Comments and Addenda

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Comment on the cascade model for quark fragmentation

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We discuss a particular solution of the Field-Feynman cascade model. The physical picture associated with this solution and the relevance it might have for quark-fragmentation jets in quantum chromodynamics is stressed.

In this article we discuss in some detail one particular solution of the cascade model[1-3] for jet fragmentation. This model visualizes a jet as originated by a quark fragmenting via a cascade of $q\bar{q}$ pairs so that the parent quark leaves momentum to a leftover jet which is essentially the same as the overall jet. This chain-decay ansatz illuminates the scaling behavior of jet fragmentation although the actual dynamics of quark fragmentation may differ from this ansatz.[4]

One other aspect of the cascade model is related to the space-time evolution of the final sate. The mechanism which leads to the recursive integral equation of the model suggests that the first pair is formed near the original pair in momentum space, and then other pairs follow down in momentum. It has been pointed out,[5,6] however, that a jet should develop in a manner opposite to this, that is, from the center out ("inside-outside" cascade).

The simplest solution[1] of the recursive equation in the cascade model has a number of interesting properties which we feel have not been sufficiently emphasized in the literature. (i) Field and Feynman notice that this solution is reminiscent of the boson distribution in the one-dimensional field theory of Ref. 5. To our knowledge, a detailed demonstration that this boson distribution is in fact the simplest solution of the cascade model has not been presented previously. We collect some previous results in order to sketch this derivation below and suggest that the physical picture which emerges may be closer to the actual dynamics of quark fragmentation than the chain-decay ansatz of the cascade model. (ii) The space-time evolution of the final state in Ref. 5 corresponds to an "inside-outside" cascade. Thus the simplest solution of the cascade model is, from this point of view, compatible with the correct dynamical evolution of the system. The recursive model and jet development from the center out are not, therefore, necessarily exclusive.[7] (iii) The jet-fragmentation predictions obtained from the cascade model use in fact this simplest solution plus a small constant.[1] Other authors use simply a function of this type.[2] Since the predictions thus obtained show a reasonable multihadron structure and are compatible with the available observations it is obviously important to grasp a better insight of this solution.

Consider that the initial state of a system is formed by a fermion-antifermion pair radiating a large number of bosons all together emanating from the space-time point $x=t=0$. The space-time evolution of this fermion-boson system, within the context of quantum electrodynamics in one space and one time dimension, may be *visualized* as follows.[5,6,8] The emitted bosons have a characteristic mean lifetime in their own rest frames, thereafter decaying into fermion-antifermion pairs. An observer in the center-of-mass frame of the original pair would then see, because of the time dilation effect, that the less energetic pairs decay first and the faster ones materialize last. Thus the final state evolves in the form of an inside-outside cascade. We next point out that the initial state of this system, i.e., the outgoing fermion-antifermion pair and the radiated bosons, satisfies the simplest solution of the cascade model.

There are two distributions of interest for the case of a single fermion in a process of radiation (boson bremsstrahlung): (i) the probability $h(z)$ that any radiated boson ends up with a fraction z of the original fermion momentum and (ii) the probability $f(z)$ that the fermion, after emission, has a fraction z of its initial momentum. The form of these functions can be borrowed from previous calculations[9] but it is nevertheless important to stress how the derivation follows within the contest of the present discussion.

Let us take first the distribution $f(z)$. Since the radiation field is coherent, the total probability $P(N)$ of radiating N bosons in the energy interval $\omega, \omega+d\omega$ follows a Poisson distribution,[5,10]

$$P(N)=\frac{(\lambda\, d\omega/\omega)^N}{N!}\exp\left(-\frac{\lambda}{\omega}d\omega\right), \tag{1}$$

where the bosons populate the rapidity axis with a density λ. The probability that the fermion has radiated N particles, with N_i of them having energy ω_i, can thus be written as

$$f_N(\epsilon)\approx\sum_{N_1,N_2,\cdots}\delta(\epsilon-N_1\omega_1-N_2\omega_2-\cdots)\times\delta(N-N_1-N_2-\cdots)P(N_1)P(N_2)\cdots, \tag{2}$$

where ϵ is the energy lost by the outgoing fermion. One then has[9]

$$f_N(\epsilon)\approx\frac{\lambda}{\epsilon}\left(\frac{\epsilon}{E}\right)^{\lambda}\frac{[\lambda\ln(\epsilon/m)]^{N-1}}{(N-1)!}e^{-\lambda\ln(\epsilon/m)} \tag{3}$$

and

$$f(\epsilon)=\sum_N f_N(\epsilon)=\frac{\lambda}{\epsilon}\left(\frac{\epsilon}{E}\right)^{\lambda}, \tag{4}$$

where E is the initial fermion energy and m is the boson mass. In terms of the scaling variable z, $\epsilon/E\approx 1-z$, Eq. (4) is written as

$$f(z)=\lambda(1-z)^{\lambda-1}, \tag{5}$$

normalized to one,

$$\int_0^1 f(z)dz=1\,.$$

We now turn our attention to the distribution $h(z)$. First notice that the Poisson relation (1), $(\lambda\,d\omega/\omega)^{N+1}/(N+1)!$, to radiate $N+1$ bosons of energy ω remains Poisson after any one of them is singled out, $(\lambda\,d\omega/\omega)(\lambda\,d\omega/\omega)^N/N!$. Thus by a derivation parallel to that leading to (4) one obtains for the joint fermion–any-boson distribution

$$f(x,z)=\frac{\lambda^2}{z}(1-x-z)^{\lambda-1}, \tag{6}$$

where x and z are the fractional momenta of the fermion and boson, respectively. The corresponding probability for any final-state boson is then

$$h(z)=\int_0^{1-z}f(x,z)dx$$

or

$$h(z)=\frac{\lambda}{z}(1-z)^{\lambda}. \tag{7}$$

With (5) and (7) one can evaluate the probability $F(z)$ that either the fermion or any radiated boson is found in the jet with scaled momentum z:

$$F(z)=f(z)+h(z)=\frac{\lambda}{z}(1-z)^{\lambda-1}. \tag{8}$$

It then follows that both distributions $f(z)$ and $F(z)$ satisfy identically the cascade-model recursive integral equation

$$F(z)=f(1-z)+\int_z^1 f(\eta)F\left(\frac{z}{\eta}\right)\frac{d\eta}{\eta}. \tag{9}$$

In fact the distributions (5) and (8) form the simplest solution of Eq. (9).

The particular solution of the cascade model considered above has therefore a familiar physical picture. It is simply a system of a fermion radiating bosons in a bremsstrahlung process. The fermion-boson system evolves to fermion-antifermion pairs in the final state and this jet develops from the center out. Both distribution functions $f(z)$ and $F(z)$ are predicted by the model and the recursive decay scheme implicit in Eq. (9) is not necessary either to obtain these distributions or to interpret them. We emphasize that within our approach the recursive equation is satisfied not by the final-state jet but instead by the initial fermion-boson state.[11]

It is known that quantum chromodynamics (QCD) is subjected to infrared divergences analogous to those in QED. Thus the physical picture emerging here is suggestive of the analog phenomenon in QCD. As a colored quark produced in a hard process leaves the interaction point, colored gluons are radiated away and live for a characteristic proper time before they decay into $q\bar{q}$ pairs. Neighboring quarks and antiquarks from adjacent gluons annihilate into color singlets so that hadrons are formed. A simple phenomenological model corresponding to this picture has been proposed.[8] However, one would like to know to what extent this naive analogy fits with the picture[4] of jets as complicated branching processes emerging from perturbative QCD.

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[9]E. Ugaz, Phys. Rev. D 17, 2475 (1978); L. Stodolsky, Phys. Rev. Lett. 28, 60 (1972). See also J. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971), who noticed the fact that independent-emission distributions like those in Eqs. (1) and (2) are strongly reminiscent of the photon distribution in the field of a fast-moving electric charge.

[10]This is also correct in four-dimensional QED. See, e.g., J. Bjorken and S. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), p. 202.

[11]From the parent-child relation one obtains the fermion distribution $\tilde{h}(x)$ of the final-state pairs. Obviously if the boson just splits in two in its rest frame, $\tilde{h}(x) = h(x)$. In general $\tilde{h}(x)$ is not too different from $h(x)$ if the Q value of the decay is small.