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Effective Lagrangian with two color-singlet gluon fields

Joseph Schechter
Physics Department, Syracuse University, Syracuse, New York 13210
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An effective Lagrangian with scalar and pseudoscalar "matter" fields as well as scalar and pseudoscalar (gauge-invariant) color-singlet gluon fields is constructed. It is a representation of quantum chromodynamics in that it has the same symmetry structure as that theory. The present model is a generalization of a one-(pseudoscalar) gluon-field effective Lagrangian which compactly summarizes the results of the $1/N_c$ approximation, gives a simple picture for the "U(1) problem," and essentially reduces to a type of generalized σ model. The new feature here is that, in addition to the chiral-anomaly equation, the trace-anomaly equation is automatically fulfilled. This gives a framework for discussion of the properties of a 0^+ color-singlet gluon field. The model may be formulated either so that this field is eliminated by a constraint equation or so that it is associated with a physical particle. Probably, experiment is the best way at present of deciding between the two possibilities. A number of possibilities for further generalization of the effective Lagrangian, including a hint of a possible connection with the "confinement" problem, are mentioned.

I. INTRODUCTION

The theory[1] of quantum chromodynamics (QCD) is expected to have the unique feature that the fundamental fields (quarks and gluons) do not appear as free particles. This makes it interesting to construct an effective Lagrangian for describing the properties of the observed particles. The effective Lagrangian is supposed to be used for low-energy processes and in the tree approximation. In the past, a very large amount of work has been done on Lagrangians of this type. The guiding principle has been to construct a Lagrangian which has a main piece invariant under chiral $U(N_F)\times U(N_F)$ (N_F = number of flavors) and an auxiliary symmetry-breaking piece which has the same transformation properties as the quark mass terms. In addition, a piece which breaks the conservation of the axial-vector baryon-number [or "U(1)"] current is needed for the predicted pseudoscalar mass spectrum to agree with experiment. Now a general Lagrangian constructed according to the scheme mentioned above will automatically satisfy the Ward identities (which hold in QCD) corresponding to the conservation and partial conservation of the appropriate $SU(N_F) \times SU(N_F)\times$ (ordinary baryon number) currents. This feature holds in the tree approximation and can be considered, since it leads easily to the "current-algebra" formulas for various processes, as a strong motivation for the effective-Lagrangian approach. Recently, it has been stressed by Witten[2] that the "$1/N_c$" (N_c = number of colors) approximation method applied to QCD provides another motivation for using an effective Lagrangian at the tree level. Actually, it seems very likely that any self-consistent approximation to QCD which satisfies crossing symmetry will lead to a tree-level effective Lagrangian. This is because if the opposite were true and, say, the computation of loop diagrams were required, that particular approximation would by definition be a bad one. Good approximations should yield reasonable results in lowest order.

A new feature which QCD brings into the picture is the possibility of gauge-invariant combinations of gluon fields (or "glueballs") appearing in the effective Lagrangian in addition to the "matter" (or ordinary particle) fields. Witten[2] noted that the presence of a pseudoscalar glueball field could provide a solution to the "U(1) problem" but he did not give the appropriate effective Lagrangian. Witten's work was further clarified in interesting papers by Veneziano[3] and by di Vecchia.[4] More recently, a chiral effective Lagrangian which includes a pseudoscalar glueball and gives the correct *anomalous* conservation law for the U(1) current was found.[5] Here the glueball G is identified with the gauge-invariant combination $F\tilde{F}$, F being the QCD field-strength tensor and $\tilde{F}$ its dual. Interestingly, in order to provide mass for the $\eta'(960)$ meson [i.e., solve the U(1) problem], it turns out to be necessary for the effective Lagrangian to contain terms which lead to the elimination of the field G in terms of the matter fields. Effectively then the theory becomes equivalent to a type of old-fashioned σ model.

In the present note we would like to indicate the results of a start on the problem of including other than pseudoscalar glueballs in the chiral effective Lagrangian. One can imagine adding glueballs with spin-parity $0^+, 1^\pm, 2^\pm, \ldots$, but for simplicity we shall restrict our attention to the 0^+ glueball, H corresponding to F^2. For the pseudoscalar glueball effective Lagrangian, the guiding principle was to have the anomalous conservation law for the U(1) current (in the massless-quark theory)

$$\partial_\mu J_\mu^5 = \frac{g^2}{16\pi^2} N_F F\tilde{F} \equiv \partial_\mu K_\mu \equiv G\,, \tag{1}$$

emerge automatically from the equations of motion. Here we shall make use of the fact that an anomaly equation very similar to (1) holds for the trace of the energy-momentum tensor $\theta_{\mu\mu}$ in QCD. This equation[6] is (in the massless-quark theory)

$$\theta_{\mu\mu} = \partial_\mu D_\mu = -\frac{\beta(g)}{2g} FF \equiv H\,. \tag{2}$$

In (2), $\beta(g)$ is the renormalization-group function equal in perturbation theory to $(-g^3/16\pi^2)(11 - \frac{2}{3}N_F) + \cdots$. Furthermore, D_μ is the dilation current. We will require our effective Lagrangian involving both G and H to satisfy (2) as well as (1) automatically. The structural similarity of (1) and (2) seems very intriguing and might have a "deep meaning." However, there is a significant difference between the two equations. In the limit where the matter fields are removed, the $\partial_\mu J_\mu^5$ anomaly vanishes while the $\theta_{\mu\mu}$ anomaly persists. This may possibly be related to a difference between the properties of the pseudoscalar and scalar glueballs.

II. THE PSEUDOSCALAR GLUEBALL LAGRANGIAN

First we briefly review the chiral effective Lagrangian[5] with only G present and make a few additional remarks. For simplicity we restrict our attention to spin-zero matter fields. These are incorporated into an $N_F \times N_F$-dimensional (flavor space) matrix M_{ab} which transforms like the quark field combination $\bar{q}_b(1+\gamma_5)q_a$. The independent chiral $\mathrm{U}(N_F)\times \mathrm{U}(N_F)$ invariants, not involving derivatives, which can be made[7] from M and $M^\dagger$ are $I_n = \mathrm{Tr}(MM^\dagger)^n$, where n goes from 1 to N_F. Including the glueball field G of Eq. (1), the effective Lagrangian can be written as

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu M \partial_\mu M^\dagger) + f(I_n, G)\\ &+ \frac{i}{4N_F} G(\ln\det M - \ln\det M^\dagger)\\ &+ \mathrm{Tr}[A(M+M^\dagger)]\,.\end{aligned} \tag{3}$$

Here the matrix $A_{ab} = \delta_{ab}A_b$ is taken to be proportional to the real diagonal matrix of quark masses. $f(I_n, G)$ is an arbitrary function of I_n and G. Requiring parity invariance gives the additional condition $f(I_n, G) = f(I_n, -G)$. Note that the field G here is taken for convenience to be $\sqrt{N_F}$ times larger than the corresponding field G' $= \partial_\mu K_\mu$ defined in Ref. 5. The axial divergence $\partial_\mu J_\mu^5 = -i\,\mathrm{Tr}(M^\dagger \overleftrightarrow{\Box} M)$ here.

There are two main new aspects to the Lagrangian (3). The first is that, owing to the presence of the third term, the anomalous partial conservation law is, as previously discussed,[5] satisfied automatically. The second is that the equation of motion for G, $\partial f/\partial G + (i/4N_F)(\ln\det M - \ln\det M^\dagger) = 0$, leads to the elimination of G in terms of the matter field combination $(\ln\det M^\dagger - \ln\det M)$. Substituting this relation back into $\mathcal{L}$ gives, once more, the old linear σ model[7] which is described by

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu M\partial_\mu M^\dagger) - V_0(I_n, J)\\ &+ \mathrm{Tr}[A(M+M^\dagger)]\,,\\ J = &\det M + \det M^\dagger\,.\end{aligned} \tag{4}$$

In (4), V_0 is an arbitrary function of I_n and J. To see that only functions of J appear, first note that by parity invariance only the combination $(\ln\det M - \ln\det M^\dagger)^2$ will appear in $\mathcal{L}$ after G is eliminated. Further, note that $\det(MM^\dagger)$ is a $\mathrm{U}(N_F)\times\mathrm{U}(N_F)$-invariant quantity which can be expressed[8] in terms of the I_n by using the characteristic equation for the matrix $(MM^\dagger)$. Then

$$\begin{aligned}(\ln\det M - \ln\det M^\dagger)^2 &= \left[\ln\frac{\det^2 M}{\det(MM^\dagger)}\right]^2\\ &= -4\left[\cos^{-1}\frac{J}{2[\det(MM^\dagger)]^{1/2}}\right]^2,\end{aligned} \tag{5}$$

which depends only on J and the I_n. There actually are implicit restrictions on V_0 in (4) or, equivalently, on f in (3). These arise because V_0 must be such that the chiral symmetry be spontaneously broken. Also, in order to solve the U(1) problem [i.e., give mass to the $\eta'(960)$ in the three-flavor case, etc.] one must have $\langle \partial V_0/\partial J\rangle_0$ to be negative and of sufficient magnitude, as discussed elsewhere.[7]

The following additional remarks may be helpful:

(i) The dependence of amplitudes on the vacuum angle[1] θ can be incorporated by adding to (3) the term

$$-\frac{\theta G}{2N_F}\,. \tag{6}$$

The constraint equation for G is now modified to

$$\frac{\partial f}{\partial G} + \left[\frac{i}{4N_F}(\ln\det M - \ln\det M^\dagger) - \frac{\theta}{2N_F}\right] = 0\,. \tag{7}$$

This means that, after substituting back, θ always appears in combination with $(\ln\det M - \ln\det M^\dagger)$. This latter quantity is seen[5] in the approximation where the scalar fields $(M+M^\dagger)$ become very massive to behave as, in the three-flavor case, for example, the $\eta'(960)$ field. Equation (7) is a generalization of Witten's "soft η'" theorem.[2]

(ii) The quark-mass matrix can always be brought to the diagonal form A in (3) by making a chiral $\mathrm{U}(N_F)\times\mathrm{U}(N_F)$ transformation on the fields $M\to U_L M U_R^\dagger$, where U_L and U_R are two unitary matrices. The net effect of this is to add, via the third term of (3), the additional quantity $(i/2N_F)G\ln\det U_L U_R^\dagger$ to $\mathcal{L}$. This piece can be lumped into (6) however, giving a new θ_{eff}. We interpret the θ in (6) and (7) to already include such a possible contribution.

(iii) The presence of a nonzero θ in (6) gives rise to the problem of strong T violation.[9] A detailed treatment in the present model has been given elsewhere.[10]

(iv) Instead of writing the effective Lagrangian in terms of the glueball G, it is possible[3-5] to use the pseudovector field K_μ satisfying $\partial_\mu K_\mu = G$ as in (1). K_μ would also get eliminated. However, for the purpose of the next Section it is better to use G. Furthermore, for illustrative purposes, it is possible to keep G from getting eliminated by adding to $\mathcal{L}$ a kinetic term $-\frac{1}{2}\lambda^2(\partial_\mu G)^2$ and taking $\lambda\to 0$ at the end. Then it is seen[5] that the field (λG) behaves as an infinitely "heavy" tachyon which decouples but leaves as a finite residue a mass term for the η'.

(v) In his well-known calculation[11] of the effects of an instanton in the semiclassical approximation, 't Hooft found that his result could be roughly summarized as the addition of an extra term such as $J=\det M+\det M^\dagger$. One might now guess that if his calculation is carried to higher orders an effective term more like $(\ln\det M - \ln\det M^\dagger)^2$ might emerge.

III. THE TWO-GLUEBALL LAGRANGIAN

Our goal in this Section is to find a Lagrangian constructed out of M, $M^\dagger$, the 0^- glueball G, and a 0^+ glueball H which automatically satisfies the anomalous conservation laws (1) and (2) at the tree level. It also should be $\mathrm{SU}(N_F)\times\mathrm{SU}(N_F)\times$ (baryon number) symmetric. We will then consider the question of whether the new glueball field must get eliminated effectively from the theory, as was the case for G, or whether it may be related to a physical 0^+ particle.

As a preliminary, let us consider two sets of real scalar fields η_A and ξ_A, which transform respectively as objects of (mass) dimension 1 and 4 under an infinitesimal scale transformation,[12]

$$\delta\eta_A = \eta_A + x_\mu\partial_\mu\eta_A\,,\qquad \delta\xi_A = 4\xi_A + x_\mu\partial_\mu\xi_A\,. \tag{8}$$

[An overall infinitesimal factor on the right-hand side (RHS) has been set equal to one.] Consider the Lagrangian density

$$\mathcal{L} = -\tfrac{1}{2}\sum_A \partial_\mu\eta_A\partial_\mu\eta_A - V(\eta,\xi)\,, \tag{9}$$

where $V(\eta,\xi)$ is an arbitrary function of η and ξ, not containing derivatives. Note that there are no kinetic terms for the ξ fields. The "new improved" energy-momentum tensor[12] for (9) is

$$\theta_{\mu\nu} = \delta_{\mu\nu}\mathcal{L} + \sum_A \partial_\mu\eta_A\partial_\nu\eta_A - \tfrac{1}{6}\sum_A(\partial_\mu\partial_\nu - \delta_{\mu\nu}\Box)\eta_A{}^2 \tag{10}$$

and the dilation current D_μ corresponding to (8) may be written as

$$D_\mu = \theta_{\mu\nu}x_\nu \tag{11}$$

so that

$$\partial_\mu D_\mu = \theta_{\mu\mu}\,. \tag{12}$$

$\theta_{\mu\mu}$ in (12) thus must be equal to a quantity which vanishes when V is scale invariant. That quantity is $\partial_\mu(x_\mu\mathcal{L}) - \delta\mathcal{L}$, which works out to be

$$\theta_{\mu\mu} = \sum_A\left(\eta_A\frac{\partial V}{\partial\eta_A} + 4\xi_A\frac{\partial V}{\partial\xi_A}\right) - 4V\,. \tag{13}$$

This is the basic equation for our present purpose. As an aside, we remark that there would be a technical problem with the present approach if one were to use a pseudovector glueball K_μ rather than the 0^- glueball G. This arises because one would then have quantities like $(\partial_\mu K_\mu)$ appearing in $\mathcal{L}$ and it is known[12] that (11) does not hold in that case.

Now we modify the effective Lagrangian (3), with the help of (13) above so that the trace-anomaly equation (2) holds automatically. We consider first the possibility that H gets eliminated from the theory, and second the possibility that it remains.

A. The case when H disappears

Here we identify the fields η_A with the Hermitian combinations of M and $M^\dagger$ and the fields ξ_A with the 0^- and 0^+ glueballs G and H. Equation (13) may then be written as

$$\theta_{\mu\mu} = \mathrm{Tr}\left(M\frac{\partial V}{\partial M} + M^\dagger\frac{\partial V}{\partial M^\dagger}\right) + 4G\frac{\partial V}{\partial G} - 4V\,. \tag{14}$$

Immediately, we note from (3) that the characteristic $G(\ln\det M - \ln\det M^\dagger)$ term is scale invariant and will not contribute to the right-hand side of (14). This is an encouraging start because if it were not so, we could not proceed further. Clearly the chiral-invariant term $f(I_n, G)$ in (3) may be generalized to $f(I_n, G, H)$ and should be scale invariant; i.e., it should satisfy

$$\mathrm{Tr}\left(M\frac{\partial f}{\partial M} + M^\dagger \frac{\partial f}{\partial M^\dagger}\right) + 4G\frac{\partial f}{\partial G} + 4H\frac{\partial f}{\partial H} = 4f\,, \tag{15}$$

in addition to

$$f(I_n, G, H) = f(I_n, -G, H) \tag{16}$$

for parity invariance. The main problem is to find a new term involving H which will lead to a $\theta_{\mu\mu}$ satisfying (2). However, it is easy to see that a term equal to H multiplied by the logarithm of a homogeneous function will have this property. The two-glueball effective Lagrangian may then be written as

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu M \partial_\mu M^\dagger) + f(I_n, G, H)\\ &+ \frac{i}{4N_F} G(\ln\det M - \ln\det M^\dagger + 2i\theta)\\ &- H\sum_m \frac{c_m}{m}\ln\left(\frac{R_m}{\Lambda^m}\right).\end{aligned} \tag{17}$$

In (17) R_m is any function of I_n, G, and H satisfying

$$\begin{aligned}&\mathrm{Tr}\left(M\frac{\partial R_m}{\partial M} + M^\dagger \frac{\partial R_m}{\partial M^\dagger}\right) + 4G\frac{\partial R_m}{\partial G} + 4H\frac{\partial R_m}{\partial H} = mR_m\,,\\ &R_m(I_n, G, H) = R_m(I_n, -G, H)\,.\end{aligned} \tag{18}$$

In other words, it is of scale dimension m and parity invariant. The real coefficients c_m obey the equation

$$\sum_m c_m = 1\,. \tag{19}$$

Finally, Λ is a scale parameter with dimensions of mass, inserted to preserve naive dimensional consistency for the Lagrangian (17). Using (14) it is straightforward to verify that (neglecting the "quark mass" matrix A) the anomaly equation (2) is satisfied. The necessary quark mass term will be discussed later.

It is interesting that (17) makes manifest an interpretation for the anomalies. When there is an anomaly so that the classical conservation law is not the same as the quantum one, it is an indication that the classical Lagrangian is incomplete and must be supplemented by an additional term possibly containing a new parameter. For the axial-vector-current anomaly the new parameter (vacuum phase angle) is θ and the new term $\sim G\ln e^{i\theta} \propto i\theta(F\tilde{F})$. For the scale anomaly the new parameter is a characteristic scale $\Lambda \equiv \epsilon^\tau$ and is contained in a term $\sim H\ln e^\tau \propto \tau(FF)$. The similarity of the two terms is striking in spite of the fact that the θ anomaly is associated with a compact (axial-vector current) transformation, while the τ anomaly is associated with a noncompact (scale) transformation.

Since there are no $\partial_\mu G$ and $\partial_\mu H$ terms in (17) the equations of motion for the G and H fields just lead to their elimination in terms of the matter fields M and $M^\dagger$. These explicit constraint equations are

$$\begin{aligned}&\frac{\partial f}{\partial G} + \frac{i}{4N_F}(\ln\det M - \ln\det M^\dagger + 2i\theta) - H\sum_m \frac{c_m}{m}\frac{\partial}{\partial G}\ln R_m = 0\,,\\ &\frac{\partial f}{\partial H} - \sum_m \frac{c_m}{m}\ln\left(\frac{R_m}{\Lambda^m}\right) - H\sum_m \frac{c_m}{m}\frac{\partial}{\partial H}\ln R_m = 0\,.\end{aligned} \tag{20}$$

After G and H are eliminated by solving (20) and substituting back into (17), one would again be left with the general linear σ model. What has been achieved then is a better justification and interpretation of this old model in the light of QCD. This is welcome because a number of fairly successful experimental predictions[13] can be taken over and also extended further.

Let us consider, for simplicity of further discussion, a particular example of the two-glueball Lagrangian:

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu M\partial_\mu M^\dagger) + f_0(I_n) + a_1(I_1)^{-2}G^2 + a_2(I_1)^{-2}H^2\\ &+ \frac{i}{4N_F}G(\ln\det M - \ln\det M^\dagger + 2i\theta)\\ &- \frac{b}{2N_F}H\left(\ln\det\frac{M}{\Lambda^{N_F}} + \ln\det\frac{M}{\Lambda^{N_F}}\right)\\ &- \frac{(1-b)}{4}H\ln(H/\Lambda^4)\,.\end{aligned} \tag{21}$$

Here a_1, a_2, and b are dimensionless constants while $f_0(I_n)$ is some scale-invariant function of the I_n. Equation (21) corresponds to the choice of the R_m in (17) which gives a characteristic H term most nearly similar to the characteristic G term. A suitable quark-mass symmetry-breaking term which should be added to (21) will be given in (28) and discussed there. Note that we have included in (21) a term like $H\ln H$ which contains only glueball fields. The reason for this is that (in lowest-order perturbation theory)

$$H = \frac{g^2}{32\pi^2}\left(11 - \frac{2N_F}{N_c}\right)F^2 + \cdots\,,$$

and so in the limit when the matter fields are not present the trace anomaly still remains. In this formula we have reinstated N_c = number of colors = 3. (The factor $1/N_c$ in the second term reflects the fact that the quark-loop contribution to the vacuum polarization is down by $1/N_c$ compared to the gluon contribution.) We may try to use this to estimate in perturbation theory the quantity $b/(1-b)$ which measures the relative contribution of matter versus glueballs to the trace-anomaly term

$$\frac{b}{1-b} \approx -\frac{2}{11}\frac{N_F}{N_c}\,. \tag{22}$$

It is seen that the relative contribution of the matter term becomes less important in the high-N_c limit. Although in the present note we are focusing on those aspects of the effective Lagrangian which follow directly from the symmetry structure and anomalous conservation laws, it is interesting to observe that the $1/N_c$ approximation would seem to provide a way of distinguishing between the assumption of this section—that H gets eliminated—or the assumption of the next section—that H remains. If we accept the validity of the $1/N_c$ approach we expect that H should remain. Thus from our point of view the experimental discovery of a fairly low-lying 0^+ glueball state would reinforce the status of the $1/N_c$ approximation. We may understand this feature by mimicking the discussion given around Eq. (11) of Ref. 2. Note first that unlike the chiral anomaly in Eq. (1), which is of order $1/N_c$ for large N_c, the trace anomaly in Eq. (2) is of order unity. Now a 0^+ state would be characterized by a squared mass parameter which is related to the zero-momentum limit of the Fourier transform of the matrix element $\langle 0|TH(x)H(0)|0\rangle$. In order for the glueball field to get eliminated in terms of matter fields we would expect that both gluon and matter contributions to $\langle 0|TH(x)H(0)|0\rangle$ should give the same result and so should be of the same order in N_c. However, the only way this can happen is for the matter field to behave like a dilaton whose mass is proportional to $1/N_c$. The mass of a dilaton should, however, be proportional to the trace anomaly, which we have just noted to be of order unity.

One point of interest arises from the necessity of having a spontaneous breakdown of chiral symmetry for the σ-model Lagrangian. This means that the vacuum value $\langle M\rangle \neq 0$; neglecting θ we would have $\langle M\rangle = \langle M^\dagger\rangle$. This does not give anything striking for the one-glueball Lagrangian. In that case the constraint Eq. (7) says that G is eliminated in terms of $\ln(M/M^\dagger)$. Thus one predicts $\langle G\rangle \propto \ln(\langle M\rangle/\langle M^\dagger\rangle) = 0$, which is consistent with the requirement of parity invariance. On the other hand, parity invariance places no restriction on the vacuum value of the scalar glueball $\langle H\rangle$. In fact, if one makes the assumption (as we are doing in this section) that H should get eliminated in terms of the matter fields it is unnatural to avoid predicting $\langle H\rangle \neq 0$. For example, in the case of Eq. (21) (with the further simplifying approximation $b=0$), the constraint equation gives

$$(\mathrm{Tr}\langle MM^\dagger\rangle)^2 = 8a_2\,\frac{\langle H\rangle}{\ln(\langle H\rangle/\Lambda^4)+1}\,,$$

so one must have $\langle H\rangle \neq 0$ for consistency. In turn this implies $\langle F^2\rangle$ to be nonzero. We note that a number of authors[14] have recently made speculations along these lines. The possibility of a magnetic vacuum is favored and is closely related to a mechanism for quark confinement. In any event, it is clear that the assumption requiring H to get eliminated in terms of matter fields has very strong consequences. Amusingly, as we shall see, even if we do not make this assumption, $\langle F^2\rangle \neq 0$ is still plausible.

B. The case when H remains

In this case we want to have an actual physical 0^+ glueball in the theory. In order to get a scale-invariant kinetic term we define (a possible dimensionless factor has been set to 1 for simplicity)

$$H = \pm h^4\,, \tag{23}$$

where h is identified as the field of scale dimension 1 associated with the glueball. Remembering that $H \propto 2F^2 = B^2 - E^2$ (E and B are Yang-Mills electric and magnetic fields), we see that it is not automatic for H considered as a time-varying field quantity to have a unique sign as required by (23). The sign will be unique, however, if there is spontaneous breakdown giving $H = \langle H\rangle + \tilde{H}$, with $|\tilde{H}| \ll |\langle H\rangle|$. Then we have a plus or minus sign in (23) depending on whether the vacuum is of magnetic or electric type. It is interesting to note that the requirements for constructing a consistent effective QCD Lagrangian seem again to have pushed us to a situation where $\langle F^2\rangle \neq 0$.

To proceed, we now identify the fields η_A in (9) with Hermitian combinations of M and $M^\dagger$ *and* with h. The field ξ in (9) is identified as before with the pseudoscalar glueball G. Equation (14) is now replaced by

$$\theta_{\mu\mu} = \mathrm{Tr}\left(M\frac{\partial V}{\partial M} + M^\dagger\frac{\partial V}{\partial M^\dagger}\right) + h\frac{\partial V}{\partial h} + 4G\frac{\partial V}{\partial G} - 4V\,, \tag{24}$$

and the two-glueball effective Lagrangian becomes

$$\mathcal{L} = -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu M \partial_\mu M^\dagger) - \tfrac{1}{2}(\partial_\mu h)^2 + f(I_n, G, h) + \frac{i}{4N_F} G(\ln\det M - \ln\det M^\dagger + 2i\theta) \mp h^4 \sum_m \frac{c_m}{m} \ln\left(\frac{R_m}{\Lambda^m}\right). \qquad (25)$$

Note the presence of the kinetic term $-\frac{1}{2}(\partial_\mu h)^2$ in (25). As before, $f(I_n, G, h)$ must be scale invariant and invariant under the interchange $G \to -G$. Furthermore, R_m is any function of scale dimension m of the quantities I_n, G, and h; it is also invariant under $G \to -G$. Using (24 and (23), we verify that the trace-anomaly equation (2) is satisfied (for zero quark masses A). As discussed above, f and the R_m must be arranged so that $\langle h\rangle \neq 0$ emerges; this is, however, a weak restriction.

The scale-invariant part of the potential $f(I_n, G, h)$ may contain terms such as $h^2\,\mathrm{Tr}(MM^\dagger)$ and $h[\mathrm{Tr}(MM^\dagger)]^{3/2}$, for example. This means that we must expect $\langle \partial^2 f/\partial h^2\rangle$ and $\langle \partial^2 f/\partial h \partial\sigma\rangle$ (where σ is an isoscalar 0^+ particle) to be nonzero. In other words, from a phenomenological standpoint the particle h can mix with the matter isoscalar scalars of the theory and would have decay modes like $\pi^+\pi^-$, k^+k^-, etc. There is no special reason to expect its mass to be anything other than of the order of the usual hadronic masses.

Distinguishing between case A where H gets eliminated from the effective theory and case B where it is associated with the particle field h can be accomplished most directly by finding three rather than two 0^+ isoscalar resonances[15] below 1.5 GeV or so. This could be done either by studying $\pi^+\pi^-$ and k^+k^- partial-wave phase shifts or by directly searching for decays[16] like $\psi \to h\gamma$. In this analysis one is hampered by the fact that the σ-type resonances are rather broad. An additional possible source of confusion arises from the possibility of radially excited quark-antiquark states or of baryoniumlike states. A deeper theoretical study of this question may also help one to decide between the two possibilities. For example, it is known[17] that pure classical QCD cannot sustain a glueball excitation. However, the relevance of this result to the quantum case is not at all clear. In the present context we should remark on the difference between 0^- and 0^+ glueballs. The special situation in the 0^- channel is related to the spontaneous breakdown of chiral symmetry. Thus, when for simplicity the quark masses are not present, the η'-type particle is expected to be a Nambu-Goldstone boson. If one allows the field associated with G to mix with η' in order to boost its mass to a suitable nonzero value, it is seen (as outlined in Ref. 5) that G should behave as a tachyon. An alternative formulation presented here leads to its elimination. In contrast, the spontaneous breakdown of chiral symmetry does not affect the (isoscalar) σ-type mesons; they have the wrong quantum numbers to be associated with the divergences of any spontaneously broken currents. Thus the σ mesons are expected to have suitable masses without requiring any special mechanism.

Finally, we mention that the quark mass term in (3) should be slightly modified if our effective Lagrangian is to exactly mock-up $\theta_{\mu\mu}$ in the broken theory. The chiral-symmetry-breaking term in the fundamental QCD Lagrangian is

$$-V_{\mathrm{SB}}(\text{quarks}) = -\sum_{a=1}^{N_F} m_a \bar{q}_a q_a, \qquad (26)$$

where the m_a are the current-algebra quark masses. The trace-anomaly equation (2) should be extended to read[6]

$$\theta_{\mu\mu} = H - [1 + \gamma(g)]\, V_{\mathrm{SB}}(\text{quarks}), \qquad (27)$$

where $\gamma(g)$ is the anomalous dimension of the $\bar{q}q$ operator. It is easy to verify that the term in (3) $V_{\mathrm{SB}} = -\mathrm{Tr}[A(M + M^\dagger)]$ does not yield this property. In fact, it gives the equation $\theta_{\mu\mu} = H - 3V_{\mathrm{SB}}$. We must modify this V_{SB} in such a way that the behavior of $\partial_\mu J_\mu^5$ in the broken theory is not upset. However, the equation for $\partial_\mu J_\mu^5$ only requires that V_{SB} transform according to the $[(N_F, N_F^*) + (N_F^*, N_F)]$ representation of chiral $\mathrm{SU}(N_F) \times \mathrm{SU}(N_F)$ and not contain a factor like J of Eq. (4). A simple but not unique choice of a symmetry-breaking term which will satisfy both the $\partial_\mu J_\mu^5$ and $\theta_{\mu\mu}$ equations is

$$-V_{\mathrm{SB}} = [\mathrm{Tr}(MM^\dagger)]^{(1-\gamma/2)}\,\mathrm{Tr}[A(M + M^\dagger)]. \qquad (28)$$

The first factor in (28) is a chiral invariant so that our new V_{SB} has the same chiral transformation properties as our old one. Equation (28) should be added to (17), (21), and (25) to complete the effective Lagrangian. Consequences of using (28) rather than the conventional one will be discussed elsewhere.

IV. DISCUSSION

Generally, theorists studying nonperturbative QCD have attempted to derive the properties of the observed particles, including the fact that they are color singlets, from the quark and gluon field equations. This is of course a difficult but fascinating project. Here we have attempted to work backwards. We have assumed that composites of quarks and gluons appear in the effective theory as, respectively, mesons and (gauge-invariant) glueballs. For simplicity only spin-zero objects were considered; this is, however, sufficient to make contact with a large part of the previous work on chiral symmetry. The QCD "ingredient" in our approach is the symmetry structure of the theory. Thus we have demanded that (in the limit of zero quark masses) the theory be $SU(N_F) \times SU(N_F) \times$ (baryon number) invariant and that the chiral and trace anomalies be fulfilled exactly in terms of 0^- and 0^+ glueball fields. It seems interesting that so much structure for the effective Lagrangian emerges from such simple requirements. In particular, (i) The nature of the U(1) problem and the related "strong T violation" problem has been illuminated; (ii) The connection of the effective QCD Lagrangian with an old σ model has been made; (iii) The parallel structure of the chiral and trace-anomaly effective terms has emerged; (iv) The possibility of a nontrivial gauge-theory vacuum characterized by $\langle F^2 \rangle \neq 0$ has been suggested; and finally (v) a nontrivial framework for discussing the properties of glueball fields and their interaction with matter fields has been laid down.

There are many directions for future work:

(a) Especially that part of the Lagrangian involving the scalar glueball still has a fair amount of arbitrariness. It seems likely that various approximation schemes will give further restrictions. The $1/N_c$ approximation (which provided the motivation[2] for the introduction of effective glueball fields in the present context) seems a likely candidate. It might also be interesting to try to relate the results of the lattice, bag, instanton, and perturbation approaches[1] to the present model.

(b) One can consider both matter and glueball fields of spin greater than zero to be present in $\mathcal{L}$. A lot of work in this direction was done for chiral Lagrangians without glueball fields. This might be now extended. If glueball fields of spin zero remain as physical excitations (Sec. III B), appropriate modifications of the old chiral-symmetry results should be made. Some relevant calculations might be carried out for scalar-meson decays, $\eta' \to \eta 2\pi$ decays, $\pi N\, \sigma$ terms, etc. Extending $\mathcal{L}$ by including higher-spin glueballs would seem to require a new method, since we have already used up the information of the standard anomaly equations. For example, one might study the possibility of QCD anomalies associated with more complicated gauge-invariant gluon operators and then assume that these operators are dominated by appropriate glueballs.

(c) The phenomenological analysis[15,16] of resonances which are suspected glueballs might provide useful information which can be fed back as constraints on the effective $\mathcal{L}$.

(d) A difficulty common to many approaches is that it is unclear how many flavors (N_F) are relevant for an effective low-energy theory. The answer to this question would seem to be related to a more satisfactory understanding of the quark mass terms. Some preliminary work has been done in collaboration with Per Salomonson.

Further work along the lines above will be reported elsewhere.

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[1]Reviews of the subject are provided by R. Crewther, in *Facts and Prospects of Gauge Theories*, proceedings of the XVII International Universitatswochen für Kernphysik, Schladming, 1978, edited by P. Urban (Springer, New York, 1978) [Acta Phys. Austriaca Suppl. 19, (1978)], p. 47; S. Coleman, "Ettore Majorana lectures, 1977 (unpublished); W. Marciano and H. Pagels, Phys. Rep. 36C, 137 (1978); V. A. Novikov *et al.*, *ibid.* 41C, 1 (1978).

[2]E. Witten, Nucl. Phys. B156, 269 (1979) and references therein.

[3]G. Veneziano, Nucl. Phys. B159, 213 (1979).

[4]P. Di Vecchia, Phys. Lett. 85B, 357 (1979).

[5]C. Rosenzweig, J. Schechter, and G. Trahern, preceding paper, Phys. Rev. D 21, 3388 (1980). We were informed (private communication from G. Veneziano to C. Rosenzweig) that Di Vecchia, Veneziano, and Witten have been jointly working on a similar Lagrangian.

[6]N. K. Nielsen, Nucl. Phys. B120, 212 (1977); J. C. Collins, A. Duncan, and S. D. Joglekar, Phys. Rev. D 16, 438 (1977). Related earlier work includes S. L. Adler, J. C. Collins, and A. Duncan, Phys. Rev. D 15, 1712 (1977); M. S. Chanowitz and J. Ellis, *ibid.* 7, 2490 (1973); R. J. Crewther, Phys. Rev. Lett. 28, 1421 (1972).

[7]A detailed discussion of the present notation, etc., is given in J. Schechter and Y. Ueda, Phys. Rev. D 3, 176 (1971); 3, 2874 (1971); 8, 987(E) (1973). The literature on chiral symmetry and effective Lagrangians is, of course, vast. The connection with the current-algebra approach and an attempt to supply enough representative references so that other works in the field can be traced is made in J. Kandaswamy, J. Schechter, and M. Singer, Phys. Rev. D 17, 1430 (1978). Apology is made to the authors of the many interesting papers we have not mentioned.

[8] See, for example, the first paper in Ref. 7.

[9] R. D. Peccei and H. R. Quinn, Phys. Rev. D 16, 1791 (1977).

[10] J. Kandaswamy, Per Salomonson, and J. Schechter, Phys. Rev. D 19, 2757 (1979). Section VII B of this reference contains a discussion of how the vacuum structure of the pure (no extra Higgs mesons) σ model is modified for nonzero θ. An attempt to give all the references through the summer of 1978 on strong T violation is made in this reference.

[11] G. 't Hooft, Phys. Rev. D 14, 3432 (1976). This is based on the idea of A. A. Belavin, A. M. Polyakov, A. S. Schwartz, and Xu. S. Tyupkin, Phys. Lett. 59B, 85 (1978). Many additional references are contained in Ref. 1.

[12] Useful references are C. G. Callan, Jr., S. Coleman, and R. Jackiw, Ann. Phys. (N.Y.) 59, 42 (1970); G. Mack and A. Salam, *ibid.* 53, 174 (1969).

[13] In fact, a number of topics which have recently attracted attention were discussed previously in this model. For example, the need for an effective determinant term was stressed in the first paper in Ref. 7. Furthermore, the fact that the current-algebra mass ratio $(A_1 - A_2)/(A_1 + A_2)$ is of order unity rather than α, as had been expected, was stressed by J. Schechter and Y. Ueda, Phys. Rev. D 4, 733 (1971). This ratio was computed and used to obtain a satisfactory understanding of meson and baryon electromagnetic mass differences, $\eta \to 3\pi$ decays, and similar processes. See also W. Hudnall and J. Schechter, *ibid.* 9, 2111 (1974) and D. N. Goswami *et al.*, *ibid.* 5, 2276 (1972).

[14] The most recent reference seems to be R. Fukuda, Phys. Rev. D 21, 485 (1980). Earlier references include C. G. Callan, R. Dashen, and D. J. Gross, Phys. Rev. D 19, 1826 (1979); J. Kogut and L. Susskind, *ibid.* 9, 3501 (1974); G. 't Hooft, in *Recent Progress in Lagrangian Field Theory and Applications*, proceedings of the Marseille Colloquium, 1974, edited by C. P. Korthes-Altes *et al.* (Centre de Physique Theorique, Marseille, 1975), p. 58; I. A. Batalin, S. G. Matinyan, and G. K. Savvidi, Yad. Fiz. 26, 407 (1977) [Sov. J. Nucl. Phys. 26, 214 (1977)]; N. K. Nielsen and P. Olesen, Nucl. Phys. B144, 376 (1978).

[15] A review of the search for particles which might be glueballs has been made by D. Robson, Nucl. Phys. B130, 328 (1977). Other related references include H. Fritzsch and P. Minkowski, Nuovo Cimento 30A, 393 (1975); P. G. O. Freund and Y. Nambu, Phys. Rev. Lett. 34, 1645 (1975); R. L. Jaffe and K. Johnson, Phys. Lett. 60B, 201 (1976); J. F. Bolzan, W. F. Palmer, and S. S. Pinsky, Phys. Rev. D 14, 3203 (1976); N. Fuchs, *ibid.* 14, 1912 (1976); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. 86B, 347 (1979); H. Goldberg, Northeastern Report No. NUB 2397, 1979 (unpublished).

[16] J. D. Bjorken, SLAC Report No. PUB-2366, 1979 (unpublished); M. Chanowitz, Phys. Rev. D 12, 918 (1975); S. Brodsky, D. Coyne, T. DeGrand, and R. Horgan, Phys. Lett. 73B, 203 (1978); M. Krammer, H. Krammer, and H. Krasemann, DESY Report No. DESY 79/20 (unpublished).

[17] S. Coleman, Commun. Math. Phys. 55, 113 (1977).