

States and observables in relativistic quantum field theories

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We discuss two related issues pertaining to the foundations of relativistic quantum field theories. First: the question of observables; more particularly the question of what constraints, if any, are imposed upon the set of observables by the requirement of relativistic microcausality. It turns out that this requirement is in fact considerably less restrictive than has been supposed in the past. Second: the issue of physical states. We find that, in contrast to the nonrelativistic case, it is not possible to define the quantum state of a system in relativistic quantum field theories, because in this latter case no consistent description of how the state changes as the result of a measurement can be developed.

I. INTRODUCTION

Any nonrelativistic quantum-mechanical system may at any time in its history be associated with well-defined values of some complete commuting subset of the observables of that system, and these values may always be verified directly by experiment.

If, for example, some isolated physical system is initially observed to have a total momentum \vec{P} , and if the system is not subsequently disturbed in any way, then the theory will describe this system as being in a state of momentum \vec{P} even at times when the system is no longer being directly observed; furthermore, the theory will predict that any future measurement of the momentum will with certainty confirm that this description is correct.

Our purpose in the present paper is to consider to what extent these circumstances may persist in the case of relativistic quantum field theories, and this will take its natural form from two basic questions: First, what kinds of quantities may in principle be measured in these theories? Second, what are the states, or rather, can these theories support the idea of a physical state at all?

We will begin in Sec. II by looking more carefully at nonrelativistic quantum mechanics with an eye towards the transition to the relativistic case. The nonrelativistic equations of motion and the collapse postulate, which dictates how the state of the system changes as the result of a measurement, will together define a unique state history for any system subject to given initial conditions. Section II will briefly review the formal properties of the nonrelativistic state, how the state may be checked by appropriate experiments, and how the probabilities of the results of experiments are

calculated in the nonrelativistic case; and in connection with this last point it will be noted that the capacity of the theory to predict probabilities is to some extent independent of its capacity to define a state for a given system. Indeed the main result of the present work is that although relativistic field theories have the former capacity, they lack the latter one.

Section III will be concerned with the problem of relativistic quantum-mechanical observables and in particular with one central question, namely, what conditions are imposed upon the set of observables by the requirement of relativistic microcausality? This requirement has been thought to give rise to new uncertainty relations, without precedent in the nonrelativistic theory, which restrict the set of observables to purely local ones. However, Sec. III will show explicitly how, through purely local interactions between the measuring apparatus and the physical system, these uncertainty relations may be violated and various non-local properties of certain systems directly measured.

In Sec. IV we will study a simple nonlocal system for which a complete commuting subset of measurable observables exists, and consider whether, as in the nonrelativistic case, the values of these observables may be incorporated into a covariant definition of the state of this system. Section IV will show that in fact no such state can be constructed, because no relativistically satisfactory version of the collapse postulate can be found. Those state histories which may be checked by experiment will not transform correctly between different frames and, conversely, those which are defined so as to transform correctly will lack the capacity to be verified by experiments. We give our conclusion in Sec. V.

II. THE NONRELATIVISTIC THEORY

Let us first introduce some notation. We will denote both the observables of a physical system and the associated operators on the Hilbert space of states by capital letters A, B, C, \dots ; a_i, b_i, c_i, \dots will denote the possible results of experiments which measure those observables (a_i, b_i, c_i, \dots are, then, the eigenvalues of A, B, C, \dots); and $P(a, b, c, \dots; T_1, T_2, T_3, \dots; g, h, i, \dots; t_1, t_2, t_3, \dots)$ will denote the probability that, if the results of measurements of A, B, C, \dots carried out at times T_1, T_2, T_3, \dots , respectively, are given to be a, b, c, \dots , respectively, then the results of measurements of G, H, I, \dots carried out at times t_1, t_2, t_3, \dots will be g, h, i, \dots . For the moment it will be convenient to allow each of the capital letters to represent some complete set of commuting observables; the lower-case letters will then denote the corresponding set of eigenvalues which will uniquely specify some physical state.

Now consider a simple example. Some physical system is prepared in a specified state $|a\rangle$ by some complete measurement A carried out at time T_1 , and is later subjected to some other complete measurement B , the result of which is $|b\rangle$, at time T_2 . We can make a list of some of the properties of the nonrelativistic Schrödinger picture as follows (some of these will require further explanation below).

(i) The theory contains a covariant prescription for calculating $P(a, b; T_1, T_2 / c, d, e, \dots; t_1, t_2, t_3, \dots)$ where $t_1, t_2, t_3, \dots > T_1$ and where P must satisfy the usual requirements of a probability distribution.

Also, it will be possible everywhere in the future of T_1 to define a unique succession of states of the system, that is, to define a state history $\Psi(a, b; T_1, T_2 / c; t)$ such that:

(ii) $\Psi(t)$ evolves in t in accordance with causal, covariant dynamical equations of motion at all times except when the system is being measured.

(iii) Ψ transforms in accordance with the requirement that the equations of motion be covariant.

(iv) If $C|\Psi(t)\rangle = c|\Psi(t)\rangle$ then $P(a, b; T_1, T_2 / c; t) = 1$. (Note that the operator C which satisfies this relation will depend both on the time and the frame of reference.)

By "covariance" here we mean of course Galilean covariance, and similarly when we speak of transformations between different frames we are referring to Galilean transformations.

Satisfying property (i), then, is certainly the primary business of any physical theory. The other three properties have a less directly experimental character. Their role in the theory is to paint a picture around the experimental results,

out of which the results can be seen to emerge in a clear and natural way. This picture, the state, is characterized by certain transformation properties and conservation laws and dynamical equations of motion.

Property (iv) guarantees that (ii) and (iii) can be directly verified by experiment. (iv) asserts that if we do go and look at the system, or more specifically if we look at it in the way prescribed by property (iv), i.e., by measuring the complete set of commuting observables C of which $\Psi(t)$ is a simultaneous eigenstate, then we will with certainty observe that it is what (ii) and (iii) predict it to be. Moreover, the measurement prescribed in (iv) will not disturb the history of the state in any way; it will project the state exactly onto itself, and therefore it is referred to as a non-demolition experiment. Finally, we can obviously do as many nondemolition experiments as we like within a given interval and check to any desired degree of accuracy the complete history of the system. Indeed a limit can be approached in which the state is checked at every instant by a non-demolition experiment, and this we will call a monitoring of the state.

So in nonrelativistic quantum mechanics one can write down a complete history of the physical system, even for times when the system was not being observed, which transforms and evolves in the appropriate ways, and furthermore this history can be fully verified by experiment. We say of such a theory that it can support the idea of a physical state.

All of the probability distributions which are calculable in the theory can be built up out of the fundamental probability $P(a; T/b; t)$, which is in turn derived from quantum mechanics via the relation

$$P(a; T/b; t) = |\Psi(a; T/b; t)|^2, \quad (1)$$

where $\Psi(t)$ satisfies (ii) and (iii) (the equation of motion here being the Schrödinger equation) and the boundary condition

$$A\Psi(a; T/b; t=T) = a\Psi(a; T/b; t=T). \quad (2)$$

More elaborate predictions may now be built up purely on the basis of probability theory. If, for example, a measurement of C is carried out after a measurement of A and before a measurement of B then

$$P(a, b; T_1, T_2 / c; t) = \frac{P(a; T_1 / c; t)P(c; T_3 = t/b; t_2 = T_2)}{P(a; T_1/b; t_2 = T_2)}. \quad (3)$$

What will be of interest for us in the above relation is that one need not be able to write down

$\Psi(a, b; T_1, T_2/c; t)$ in order to calculate $P(a, b; T_1, T_2/c; t)$. Each of the factors on the right-hand side of (3) is a fundamental probability, and these are calculated not from the full state history, but, through (1), from the two-point propagators $\Psi(a; T_1/c; t)$, $\Psi(c; t/b; T_2)$, and $\Psi(a; T_1/b; T_2)$. The generalization to still more elaborate circumstances (i.e., to still larger numbers of successive measurements) is now straightforward: Given a complete set of two-point propagators, and nothing more, one can calculate a complete set of probabilities.

Finally, all that needs to be said about incomplete measurements here is that introducing them does not change the foregoing or subsequent discussion in any important way. When incomplete measurements are brought into the picture then Ψ , as we have used it above, must be taken to represent an entire subspace of the Hilbert space of states, rather than a single state, but with this understood, the various arguments and conclusions retain essentially the same form insofar as they will be relevant to our concerns here.

III. MONITORING EXPERIMENTS

In Sec. II we have reviewed some of the properties of states and measurements in non-relativistic quantum mechanics, and now we are prepared to begin the real work of the present paper, which is to examine whether, and in what forms, these properties are also characteristic of relativistic quantum field theories.

This section will be concerned with the subject of monitoring experiments; that is, with property (iv), which reads "If $\Psi(t)$ is an eigenstate of $C(t)$ with eigenvalue c , then $P(a, b; T_1, T_2/c; t) = 1$." We do not know yet how big the set of relativistic observables is, and this is clearly going to give rise to problems with (iv), which simply has no meaning unless C is an observable. In the non-relativistic case any physically realizable $\psi(t)$ will, at any time t , be a simultaneous eigenfunction of some complete commuting subset of the observables, and so in that case (iv) does indeed guarantee that any state history will be monitorable. The relativistic circumstances are, however, a good deal more involved. We should like to know then what, in general, the relativistic observables are and, in particular, whether there are enough of them to do the experiments referred to in (iv).

It has been noticed before that a new and serious problem may arise here in the relativistic case. The source of the trouble is the requirement of relativistic causality, and the trouble is that this requirement certainly imposes new limitations on

the kinds of measurement procedures that can be carried out.

Imagine, for example, that it is possible to design a nondemolition experiment for an arbitrary single-particle momentum eigenstate. That is, we have some experimental apparatus which interacts with the particle only for an arbitrarily short time and in such a way that, after the interaction has ended, the state of the particle is entirely undisturbed and the apparatus is with certainty in a state $f(p)$, where p is the initial (and final) momentum of the particle, such that

$$f(p') \neq f(p) \text{ if } p' \neq p. \quad (4)$$

Now consider another one-particle system which is initially localized to within some finite region of space-time A (where A is taken to be large enough to permit the number of particles to be well defined, i.e., its spatial dimensions are of the order of the Compton wavelength of the particle). Since such a localized state is a linear superposition of momentum eigenstates, and since the quantum-mechanical equations of motion are themselves linear, the device which we have imagined above is obviously capable of measuring the momentum of such a particle. Suppose, then, that at some well-defined time t_1 we carry out such a measurement. Whatever the result is, this measurement will instantaneously spread the probability density uniformly all over space, so that if another localization measurement is carried out at time $t_1 + \epsilon$, this will have a nonzero probability of localizing the particle in a region which is entirely spacelike separated from A . Such a measurement, then, can move a particle around at superluminal velocities, and such a particle, or certainly an ensemble of such particles, can carry information between spacelike-separated points, and this is a direct violation of the relativistic principle of causality. So this sort of nondemolition experiment is certainly impossible.

In the past it has been thought that this represents a restriction on the set of relativistic quantum-mechanical observables.¹ That is, the impossibility of carrying out these particular experiments has been taken to mean that the momentum of a system at some well-defined instant cannot be assigned any experimental meaning at all, and more generally that the measurement of any state of spatial extension ΔX must require a time interval at least as large as ΔT to be carried out, where

$$\Delta T > \Delta X/c, \quad (5)$$

or, combining (5) with the usual x - p uncertainty relation $\Delta x \Delta p > \hbar$, we have

$$\Delta p \Delta T > \hbar/c. \quad (6)$$

If (6) is a new relativistic uncertainty relation, that is, if it represents a real restriction on the set of relativistic observables, then it must be true of *all* possible experimental procedures. But the argument leading to (6) assumes that the single procedure in question is capable of verifying a state $|p\rangle$ for *any* value of p . If, say, we were to design a procedure which verifies the state $|p = \alpha\rangle$ but which, when applied to another momentum eigenstate $|p \neq \alpha\rangle$, may alter or destroy it (it may, for example, create new particles), then the argument leading to (6) will collapse. Indeed the possibility of carrying out such an experiment is entirely consistent with the relativistic causality principle as long as all of the interactions between the measuring devices and the system are purely local ones, i.e., as long as we restrict ourselves purely to coincidence interactions, or interactions with local densities, or interactions involving internal variables of the system which commute with the position. The work of the remainder of this section will be to construct a few explicit examples of such a procedure.

The trick is to design a procedure which combines several local interactions in such a way as to end up measuring some nonlocal property of the system.² To begin with, imagine a continuous array of measuring devices designed to collectively measure the position of a particle in the following way: Each device interacts locally with the particle through a term in the Hamiltonian of the form

$$h^{(i)} = g(t)q^{(i)}x\delta(x - x^{(i)}), \quad (7)$$

where x is the position of the particle, $x^{(i)}$ is the position of the i th measuring device, $q^{(i)}$ is some internal variable associated with that device, and $g(t)$ is a coupling which is nonzero only during a short interval $t_1 < t < t_2$, when the device is switched on. The interaction of the particle with the full array is then

$$H_{\text{int}} = \int di h^{(i)}. \quad (8)$$

So, in the Heisenberg picture,

$$\frac{\partial \pi}{\partial t} \equiv \int di \frac{\partial \pi^{(i)}}{\partial t} = - \int di \frac{\partial H_{\text{int}}}{\partial q^{(i)}} = -g(t)x, \quad (9)$$

where $\pi^{(i)}$ is the momentum conjugate to $q^{(i)}$, and $\pi \equiv \int di \pi^{(i)}$. Thus, if we take a very short interval ($t_1 \approx t_2$) during which we may approximate x and $q^{(i)}$ by constants, then we have

$$x(t_2) = \frac{\pi(t < t_1) - \pi(t > t_2)}{\int_{t_1}^{t_2} dt g(t)}, \quad (10)$$

and this is how we use our device to measure the

position.

Now consider a two-particle system. What we would like to do is to design some combination of local experiments which will collectively measure some nonlocal variable of the system. Here, in particular, we will, *without* measuring any local variables such as x_1 or x_2 , measure the nonlocal variable $x_1 - x_2$.

We will work here with two arrays of devices of the kind just described, which interact with the system through the Hamiltonian:

$$H_{\text{int}} = \int di (h_1^{(i)} + h_2^{(i)}), \quad (11)$$

where

$$h_j^{(i)} \equiv g_j(t)q_j^{(i)}x_j\delta(x_j - x^{(i)}) \quad (j = 1, 2). \quad (12)$$

For the moment we will set $g_1(t) = g_2(t)$. Now imagine that at some time $t_0 < t_1$ [t_1 is the time at which $g_1(t)$ begins to be nonzero] we bring the two devices together and prepare them in an initial state which has the properties

$$\pi_1^{(i)}(t_0) - \pi_2^{(i)}(t_0) = 0 \quad \forall i, \quad (13a)$$

$$q_1^{(i)}(t_0) + q_2^{(i)}(t_0) = 0 \quad \forall i, \quad (13b)$$

or, integrating over i ,

$$\pi_1(t_0) - \pi_2(t_0) = 0, \quad (14a)$$

$$q_1(t_0) + q_2(t_0) = 0. \quad (14b)$$

Then we separate the arrays again and allow them to interact with the particles. When the interaction is over, the devices will have measured $x_1 - x_2$; that is,

$$x_1(t) - x_2(t) = \frac{\pi_2(t > t_2) - \pi_1(t > t_2)}{\int_{t_1}^{t_2} dt g(t)} \quad (15)$$

[where we have made use of (14a) to eliminate $\pi_1(t < t_1) - \pi_2(t < t_1)$]. But they will not have measured x_1 , x_2 , or $x_1 + x_2$. In order to measure, say, x_1 , we need to know $\pi_1(t < t_1) - \pi_1(t > t_2)$; however, Π_1 does not commute with $Q_1 + Q_2$, that is, π_1 is not well defined for the state [(13) or (14)] in which the devices have been prepared initially. So no measurement of x_1 has occurred, i.e., no information about $x_1(t_2)$ can be discerned from the devices. Similarly,

$$[\Pi_2, Q_1 + Q_2] \neq 0, \quad (16)$$

$$[\Pi_1 + \Pi_2, Q_1 + Q_2] \neq 0,$$

so no measurement of x_2 or $x_1 + x_2$ has taken place, either. So we have succeeded in designing a system of purely local experiments which measures a nonlocal property of this physical system.

It is worthwhile to push this particular example somewhat further along the following lines: $X_1 - X_2$ and $P_1 + P_2$ form a complete set of commuting ob-

servables for this system; if we can measure them both we can fully verify such a state. For this purpose we employ now four different measuring devices, which interact with the particles via³

$$H_{\text{int}} = g_1(t)(q_1x_1 + q_2p_1) + g_2(t)(q_3x_2 + q_4p_2). \quad (17)$$

This needs some explanation, since none of the four interactions in (17) are really local ones. They may all be considered approximately local in the following sense: Let the order of magnitude of the separation of the two particles be large enough so that even if the momentum of each of the particles is reasonably well defined, the resulting uncertainty in this separation is only a very small fraction of the separation itself. We design our momentum detectors, then, to measure the momentum with no greater accuracy than this, and so our position detectors need only be as large as the resulting position uncertainty. Thus, provided we make the separation large enough, the four measuring devices and their various interactions may be considered, on the scale of the size of the two-particle system, to be local. We have chosen (17) for the sake of the simplicity of the subsequent calculations, but later on in this section, when we discuss a more realistic sort of experiment, we will take care to make use only of *purely* local interactions.

Now then, we prepare the initial state of the devices referred to in (17) so that

$$\pi_1(t_0) - \pi_3(t_0) = 0, \quad (18)$$

$$q_1(t_0) + q_3(t_0) = 0,$$

$$\pi_2(t_0) + \pi_4(t_0) = 0, \quad (19)$$

$$q_2(t_0) - q_4(t_0) = 0.$$

So, if we initially prepare our system in a state $|\alpha\rangle$ with $(X_1 - X_2)|\alpha\rangle = l|\alpha\rangle$ and $(P_1 + P_2)|\alpha\rangle = p|\alpha\rangle$, then (17) [with $g_1(t) = g_2(t)$], (18), and (19) define a procedure whereby $|\alpha\rangle$ may be verified. Furthermore, the interaction by means of which this measurement occurs may in principle be confined to a single arbitrarily short time interval⁴; that is, the procedure can serve as a nondemolition experiment for this state (i.e., an experiment that does not disturb the state history, in principle, at all), and so by means of a sequence of many such experiments we can fully monitor the state history.

Now we are in a position to consider a question of great importance for the problem of defining a covariant state: What does this process look like in another Lorentz frame? The two sets of interactions in (17), governed by $g_1(t)$ and $g_2(t)$, occur in spacelike-separated regions. In our original frame they are simultaneous; in another frame, then, they will not be, and this will change the character of the procedure, as seen in this new

frame, in an important way.

Here again in the interest of simplicity we will make one more approximation, namely, that all of the masses in the problem are large enough so that, for the time scale in which we are interested here, the kinetic terms in the Hamiltonian may be ignored. We begin, then, with an initial two-particle state described by

$$\begin{aligned} (X_1 - X_2)|\alpha'\rangle &= l'|\alpha'\rangle, \\ (P_1' + P_2')|\alpha'\rangle &= p'|\alpha'\rangle. \end{aligned} \quad (20)$$

Our Hamiltonian is

$$H = g_1(t')(q_1'x_1' + q_2'p_1') + g_2(t')(q_3'x_2' + q_4'p_2'), \quad (21)$$

where now

$$\begin{aligned} g_1(t') &= 0 \text{ for } t' < t_1, t' > t_2, \\ g_1(t') &= g \text{ for } t_1 < t' < t_2, \\ g_2(t') &= 0 \text{ for } t' < t_3, t' > t_4, \\ g_2(t') &= g \text{ for } t_3 < t' < t_4, \\ t_1 < t_2 < t_3 < t_4 \text{ and } t_2 - t_1 &= t_4 - t_3, \end{aligned} \quad (22)$$

and we initially prepare our measuring devices in accordance with the transformed versions of (18) and (19). The equations of motion of the system of particles and measuring devices are then

$$\begin{aligned} \dot{q}_i' - \dot{q}_2' = \dot{q}_3' = \dot{q}_4' &= 0, \\ \dot{\pi}_i' &= -\frac{\partial H}{\partial q_i'}, \quad \dot{p}_i' = -\frac{\partial H}{\partial x_i'}, \quad \dot{x}_i' = \frac{\partial H}{\partial p_i'}, \end{aligned} \quad (23)$$

which can all be integrated directly. Here we will spare the reader a little algebra and simply note some properties of the solution. First of all, the process, as viewed in this new frame, remains a measurement which verifies and eventually reconstructs the initial two-particle state. That is, for time $T_2 > t_4$,

$$x_1'(T_2) - x_2'(T_2) = l', \quad (24)$$

$$p_1'(T_2) + p_2'(T_2) = p',$$

so the two-particle state is the same at the end of the process as it was at the beginning, and furthermore the state has been recorded in the measuring devices, for example,

$$l' = x_1'(T_2) - x_2'(T_2) = \frac{\pi_3'(T_2) - \pi_1'(T_2)}{\int_{t_1}^{t_2} dt g_1(t)}, \quad (25)$$

and so on. However, the process does not leave the full history of the two-particle system entirely undisturbed, as it did in the old frame. Consider, for example, the interval $t_2 < T_1 < t_3$, where the solution reads

$$\begin{aligned}
x'_1(T_1) - x'_2(T_1) - Gq'_2(T_1) &= l', \\
p'_1(T_1) + p'_2(T_1) + Gq'_1(T_1) &= p', \\
-\pi'_1(T_1) + \pi'_3(T_1) + Gx'_1(T_1) - \frac{1}{2}G^2q'_2(T_1) &= 0, \\
\pi'_2(T_1) + \pi'_4(T_1) + Gp'_1(T_1) + \frac{1}{2}G^2q'_1(T_1) &= 0,
\end{aligned} \tag{26}$$

with

$$G \equiv \int_{t_1}^{t_2} dt g_1(t).$$

This is not what the history would have been without the intervention of the measuring devices. During this interval the full state cannot be separated into a two-particle state and a state of the measuring devices (that is, the full state cannot be written as a direct product of two such states), rather, the two systems are inexorably entangled here, and the interesting thing about this entanglement is that it is purely a product of the Lorentz transformation. In the old frame the two systems never get tangled at all, or more precisely the process of getting tangled and that of getting untangled, which occur in the intervals $t_1 \rightarrow t_2$ and $t_3 \rightarrow t_4$, respectively, in the new frame, are simultaneous in the old one.

In the old frame, then, this procedure, without disturbing in any way the state history in that frame, has changed the transformation properties of that history. So although the capacity of some experimental procedure to verify a given state is preserved under Lorentz transformations, the property of being a nondemolition experiment is not. This kind of procedure cannot monitor the history covariantly. In Sec. IV we will show that indeed there can be in principle no process of any kind capable of covariantly monitoring such a state history.

Now we proceed to the more realistic and familiar case of an Einstein-Podolsky-Rosen (EPR) experiment. We begin with some spin-0 system which decays into two spin- $\frac{1}{2}$ particles in a state $|\alpha\rangle$ where

$$|\alpha\rangle \equiv (|+\frac{1}{2}\rangle_1 |-\frac{1}{2}\rangle_2 - |-\frac{1}{2}\rangle_1 |+\frac{1}{2}\rangle_2) \frac{1}{\sqrt{2}}, \tag{27}$$

where $\sigma_x^{(1)} |+\frac{1}{2}\rangle_1 = +\frac{1}{2} |+\frac{1}{2}\rangle_1$, and so on. We want then to design a nondemolition experiment for $|\alpha\rangle$. This is simple enough. First, we note that $|\alpha\rangle$ may be uniquely defined by the requirements

$$\begin{aligned}
\sigma_x^{(1)} + \sigma_x^{(2)} &= 0, \\
\sigma_y^{(1)} + \sigma_y^{(2)} &= 0, \\
\sigma_x^{(1)} + \sigma_x^{(2)} &= 0,
\end{aligned} \tag{28}$$

and we will design our experiment to measure these quantities. So, in the now-familiar way, we design our measuring devices so as to interact with the particles through the Hamiltonian

$$\begin{aligned}
H_{\text{int}} &= g(t) (\sigma_x^{(1)} q_1 + \sigma_y^{(1)} q_2 + \sigma_z^{(1)} q_3 \\
&\quad + \sigma_x^{(2)} q_4 + \sigma_y^{(2)} q_5 + \sigma_z^{(2)} q_6)
\end{aligned} \tag{29}$$

and we prepare the device initially so that

$$\begin{aligned}
q_1 &= q_4, \quad q_2 = q_5, \quad q_3 = q_6, \\
\pi_1 &= -\pi_4, \quad \pi_2 = -\pi_5, \quad \pi_3 = -\pi_6.
\end{aligned} \tag{30}$$

With the aid of (30), we can rewrite (29) as

$$\begin{aligned}
H_{\text{int}} &= g(t) [(\sigma_x^{(1)} + \sigma_x^{(2)}) (q_1 + q_4) \\
&\quad + (\sigma_y^{(1)} + \sigma_y^{(2)}) (q_2 + q_5) \\
&\quad + (\sigma_z^{(1)} + \sigma_z^{(2)}) (q_3 + q_6)] \frac{1}{2}
\end{aligned} \tag{31}$$

and the resulting equations of motion for the σ^{-s} are

$$\begin{aligned}
\frac{\partial}{\partial t} (\sigma_x^{(1)} + \sigma_x^{(2)}) &= g(t) [(q_2 + q_5) (\sigma_x^{(1)} + \sigma_x^{(2)}) \\
&\quad - (q_3 + q_6) (\sigma_y^{(1)} + \sigma_y^{(2)})] \frac{1}{2}, \\
\frac{\partial}{\partial t} (\sigma_y^{(1)} + \sigma_y^{(2)}) &= g(t) [-(q_1 + q_4) (\sigma_x^{(1)} + \sigma_x^{(2)}) \\
&\quad + (q_3 + q_6) (\sigma_x^{(1)} + \sigma_x^{(2)})] \frac{1}{2},
\end{aligned} \tag{32}$$

$$\begin{aligned}
\frac{\partial}{\partial t} (\sigma_z^{(1)} + \sigma_z^{(2)}) &= g(t) [(q_1 + q_4) (\sigma_y^{(1)} + \sigma_y^{(2)}) \\
&\quad - (q_2 + q_5) (\sigma_x^{(1)} + \sigma_x^{(2)})] \frac{1}{2}.
\end{aligned}$$

By inspection, then, the constant functions defined in (28) are a solution of (32), so the procedure we have designed here is indeed a nondemolition experiment for $|\alpha\rangle$. The reader will note here that, as promised, we have made use strictly of local interactions in (29); that restriction has posed no difficulty in this case because all of the relevant variables commute with x_1 and x_2 .

As a final example we will apply the formalism which we have just now developed for the EPR experiment to another sort of measurement. Consider a single-particle state $|\beta\rangle$ in which the particle is in a superposition of two localized states: one at x_1 and the other at x_2 , i.e.,

$$|\beta\rangle \equiv \frac{1}{\sqrt{2}} (|x_1\rangle - |x_2\rangle). \tag{33}$$

Now we define an operator $\sigma_x^{(1)}$ by

$$\begin{aligned}
\sigma_x^{(1)} |x = x_1\rangle &= +\frac{1}{2} |x = x_1\rangle, \\
\sigma_x^{(1)} |x \neq x_1\rangle &= -\frac{1}{2} |x \neq x_1\rangle
\end{aligned} \tag{34}$$

and we define σ_y and σ_z so as to satisfy the appropriate spin commutation relations with σ_x . If the particle is a boson, then

$$[\sigma_i^{(1)}, \sigma_j^{(2)}] = 0 \tag{35}$$

and (28) will then uniquely define $|\beta\rangle$. Everything now proceeds as before; that is, we have now designed a nondemolition experiment for $|\beta\rangle$.

This example is interesting in relation to the discussion of causality at the beginning of this section. Since we have used nothing other than purely local interactions in our measuring procedure, we may be certain that the procedure cannot give rise to any violation of relativistic causality. However, if, say, we prepare the particle initially in the state $|x_1\rangle$, rather than $|\beta\rangle$, and then apply our procedure, there is certainly a nonzero probability that this measurement will instantaneously produce a nonlocalized state out of $|x_1\rangle$; and this, at first glance, would seem to lead to causal problems. But it is easy to see merely by inspection of (29) that no such problems actually arise, that is, that the probability that at the end of the measurement a particle may be located at x_2 is entirely independent of the initial conditions at x_1 . So if, at the end of the procedure, we find a particle at x_2 , it is impossible to determine whether the particle has been moved there from x_1 or created by the device at x_2 . There is not any means, then, of transferring information between x_1 and x_2 in this way, and so there is not any violation of causality.

All of this is less than sufficient, however, to determine whether there are enough relativistic-field-theoretic observables to monitor every possible system's history. The particular methods which we have outlined here have limitations. For example, the state

$$|\gamma\rangle = |x_1\rangle + |x_2\rangle + |x_3\rangle \quad (36)$$

simply cannot be characterized by the values of any complete commuting set of linear combinations of local observables, and so it cannot be verified by the sort of procedure we have discussed here.

But at least one general conclusion can be drawn, and that is that (5) and (6) are certainly not uncertainty principles; indeed we have seen explicitly how to violate them. Whether the principle of relativistic causality imposes any restrictions at all on the set of observables, however, remains to be seen.

IV. DO STATES EXIST?

The calculation of probabilities in relativistic quantum theories proceeds much as in the non-relativistic case: The fundamental probabilities $P(a/b)$ are calculated from the propagators via (1), and these probabilities may then be combined in order to build up more elaborate probability distributions such as (3). Furthermore, the Lorentz covariance of these distributions will follow from that of the propagators, and this in turn will follow from that of the equations of motion of which the propagators are defined to be solutions. There is only one question which needs to

be reconsidered in the relativistic case, namely, how are the initial conditions for the propagator determined by experimental results?

In the nonrelativistic case a measurement is taken to set initial conditions for the propagator over the equal-time hypersurface of the measurement event via (2). In the relativistic case, however, different observers will in general have different definitions of this hypersurface; presumably, then, if each observer applies (2) in his own frame, different observers may derive different sets of probabilities. That this is not the case follows from the requirement that local observables must commute at spacelike separations⁵ and from the remark that, although we may measure nonlocal properties of various physical systems, we always carry out such measurements by means of local observations on the system of the measurement apparatus.⁶ In fact, all of these different sets of initial conditions will produce identical probability distributions, and indeed a measurement may be taken to impose initial conditions over any spacelike hypersurface containing the measurement event without altering these probabilities.

So the relativistic theories satisfy property (i), whether or not they can support the idea of a state.⁷ Now, finally, we will turn our attention to this latter question.

Suppose that in a certain region of space a double-well time-independent potential of the form depicted in Fig. 1 is established so that a particle placed at x_1 will remain there, or near there, for all times, and a particle placed at x_2 will likewise remain near x_2 . At time $t = -\infty$ a charged boson is prepared in the state $|\beta\rangle$ of Eq. (33), and later at $t = 0$ the particle is localized at, say, $x = x_1$. The state history of this system will read

$$\begin{aligned} |\psi\rangle &= |\beta\rangle \quad \text{for } t < 0, \\ |\psi\rangle &= |x = x_1\rangle \quad \text{for } t > 0. \end{aligned} \quad (37)$$

The current density associated with this history

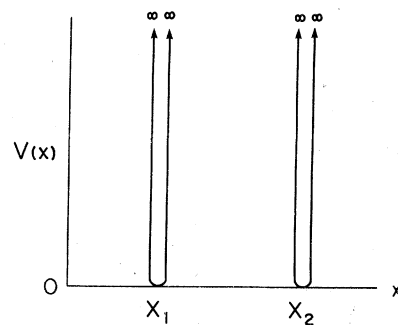


FIG. 1. An infinite double-well potential.

is

$$j^\mu(\bar{x}, t) = \langle \psi | \Psi^* \nabla^\mu \Psi - \Psi \nabla^\mu \Psi^* | \psi \rangle e \tag{38}$$

with the zeroth component

$$\rho(\bar{x}, t) = \langle \psi | \Psi^* \dot{\Psi} - \Psi \dot{\Psi}^* | \psi \rangle e, \tag{39}$$

where Ψ is the scalar field operator. This history is mapped in Fig. 2, where the integral of $\rho(\bar{x}, t) dx$ across any of the solid lines has the value $\frac{1}{2}e$.

Consider how this history will appear in another frame boosted with respect to the original one, say, in the \bar{x} direction with velocity β . $j_\mu(\bar{x}, t)$ transforms as a four-vector:

$$j_\mu(\bar{x}, t) = \Lambda_\nu^\mu j'_\nu(\bar{x}', t'), \tag{40}$$

where Λ_ν^μ is the Lorentz-transformation matrix for four-vectors and (\bar{x}', t') is the Lorentz-transformed point (\bar{x}, t) . Figure 3 maps the history in this new frame; in the figure, $\tan\theta = \beta \equiv \tanh\alpha$ and the new charge density is

$$\rho'(\bar{x}', t') = \rho(\bar{x}, t) \cosh\alpha - j_x(\bar{x}, t) \sinh\alpha. \tag{41}$$

Now the problem is obvious. The history of Fig. 3 certainly cannot be verified by experiment: In Fig. 3 the "state" at time $t' = t_3$, for example, does not carry the correct total charge. Indeed by the same token this is not a state at all, that is, it is not an element of the Hilbert space, because it cannot be assigned well-defined values of any complete commuting set of observables.

It is possible to define a monitorable state history in the primed frame; this history will collapse along $t' = t_1$. The problem is that such a history will not be the Lorentz-transformed version of Fig. 2. There are, then, no states at all which both transform properly and may be monitored in any frame; indeed in the relativistic case these two requirements [properties (iii) and (iv) of Sec. II] simply contradict one another.

It merits some emphasis that the discussion of nonlocal measurements in Sec. III has had a cen-

tral role to play in the present argument. If the nonlocal variables could not have been measured, if (5) had been a real uncertainty relation, then it would have been without any empirical meaning to assert of a system that it was in some eigenstate of a nonlocal operator. The question of monitoring these systems certainly could not have arisen at all in this case; even the introduction of such states into the formalism would arguably be superfluous to the physical content of the theory. That, in fact, these measurements can be carried out vastly changes the picture: There are indeed for this case sufficiently many nonlocal observables to uniquely characterize and to monitor nonlocal state histories, it does indeed have empirical meaning in a given frame to define such states (i.e., to attribute values of such variables to the system). The problem is not that these variables cannot be well defined, but rather that their assignment to a state does not transform properly.

Several other authors,^{8,9} noting that the postulate of collapse along an equal-time hypersurface is manifestly not a covariant one, have argued that in the relativistic case the collapse must be taken to occur along some invariant spacelike hypersurface containing the measurement event. Various different prescriptions have been proposed, all of them based on the remark first made by Bloch and later proven by Hellwig and Kraus that all possible choices of this hypersurface will lead to identical probability distributions for all local observables.

Consequently, these proposals all suffer from a common flaw: They have restricted their attention to local observables or, equivalently, they have ignored the requirements of monitorability. Bloch, for example, suggests that the collapse may be taken to occur along an equal-time hypersurface in some arbitrary frame, since any choice of this frame will lead to the same local probabilities. But this is exactly the prescription which

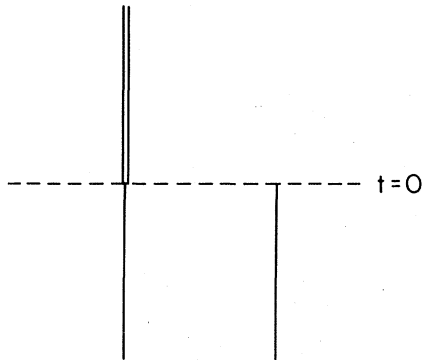


FIG. 2. The reduction of $|\beta\rangle$ to $|x=x_1\rangle$ at $t=0$.

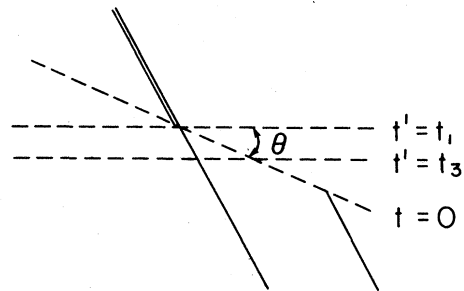


FIG. 3. The history of Fig. 2 as seen in the primed frame.

we have just now considered in detail, and which we have seen to be unsatisfactory because certain of the nonlocal variables, such as the total charge in the system,¹⁰ will not transform properly here. Hellwig and Kraus, who would prefer on aesthetic grounds that the prescription take the same form in all Lorentz frames, propose that the collapse be taken to occur along the backward light cone of the measurement event, as is illustrated here in Fig. 4. The reader can verify by inspection, however, that this prescription will not conserve the total charge even in a single frame.

V. CONCLUSION

Our central purpose here has been to consider whether relativistic quantum-mechanical systems may be described in terms of some subset of their observable physical variables, even at times when the system is not directly being observed. We have been interested, for example, in whether a system measured to have momentum \vec{P} at some time may consistently be described as being in a state of momentum \vec{P} at later times when it is no longer being observed, if it has not been disturbed in any way in the ensuing interval.

We have found that, in the relativistic case, such states do not exist; that in this case the various elements of the definition of the state are mutually exclusive. A description of the physical system in terms of its observables simply cannot consistently be written down.

This does not diminish the predictive power of these theories, but certainly it alters their conceptual content. Even if, say, one can predict with certainty that any measurement of the total charge of a system at any time in any frame will yield the value e , still neither this charge nor any other physical property may be consistently at-

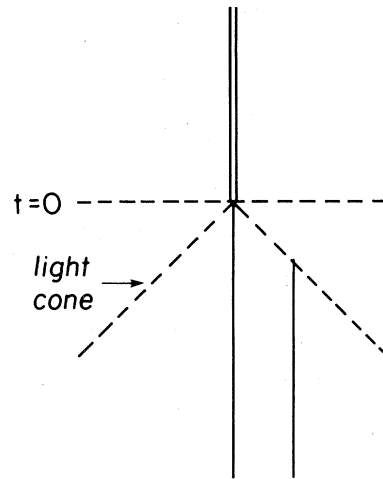


FIG. 4. The proposal of Hellwig and Kraus.

tributed to the state.

The equations of motion and the postulate of collapse enter into the calculation of probabilities exactly as they do in the nonrelativistic case, but they can no longer be thought of as describing the evolution of the physical system, because it is impossible to define a consistent description of the system which collapses, or evolves in accordance with these equations.

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¹L. D. Landau and R. Peierls, *Z. Phys.* **69**, 56 (1931).

²A sharp distinction is often drawn in the literature between "particle observables" and the more general class of "field observables" [e.g., A. S. Wightman and S. Schweber, *Phys. Rev.* **98**, 812 (1955)]. For our purposes in the present work no such distinction need be made: We will refer to any measurable dynamical property of a system as an observable. The relevant distinction here is not that between particle and field observables, but rather that between local and nonlocal ones.

³We have employed four measuring devices here only to make the discussion somewhat more transparent; the same measurements could in fact have been carried out using only two devices.

⁴Arbitrarily short, that is, compared to $\Delta T = \Delta L/c$, where ΔL is the separation of the two particles.

⁵This issue is discussed by Bloch (Ref. 8), and also by

Hellwig and Kraus (Ref. 9).

⁶That is, the nonlocal properties of these systems are measured by allowing the system to interact with the measuring apparatus in such a way that the various nonlocal properties of the system become correlated to local properties of the apparatus. The results of these experiments, then, may be characterized in terms of local properties of the apparatus. In order to calculate the probabilities of such results we require *not* a full state history, but rather only those propagators connecting pairs of local states of the apparatus.

⁷Indeed it is not difficult to formulate the theory entirely without reference to the state. See, for example, M. D. Srinivas, *J. Math. Phys.* **20**, 1593 (1979).

⁸I. Bloch, *Phys. Rev.* **156**, 1377 (1967).

⁹K. E. Hellwig and K. Kraus, *Phys. Rev. D* **1**, 566 (1970).

¹⁰The charge, that is, measured nonlocally.