PHYSICAL REVIEW D VOLUME 21, NUMBER 12 15 JUNE 1980

Are grand unified theories compatible with standard cosmology?

Martin B. Einhorn
Department of Physics, University of Michigan, Ann Arbor, Michigan 48109

D. L. Stein
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

Doug Toussaint
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 11 January 1980)

The existence of superheavy monopoles is a necessary consequence of grand unified field theories. Estimates of the number of monopoles produced in the early Universe are made under some very general assumptions. Except possibly for the case of small Higgs mass, the number produced is many orders of magnitude greater than that allowed by the standard "hot" big-bang cosmology.

A necessary consequence of grand unified theories of the strong, weak, and electromagnetic interactions is the existence of point topological solitons which may be identified as magnetic monopoles.[1] Zeldovich and Khlopov[2] and Preskill[3] have estimated the density of such monopoles in the early Universe and have found it to be larger than that allowed by cosmological observations. These estimates of monopole production involve the assumption that at some time the density of monopoles was in thermal equilibrium. In fact, monopoles are produced as the Universe is undergoing a phase transition and (as illustrated in the calculations of the monopole-antimonopole annihilation rate[2,3]) the rates of reactions which create and destroy monopoles are small. Consequently, there is no justification for assuming an equilibrium density of monopoles. In this paper we describe a general method of estimating the number of monopoles produced during symmetry breaking without assuming that the monopoles are ever in equilibrium. We estimate this number in two different ways. In both cases the number of monopoles produced is intolerably large by many orders of magnitude.

The mass of monopoles in grand unified theories is of order $M_x/\alpha \simeq 10^{16}$ GeV, where $M_x \approx 10^{14-15}$ GeV is the mass of the superheavy vector meson (the unification mass scale) and $\alpha \approx \frac{1}{50}$ is the fine structure constant. Owing to their enormous mass, a relatively small number density of monopoles may contribute significantly to the energy density of the Universe and hence affect its rate of expansion. By considering the resulting change in cosmological helium production. Preskill[3] concluded that at the time of nucleosynthesis[4] $r \equiv N_M/T^3 \lesssim 10^{-19}$, where N_M is the number density of monopoles and T is temperature. Monopoles (M) first become stable after a phase transition occurs which leaves unbroken a symmetry group containing a U(1) factor. Once produced, they may only be destroyed by $M\overline{M}$ annihilation, which has been analyzed by several authors.[2,3,5] Preskill[3] concluded that if initially $r \ll 10^{-9}$, assuming a Coulomb interaction between monopoles, then annihilation was negligible.

The high-temperature behavior of gauge theories was first discussed by Kirzhnits and Linde,[6] who argued that symmetry restoration in gauge theories occurs at high temperatures. The order of the phase transitions and the critical temperature T_c seem to be highly model dependent.

We will assume that the monopoles arise during the phase transition and, for reasons expressed earlier, in a lesser time than that necessary for the monopoles to come into equilibrium with the other species of particles present. To estimate the number of monopoles produced, we employ a method originally proposed by Kibble.[7]

After the Universe has cooled below the transition temperature, the vacuum expectation value of the Higgs field points in some direction in the manifold of degenerate vacuums. However, the Higgs field need not take the same direction throughout all of space. Suppose there is some finite correlation length ξ, so that the order parameters at two points separated by much more than ξ are uncorrelated, while the order parameter is smooth on scales much less than ξ. Now imagine dividing the universe into cells of size ξ, so that the Higgs fields in the centers of different cells are uncorrelated. Figure 1(a) illustrates this for a two-dimensional xy model in which arrows represent the direction of the order parameter at the center of each cell. Three cells surround a central region. Consider a closed con-

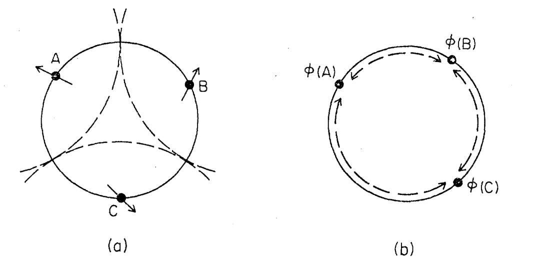

FIG. 1. (a) Kibble's construction for vortices in two dimensions. The order parameter is oriented randomly in domains A, B, and C. (b) The group space for the order parameter in (a). Values of the complex order parameter are represented by points on a circle.

tour surrounding the region and passing through the three cells. The Higgs field is continuous from one cell to the next. We assume the variation in the Higgs field is as smooth as possible, i.e., we take the shortest path in the order-parameter space between the fields at adjacent cell centers [Fig. 1(b)]. If the mapping of the path in real space onto the manifold of degenerate vacuums is topologically nontrivial, then there must be a vortex inside the circle. The probability p that this will happen is $\frac{1}{4}$.[8]

In a gauge theory the order parameter at a point can be gauge-rotated to any direction, but the topological classes of the mappings of manifolds in real space into the group space are gauge-invariant and topologically stable defects cannot be eliminated by nonsingular gauge transformations. Therefore, we may use this method in gauge theories. For monopoles in three dimensions we imagine that the order parameter is randomly oriented in domains centered at the vertices of a tetrahedron and is as smooth as possible along the edges and faces. Hence, the number density of monopoles is $N_m = p\xi^{-3}$ and so

$$r \simeq p\frac{1}{(\xi T)^3}, \tag{1}$$

where p depends on the exact structure of the order parameter; in general, we assume p to be of order $\frac{1}{10}$. (In the Georgi-Glashow model[9] it is[7] $\frac{1}{8}$.) The question now becomes how large is ξ?

The magnitude of ξ is a detailed dynamical question depending on the nature of the phase transition, among other things. However, as long as the Universe passes through the phase transition quickly, there will be insufficient time for ξ to grow indefinitely. Within the context of classical relativity and the standard cosmology,[10] an upper bound on ξ can be obtained by considering particle horizons. The maximum causal length in a radiation-dominated universe can be found using the time-temperature relation

$$t = \frac{1}{4}\left(\frac{45}{N\pi^3}\right)^{1/2}\frac{m_p}{T^2}, \tag{2}$$

where t is time, m_p the Planck mass $\approx 10^{19}$ GeV, and N = number of light-particle degrees of freedom. A photon moving along a null geodesic beginning at the initial singularity travels a proper distance $2ct$; so we may assume the Higgs fields at two points separated by more than $4ct$ are uncorrelated. Choosing $\xi = 4t\ (c=1)$, we find

$$r \sim p\frac{1}{(\xi T_c)^3} \sim p\left[\left(\frac{\pi^3 N}{45}\right)^{1/2}\frac{T_c}{m_p}\right]^3, \tag{3}$$

which, if particle horizons exist, can be considered a lower bound. For SU(5) theory $N \sim 100$, so if the phase transition occurs at the superheavy mass scale ($\sim 10^{15}$ GeV) then our causal lower bound for the monopole density is $r \sim 10^{-10}$, which is much too high. (To have $r \lesssim 10^{-19}$ requires $T_c \lesssim 10^{12}$ GeV.) However, objections may be raised against the concept of particle horizon, since quantum effects might affect the singularity at times $\lesssim 1/m_{\text{Planck}}$.[11] Moreover, it is difficult to understand how the Universe was homogeneous and in thermal equilibrium if particle horizons existed.

Estimates of the actual correlation length require a detailed dynamical analysis. A first-order transition would proceed through the formation, within the metastable vacuum, of bubbles of critical size enclosing regions of "true" vacuum, which then expand until the Universe is filled with the broken-symmetric phase.[12] ξ would then be associated with the mean size of these bubbles when they fill the Universe. Estimates of the nucleation rate and subsequent expansion of these bubbles are quite sensitive to detailed assumptions and will be reported elsewhere. However, if the transition is essentially second order, a crude estimate can be made.

Consider a temperature-dependent effective potential[13] of the form

$$V(\phi) = -\frac{\mu^2}{2}\left(-1 + \frac{T^2}{T_c^2}\right)\phi^2 + \frac{\lambda}{4}\phi^4, \tag{4}$$

with a second-order transition at $T = T_c$. Assume T varies with time according to Eq. (2). In mean field theory the equilibrium correlation length ξ_e for $T > T_c$ is

$$\xi_e^{-2} = \left.\frac{\partial^2 V}{\partial\phi^2}\right|_{\phi=0}.$$

For $T < T_c$ this expression has another interpretation: $|\xi_e|$ is the shortest wavelength for which a fluctuation around $\phi = 0$ is unstable. As T approaches T_c from above, $d\xi_e/dt$ approaches infinity. However, we expect that the actual instantaneous correlation length ξ_a, the length scale over which the field is likely to be fluctuating in one direction, cannot increase faster than some number of order c. When ξ_a reaches the length for which fluctuations are unstable, we assume that the symmetry is broken, and topological defects are well defined.

We now describe the system in terms of small oscillations about the broken-symmetric minimum and topological solitons. Assuming that $\xi_a = \xi_e$ for $d\xi_e/dt \leq 1$, and thereafter ξ_a increases at the speed of light, we find

$$r \approx p\frac{1}{4}\left(\frac{N\pi^3}{45}\right)^{1/2}\frac{\mu^2}{m_p T_c}. \qquad (5)$$

For μ and T_c of order 10^{14} GeV, the result is $r \approx 10^{-6}$. Throughout the range of Higgs mass for which the transition is approximately second order, this estimate of r is many orders of magnitude above the allowed number. However, we have assumed here that the rate of expansion of the Universe is not changed much near T_c; it is difficult to assess the accuracy of this assumption.

In principle, the nature of the transition is determined by the effective potential for the scalar field. The key points may be illustrated in the Abelian Higgs model, where the effective potential has been computed[13] to be approximately

$$V_{\text{eff}}(\phi) = \tfrac{1}{2}(-\mu^2 + \tfrac{1}{4}g^2T^2 + \tfrac{1}{3}\lambda T^2)\phi^2 - \frac{1}{24\pi}(3g^3 + g\lambda + 3\lambda^{3/2})T|\phi|^3 + \lambda\phi^4, \qquad (6)$$

where g is the gauge coupling, λ the quartic scalar coupling, and ϕ the classical scalar field. Recall that $M_A^2 = g^2\phi^2$ and $m_H^2 = 2\mu^2$. The cubic term in $|\phi|$ makes the phase transition first order, but the discontinuity in ϕ and the range of temperature of the metastable phases depend on the Higgs mass. There are three temperatures of interest (Fig. 2). At T_2 a local minimum at $\phi \neq 0$ first appears; at T_c the symmetric and asymmetric minima are degenerate; at T_1 the symmetric minimum disappears. A first-order transition occurs through the appearance of "bubbles" which can occur via quantum or thermodynamic fluctuations.[12] However, tunneling processes are slow and if the barrier is large the Universe could remain in the metastable stage almost to T_1.[6]

The formation and growth of bubbles must take place between T_c and T_1, where $T_c < T_2$. From the effective potential, one finds[13] that, for $\lambda \sim g^2$, $(T_2 - T_1) \sim O(\alpha T_1)$ and $T_1 \sim 10^{15}$ GeV. Thus, even if

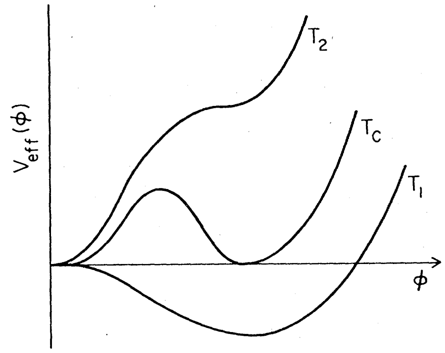

FIG. 2. Effective potential at three different temperatures $T_2 > T_c > T_1$. ϕ is the vacuum expectation value of the scalar field.

the correlation length were to grow at the speed of light throughout the interval between T_2 and T_1, ξ would not increase too much. In this regime, the first-order nature of the transition can be ignored.[14] However, if the Higgs mass is small, $\lambda \sim g^4$, the transition becomes strongly first order, higher-order corrections must be considered, and T_c could become arbitrarily small.[13,15] Postponing the phase transition to a much lower temperature may lead to a large correlation length and, hence, a small monopole density.

In summary, we have described a method of estimating the number of monopoles produced in the early Universe without assuming equilibrium density. (It is worth noting that this method should be applicable to other problems involving the formation of defects in a fast phase transition into an ordered state, as in superfluids or liquid crystals.) In all cases considered so far, a serious discrepancy has been found between the number of monopoles allowed by cosmology and those produced according to grand unified theories. Whether this problem persists for a strong first-order transition remains to be determined. Within particle physics, possible resolutions of the overabundance of monopoles include (a) the phase transition which occurs over an inordinately long time, and (b) new physics which appear at energies far below the unification scale, for example, one might arrange for the U(1) symmetry to appear first at a much lower energy. Among the cosmological modifications which might be entertained is the possibility that the Universe was never hot enough to be in the symmetric phase.

Considerations similar to those reported here have been developed by A. H. Guth and H. S. -H. Tye, whom we would like to thank for communicating their results prior to publication.[16] We also thank A. Burrows, D. Hegyi, G. Horowitz, G. L. Kane, L. Lindblom, R. Savit, and F. Wilczek for useful discussions. We gratefully acknowledge the hospitality of the Aspen Center for Physics, where this work was initiated, and the NSF ITP, where portions of this work were performed. This work was supported in part by the NSF and the U. S. Department of Energy (DOE).

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