariance can of course be checked by direct calculation using Eqs. (4) and (5).

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[1] S. L. Adler, Phys. Lett. 86B, 203 (1979); see also Phys. Rev. D 17, 3212 (1978).

