PHYSICAL REVIEW D VOLUME 21, NUMBER 11 1 JUNE 1980

The σ term revisited

R. L. Jaffe

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 3 December 1979)

A standard analysis of the pion-nucleon σ term uses partial conservation of the axial-vector current, current algebra, and a seemingly benign quark-model assumption and disagrees with experiment by a factor of 2. Here I calculate the σ term in a hybrid chiral bag model and find that the discrepancy disappears. Excepting the σ term the (successful) pattern of SU(3)- and SU(3) $\times$ SU(3)-symmetry violation in the model is essentially identical to the more standard analysis.

I. INTRODUCTION

The pion-nucleon σ term is a direct measure of chiral-SU(2)$\times$SU(2)-symmetry violation in the baryon sector. In models where chiral symmetry is broken only by explicit quark mass terms, $\Sigma_{\pi N}$ is the amount by which the nucleon mass shifts when the u- and d-quark masses are switched on. $\Sigma_{\pi N}$ can be extracted from low-energy pion-nucleon scattering. It can also be calculated to first order in the quark masses using PCAC (partial conservation of axial-vector current), current algebra, and a seemingly benign quark-model assumption. The result of this "standard analysis" differs from the measurement by a factor of 2. In this paper I calculate the σ term in a version of the quark bag model modified to incorporate approximate chiral symmetry. I find that $\Sigma_{\pi N}$ calculated in this model agrees much better with experiment. Furthermore, the remaining pattern of SU(3)- and SU(3)$\times$SU(3)-symmetry violation is essentially identical to more standard analyses.

The hybrid chiral bag (HCB) model is a phenomenological description of static hadronic sources in the chiral limit. Space is assumed to be divided into two phases: The first is the interior of hadrons, where quarks and gluons live and flavor symmetries are broken only by explicit mass terms in the quantum-chromodynamics (QCD) Lagrangian; the second is the "true vacuum" outside hadrons which expels color flux and the quarks and gluons which carry it. It is assumed that chiral symmetry is broken spontaneously by the true vacuum with the consequent appearance of massless Goldstone bosons. The theoretical and phenomenological motivation for the HCB model is discussed at length in Refs. 1 and 2. In the model, the Goldstone modes live only outside of bags and are treated as fundamental particles. Whether they are in fact fundamental degrees of freedom or bound states of quarks and gluons is not at issue. Regardless of what they are, they will cluster around static hadronic sources and couple to quarks at the bag surface in such a way that the axial-vector current is conserved (in the chiral limit). The nucleon is then a collection of quarks in a bag surrounded by a pseudoscalar-meson cloud coupled chirally. The model looks like a hybrid between a conventional quark model and a nonlinear σ model for the chiral dynamics. Similar models have been studied by Brown, Rho, and Vento[3] and by Barnhill, Cheng, and Halprin.[4]

When chiral symmetry is broken explicitly by giving the quarks in the underlying QCD Lagrangian small masses, the Goldstone bosons also become massive. The σ term which violates SU(2)$\times$SU(2) and baryon-octet mass differences which violate SU(3) receive contributions both from the quarks inside and from the Goldstone bosons outside. One can check that the interior quark contribution to these effects is very nearly linear in the quark mass up to several hundred MeV. This is not true of the exterior Goldstone-boson contribution. As the mass of some pseudoscalar boson increases, its contribution to the nucleon mass should vanish. Thus, for example, the cloud of $D\bar{D}$ pairs around a proton presumably contributes negligibly to its mass. Since the only scale in the problem is R_H, the radius of the nucleon bag, one expects the effects of a pseudoscalar boson of squared mass μ^2 to vanish when $\mu^2 R_H{}^2$ becomes large. The precise dependence of a baryon's mass on μ^2 (for fixed R_H) in a HCB model is shown in Fig. 3. The dependence is highly nonlinear for masses of order μ_K and μ_η. This is where the HCB model departs from the standard analysis: It is not sufficient to evaluate the effects of exterior Goldstone bosons to lowest order in $\mu^2 R_H{}^2$. (Note $\mu^2 \propto m_q$ in the PCAC analysis of meson masses.) If, despite this, higher orders in $\mu^2 R_H{}^2$ were ignored, then the assumptions of the standard analysis would be satisfied by the HCB model and a factor-of-2 discrepancy with $\Sigma_{\pi N}$ would remain. When the calculation is carried out to all orders in $\mu^2 R_H^2$, the effective Hamiltonian which breaks SU(3)× SU(3) and SU(3) in the HCB model is found to contain a large SU(3)-singlet term, a small octet term, and a negligibly small $\underline{27}$ term. This is exactly the form necessary to obtain agreement with experiment for the σ term while preserving the successes of the standard analysis of SU(3)-symmetry violation.

This whole discussion assumes wider significance because quark-mass ratios are predicted by grand unified theories of strong, weak, and electromagnetic interactions. The "standard analysis" of the σ term takes the ratio of strange- to nonstrange-quark masses

$$m_S/\overline{m} \cong 25 \tag{1.1}$$

from the Gell-Mann–Oakes–Renner[5] (GMOR) and Glashow-Weinberg[6] (GW) PCAC analysis of meson masses. The factor-of-2 disagreement with experiment could immediately be resolved by taking $m_s/\overline{m} \cong 12$ at the price of an inconsistency between estimates of $m_s/\overline{m}$ from meson and nucleon chiral-symmetry violation. In a recent paper[7] I argued that this discrepancy could be accounted for by remembering that quark masses are scale dependent and that the analyses of the σ term and of meson masses take place at different mass scales. That paper contains errors which invalidate that conclusion.[8] A brief analysis of the possible effects of scale on the quark-mass ratios I am using is given in the Appendix. The result is that the masses which appear in my analysis should be understood as "current" or "Lagrangian" quark masses (as distinct from "constituent" or "effective" quark masses) and that the ratios of these masses are nearly independent of scale in the region of interest. With this in mind I will drop any reference to scale in denoting the quark masses. The result of the HCB analysis is that the quark-mass ratio extracted from the σ term and baryon mass differences *is compatible* with the quark-mass ratio of Eq. (1.1) so long as external-boson effects are correctly included in the calculation.

The "standard" analysis of isospin-violating mass differences will also be modified in HCB models. Any isospin-violating effect can be separated into an electromagnetic contribution and a "tadpole" contribution arising from a nonelectromagnetic mass difference between the u and d quarks.[9] The electromagnetic contribution to baryon mass differences can be estimated from the Cottingham formula[10] and subtracted from measured mass differences leaving "tadpole mass differences" proportional to $m_d - m_u$. In HCB models, tadpole mass differences remain linear in $m_d - m_u$. In the standard analysis, the coefficient of $(m_d - m_u)$ in the symmetry-breaking Hamiltonian is $\frac{1}{2}(\overline{d}d - \overline{u}u)$. In HCB models there is another term arising from external bosons. External pions do not contribute because the $\pi^{\pm}$ and π^0 are degenerate in the absence of electromagnetism. As explained above, external kaon effects are suppressed in the HCB model so their contribution to isospin-violating mass differences is not likely to be large. In the remainder of this paper I will consider only isospin-averaged masses and σ terms. Isospin violation in HCB models is another subject, interesting in its own right but independent of the present analysis.

The rest of this paper is organized as follows. In Sec. II I define the σ term, comment on its experimental determination, and review the "standard" calculation of $\Sigma_{\pi N}$ which disagrees with the data. In Sec. III I briefly introduce the hybrid chiral bag model and construct the effective SU(3)- and SU(3)× SU(3)-symmetry-violating Hamiltonian in the HCB model. In Sec. IV I compute $\Sigma_{\pi N}$ and baryon mass differences and combine this work with other bag calculations to estimate the actual magnitude of $\overline{m}$ and m_s at typical hadronic mass scales. The result of that analysis is $\overline{m} \cong 15$ MeV and $m_s \cong 325$ MeV.

II. THE STANDARD σ-TERM ANALYSIS

The pion-nucleon σ term is defined by[11]

$$\Sigma_{\pi N} = \frac{1}{3} \sum_{a=1}^{3} \langle N | [Q_5^a, [Q_5^a, H]] | N \rangle , \tag{2.1}$$

where Q_5^a is the weak axial charge with isospin index a and H is the Hamiltonian of the strong interactions. The (isospin-averaged) nucleon state is normalized to unity. In QCD, the symmetry-breaking term in H which fails to commute with flavor-SU(3) axial charges is

$$H_{\rm SB} = \overline{m} \int d^3x [\overline{u}(x)u(x) + \overline{d}(x)d(x)] + m_S \int d^3x\, \overline{s}(x)s(x) , \tag{2.2}$$

ignoring isospin violation. Performing the commutators,

$$\Sigma_{\pi N} = \overline{m} \int d^3x \langle N | \overline{u}(x)u(x) + \overline{d}(x)d(x) | N \rangle , \tag{2.3}$$

valid up to second order in isospin violation [i.e., a term proportional to $(m_d - m_u)\langle N | \overline{d}d - \overline{u}u | N \rangle$ has been dropped]. $\Sigma_{\pi N}$ is the amount the nucleon mass changes when the u and d quarks are given a small mass $\overline{m}$.

$\Sigma_{\pi N}$ is not measured directly. Instead, a combination of isospin even πN-scattering amplitudes is analytically continued to an unphysical point (the Cheng-Dashen point)[12] where it can be shown to be proportional to $\Sigma_{\pi N}(2\mu_\pi^2)$ (Refs. 12, 13, and 14). Here

$$\Sigma_{\pi N}(q^2)=\tfrac{1}{3}\sum_{a=1}^{3}\langle N(p')|[Q_5^a,[Q_5^a,H]]|N(p)\rangle, \quad (2.4)$$

where $(p'-p)^2=q^2$ and $\Sigma_{\pi N}(0)\equiv\Sigma_{\pi N}$. In recent years several determinations of $\Sigma_{\pi N}(2\mu_\pi^2)$ (which agree reasonably well with one another) have settled down to a value of[15]

$$\Sigma_{\pi N}(2\mu_\pi^2)=65\pm 5\text{ MeV}. \quad (2.5)$$

Pagels and Pardee[16] have argued that $\Sigma_{\pi N}(2\mu_\pi^2)$ differs from $\Sigma_{\pi N}(0)$ by terms calculable in chiral perturbation theory. They find

$$\Sigma_{\pi N}(0)=\Sigma_{\pi N}(2\mu_\pi^2)-\frac{3}{8\pi}\left(\frac{g_A}{2}\right)^2\frac{\mu_\pi^3}{f_\pi^2} +O(\mu^4\log\mu^2). \quad (2.6)$$

The correction is -14 MeV. So the best present knowledge of $\Sigma_{\pi N}$ is

$$\Sigma_{\pi N}\equiv\Sigma_{\pi N}(0)=51\pm 5\text{ MeV}. \quad (2.7)$$

The "standard" QCD + quark-model calculation of $\Sigma_{\pi N}$ requires the following ingredients[17]:

(a) The GMOR-GW (Refs. 5 and 6) calculation of $m_s/\bar{m}$ from PCAC and current algebra in the meson sector.

(b) An analysis of octet baryon mass differences to lowest order in the symmetry-breaking parameters m_s and $\bar{m}$.

(c) The quark-model assumption that the strange-quark sea in the nucleon is negligible.

Together these imply $\Sigma_{\pi N}\cong 26$ MeV for $m_s/\bar{m}\cong 25$. To obtain this result one first rewrites H_{SB} as

$$H_{\text{SB}}=\int d^3x\,[c_0u_0(x)+c_8u_8(x)], \quad (2.8)$$

where

$$u_a\equiv\bar{q}\lambda_a q \quad (2.9)$$

and

$$c_0=\frac{1}{\sqrt{6}}(2\bar{m}+m_s),\qquad c_8=\frac{1}{\sqrt{3}}(\bar{m}-m_s). \quad (2.10)$$

(The λ_a are the usual 3×3 matrices of Gell-Mann normalized to $\text{Tr}\lambda_a^2=2$.) If one ignores further dependence of u_0 and u_8 on $\bar{m}$ and m_s, then they transform as flavor-SU(3) singlet and octet, respectively. Octet baryon mass differences are proportional to c_8,

$$M_\Xi-M_N=-2\sqrt{3}Fc_8,\qquad M_\Sigma-M_\Lambda=\tfrac{4}{3}Dc_8, \quad (2.11)$$

where F and D are the invariant matrix elements of the octet operator u_a in the octet representation. The form of Eq. (2.8) and the assumption that u_8 and the spin-$\frac{1}{2}$ baryons transform as octets also implies the Gell-Mann–Okubo relation

$$M_N+M_\Xi=\tfrac{1}{2}(M_\Lambda+3M_\Sigma). \quad (2.12)$$

This is well satisfied experimentally and has always been taken as evidence for ingredient (b) listed above, for if the octet matrix elements of H_{SB} are taken linearly in $\bar{m}$ and m_s, then Eq. (2.12) is automatic. A more precise statement would be that any nonlinear effects must not produce significant terms transforming as other than flavor octet or singlet.

The σ term involves both u_0 and u_8. From Eqs. (2.3), (2.8), and (2.11) one obtains

$$\frac{\Sigma_{\pi N}}{M_\Xi-M_N+\frac{1}{2}(M_\Sigma-M_\Lambda)}=\frac{1+\lambda\sqrt{2}}{2(m_s/\bar{m}-1)}, \quad (2.13)$$

where

$$\lambda\equiv\frac{\int d^3x\langle N|u_0(x)|N\rangle}{\int d^3x\langle N|u_8(x)|N\rangle}. \quad (2.14)$$

λ is unknown. To proceed further, one must now invoke a quark-model assumption, namely[18]

$$\int d^3x\langle N|\bar{s}(x)s(x)|N\rangle=0. \quad (2.15)$$

The operator $\bar{s}s$ counts strange-quark pairs (particle and antiparticle add in $\bar{s}s$ as opposed to s^+s). Everyone knows that inelastic electron-scattering experiments have demonstrated the existence of a sea of quark pairs in the nucleon and that QCD analyses indicate that the sea becomes progressively more important at asymptotically large Q^2. However, the scale for baryon matrix elements is some small μ_0 (see Appendix A) where the strange sea in the nucleon is known not to be large.[19] Accepting Eq. (2.15) for the moment, then $\lambda=\sqrt{2}$, and taking $m_s/\bar{m}$ from Eq. (1.1), one finds $\Sigma_{\pi N}\cong 26$ MeV as quoted above. To obtain agreement with experiment, one would require $\lambda\cong 3.4$ for $m_s/\bar{m}\cong 25$. This is an astonishing result. It requires a very large sea component in the nucleon. If I assume a flavor-SU(3)-symmetric sea (if anything, the sea is expected to be depleted in s quarks) and write

$$\int d^3x\langle N|\bar{u}(x)u(x)|N\rangle = \int d^3x\langle N|\bar{d}(x)d(x)|N\rangle = \tfrac{1}{2}V + \tfrac{1}{3}C\,, \tag{2.16}$$

$$\int d^3x\langle N|\bar{s}(x)s(x)|N\rangle = \tfrac{1}{3}C\,, \tag{2.17}$$

where V and C are the total valence and sea contributions, respectively, then $C/V = 1.4$ for $m_s/\bar{m} = 25$. The sea contribution to the nucleon matrix element of $\bar{q}q$ must be greater than the valence contribution.

Faced with this discrepancy one might consider a number of ways out:

(1) Give up kaon PCAC and the GMOR-GW value of $m_s/\bar{m}$.

(2) Find a failure of assumption (b), that baryon mass differences can be analyzed to lowest order in $\bar{m}$ and (in particular) m_s.

(3) Find some anomalous contribution to $u_0(x)$.

(4) Invoke a large admixture of $\bar{q}q$ pairs in the proton at typical hadronic mass scales.

As I will describe below, a hybrid chiral quark model leads one naturally to the second alternative and to a value of $m_s/\bar{m}$ in agreement with GMOR-GW. So although kaon PCAC and the GMOR-GW analysis may be suspicious, there is no reason to give them up to save the σ term. Before passing on to the HCB calculation, alternatives 3 and 4 deserve serious consideration. As for 3, one might think that $u_0(x)$ could receive an anomalous contribution from mixing with gluon operators. This is not so. The standard analysis requires $u_0(x)$ only in the limit $\bar{m} = m_S = 0$. Inside the bag, chiral symmetry is unbroken in this limit and the chirally odd operator $u_0(x)$ cannot mix with gluon operators, all of which are even. As for 4, it is very difficult to get a substantial contribution to $\langle N|\int d^3x u_0(x)|N\rangle$ from quark pairs in a quark model. The reason is an interesting interplay of confinement and chiral symmetry and is independent of the details of the model. Free, massless quarks have $\langle\bar{q}q\rangle = 0$. Quarks in hadrons have $\langle\bar{q}q\rangle \neq 0$ because they are bound. Imagine building a model for the nucleon with three valence quarks in the lowest state of some Hamiltonian and a sea of $\bar{q}q$ pairs distributed over more energetic states. Quarks in higher-lying modes are less sensitive to the confining "potential" (or boundary conditions in the case of the bag model) and behave more like free quarks. In particular, their contribution to $\langle\bar{q}q\rangle$ vanishes rapidly with excitation energy. This can be verified by explicit calculation in the bag model. It is not possible to generate a sufficient enhancement in $\int d^3x\, u_0(x)$ without having a huge amount of energy in quark pairs.

III. THE HYBRID CHIRAL BAG MODEL

The first formulation of a bag model with fundamental Goldstone bosons was given by Chodos and Thorn[20] and by Inoue and Maskawa.[21] Recently, Callan, Dashen, and Gross[1] have emphasized that a two-phase model with quarks and gluons inside bags and Goldstone bosons outside may provide an accurate phenomenological realization of chirally invariant QCD. The motivation and formulation of such a model is reviewed in Ref. 2. There it is shown that the full hybrid chiral model (QCD, Goldstone bosons, bag, and all) is very complicated, but that for large hadronic sources (such as nucleons) the model simplifies. The effects of the external Goldstone modes can be calculated perturbatively in a small parameter

$$\epsilon = \frac{g_A}{8\pi R_H{}^2 f_\pi{}^2} \cong \tfrac{1}{6}\,, \tag{3.1}$$

which is a dimensionless measure of the boson field strength at the bag's surface. Reference 2 deals exclusively with the chiral limit in which both the quark and meson masses vanish. It is straightforward to extend that work, keeping all orders in quark and meson masses but staying at lowest order in the meson field strengths. The equations of motion and boundary conditions for a spherical bag ignoring gluons are

$$(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m)q_0 = \omega_0 q_0, \quad r < R_H \tag{3.2a}$$

$$\left.\begin{aligned} -i\hat{r}\cdot\vec{\gamma}q_0 &= q_0 \\ -(\partial/\partial r)\bar{q}_0 q_0 &= 2B \end{aligned}\right\}, \quad r = R_H \tag{3.2b, 3.2c}$$

for the zeroth-order quark wave function q_0, and

$$(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m)q_1 = \omega_1 q_0 + \omega_0 q_1, \quad r < R_H \tag{3.3a}$$

$$(\Box + \mu^2)\underline{\phi}_1 = 0, \quad r > R_H \tag{3.3b}$$

$$(-i\hat{r}\cdot\vec{\gamma} - 1)q_1 = \frac{i}{f}\underline{\lambda}\cdot\underline{\phi}_1\gamma_5 q_0, \quad r = R_H \tag{3.3c}$$

$$\hat{r}\cdot\vec{\nabla}\underline{\phi}_1 = \frac{i}{2f}\bar{q}_0\gamma_5\underline{\lambda}q_0, \quad r = R_H \tag{3.3d}$$

for the first-order meson field $\underline{\phi}_1$, generated by the quarks and for the shift q_1, in the quark wave function. Color indices are suppressed throughout. q_0 and q_1 are flavor-SU(3) triplets, $\underline{\phi}$ (and $\underline{\lambda}$) are flavor octets, and m and μ^2 are diagonal matrices in flavor space. Equations (3.2) comprise the original cavity approximation to the bag model.[22] In particular, Eq. (3.2c) fixes the bag radius, R_H to balance the vacuum pressure B. f is the meson decay constant.[23] Gluon corrections can be computed perturbatively in α_s. Since both gluon and external meson corrections are in some sense small, it is consistent to ignore each while estimating the other. Lowest-order gluon corrections were estimated in the original bag model some time ago[24] and can be added to the external meson effects calculated here.

The strategy for constructing solutions to the HCB model is first to find baryon solutions to Eqs. (3.2) As usual, only the lowest cavity mode of q_0 will be considered. This limits me to baryons from the nucleon ($\frac{1}{2}+$) octet and Δ ($\frac{3}{2}+$) decuplet. Then the first-order meson field $\underline{\phi}_1$ and quark wave function shift q_1 can be calculated from Eq. (3.3), keeping all orders in quark and meson masses. The energy is

$$\begin{aligned} E = &\int_V d^3x(-\tfrac{1}{2}q_0^\dagger i\vec{\alpha}\cdot\overleftrightarrow{\nabla}q_0 + \text{H.c.} + \bar{q}_0 m q_0 + B) \\ &+ \int_V d^3x(-\tfrac{1}{2}q_0^\dagger i\vec{\alpha}\cdot\vec{\nabla}q_1 - \tfrac{1}{2}q_1^\dagger i\vec{\alpha}\cdot\vec{\nabla}q_0 + \text{H.c.} \\ &\qquad + \bar{q}_0 m q_1 + \bar{q}_1 m q_0) \\ &+ \int_{\bar{V}} d^3x(\tfrac{1}{2}|\vec{\nabla}\underline{\phi}_1|^2 + \tfrac{1}{2}\underline{\phi}_1^\dagger\mu^2\underline{\phi}_1) + O(\alpha_s), \end{aligned} \qquad (3.4)$$

where $V\,(\bar{V})$ is the interior (exterior) of the bag. Equation (3.4) simplifies considerably when the equations of motion are used to relate q_1 to $\underline{\phi}_1$:

$$E = E_0(m) + \tfrac{1}{2}\oint_{\partial V} d^2s\,\underline{\phi}_1^\dagger\cdot\frac{\partial}{\partial r}\underline{\phi}_1, \qquad (3.5)$$

where the integral is now over the bag's surface (∂V). The first term in Eq. (3.5) is the energy of the static quark bag ignoring pions but including gluon corrections. Equation (3.5) is accurate to lowest order in the meson field strengths but given that, to all orders in meson and quark masses. It will soon be evident that the interior QCD energy $E_0(m)$ is very nearly linear in the quark masses over the range of interest here. This allows me to write

$$\begin{aligned} \Delta E^{\text{HCB}} &= E - E_0^{\text{HCB}} \\ &= \int_V d^3x\{\bar{m}[\bar{u}_0(x)u_0(x) + \bar{d}_0(x)d_0(x)] \\ &\qquad + m_s\bar{s}_0(x)s_0(x)\} \\ &\quad + \oint_{\partial V} d^2s\left(\underline{\phi}_1^\dagger\cdot\frac{\partial}{\partial r}\underline{\phi}_1 - \underline{\phi}_1^{0\dagger}\cdot\frac{\partial}{\partial r}\underline{\phi}_1^0\right). \end{aligned} \qquad (3.6)$$

Here $\underline{\phi}_1^0$ is defined to be the exterior meson field in the chiral limit, so that ΔE^{HCB} vanishes that limit.

I propose to interpret Eq. (3.6) as a Hamiltonian for symmetry violation in the space of eigenstates of the cavity bag model, and then to identify its eigenvalues with hadron masses. The first term on the right-hand side of Eq. (3.6) can be expanded in a series in the QCD coupling α_s. The first

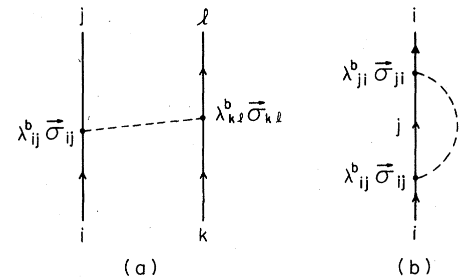

FIG. 1. Quark-line diagrams for the coupling of a meson of flavor b.

term in the series (independent of α_s) is simply the bag integral of the unperturbed quark field scalar density and is a two-quark operator. The term of order α_s is the derivative with respect to quark mass of the first-order gluon-exchange diagrams evaluated at zero quark mass. This is a four-quark operator. Terms higher order in α_s involve more complicated multiquark operators and will be ignored here. The matrix elements of $\bar{q}_0(x)q_0(x)$ can be evaluated including order α_s because the relevant gluon-exchange diagrams were computed in Ref. 24. The second term in Eq. (3.6) is a four-quark operator since the source of $\underline{\phi}_1$ is a quark bilinear. The corresponding quark-line diagrams are shown in Fig. 1. The matrix elements of Eq. (3.6) between nucleon octet states receives contributions from both octet and decuplet intermediate states.[25] The 300-MeV mass difference between the octet and decuplet which exists already in the chiral limit has so far been ignored. I have studied the effect of this splitting on chiral-symmetry violation by making an effective Yukawa model after the fashion of Brown and his collaborators.[3] Consider, for example, the nucleon. The HCB is interpreted as a model for the $NN\pi$ and $N\Delta\pi$ vertices and the mass shifts induced by the mesons are calculated in second-order perturbation theory. Because the effective momentum cutoff in this model is small ($\sim 1/R_H$), the contribution of the Δ intermediate state is suppressed. The contribution of an intermediate baryon of mass $M' = M + \Delta M$ to the nucleon mass shift is proportional to

$$I(\mu, \Delta M) \equiv \frac{1}{M'^2}\int_\mu^{(\mu^2+\kappa^2)^{1/2}} \frac{k^3 d\omega}{\omega + \Delta M}, \qquad (3.7)$$

where $k = (\omega^2 - \mu^2)^{1/2}$ and κ is the effective momentum cutoff. [To reproduce the results of the coordinate-space calculation with $M' = M$, one finds $\kappa = (3\pi)^{1/3}/R_H \approx 2/R_H$.] The suppression is measured by the ratio of the mass shift with $\Delta M = M_\Delta - M_N$ versus the shift when $\Delta M = 0$:

$$S \equiv \frac{I(\mu, \Delta M) - I(0, \Delta M)}{I(\mu, 0) - I(0, 0)}. \qquad (3.8)$$

Numerical calculation gives S of the order of $\frac{1}{8}$ for $\kappa = 400$ MeV. Similar results are obtained for other octet nucleons. So to a first-approximation decuplet intermediate states should be ignored in calculating octet matrix elements of Eq. (3.6). The effective Hamiltonian for octet symmetry breaking is then

$$H_{\rm SB}^{\rm HCB} \equiv H_{\rm SB}^{\rm quark} + H_{\rm SB}^{\rm meson}$$
$$= \int_V d^3x\{\bar{m}[\bar{u}_0(x)u_0(x) + \bar{d}_0(x)d_0(x)] + m_s\bar{s}_0(x)s_0(x)\}$$
$$+ \oint_{\partial V} d^2s[\underline{\phi}_1^\dagger(x)\cdot \mathcal{P}_8\frac{\partial}{\partial r}\underline{\phi}_1(x)$$
$$- \underline{\phi}_1^{0\dagger}(x)\cdot \mathcal{P}_8\frac{\partial}{\partial r}\underline{\phi}_1^0(r)], \qquad (3.9)$$

where $\mathcal{P}_8$ is a projection operator onto the octet. The next step is to evaluate the nucleon matrix elements of $H_{\rm SB}^{\rm HCB}$.[26]

IV. SYMMETRY VIOLATION IN THE BARYON SECTOR

The symmetry violation arising from quark masses inside hadrons is familiar. Here it is contained in the operator $E_0(m)$. In the bag model, $E_0(m)$ is nearly linear in the quark masses for masses up to several hundred MeV. To illustrate this, the matrix element of $E_0(m)$ (through order α_s) in a baryon state made of three quarks of equal mass is plotted versus m in Fig. 2 for fixed $R_H = 5\ \text{GeV}^{-1}$. As $m \to 0$ the matrix elements of $E_0(m) - E_0(0)$ vanish linearly:

$$\lim_{m\to 0}\langle E_0(m)\rangle = \langle E_0(0)\rangle + 3m\left(\frac{1}{2(x_0-1)} + \tfrac{2}{3}\alpha_s(0.05)\right), \qquad (4.1)$$

where x_0 is a cavity eigenvalue ($x_0 \cong 2.043$) and the angular brackets denote the matrix element in a hypothetical baryon made of three equal-mass quarks. The second term in Eq. (4.1) is borrowed from the analysis of Ref. 24. (The factor of 0.05 comes from an integral over cavity wave functions.) Numerically, the coefficient of $3m$ in Eq. (4.1) is roughly 0.55. As $m \to \infty \langle E_0(m)\rangle$ grows linearly with m

$$\lim_{m\to\infty}\langle E_0(m)\rangle = 3m\int d^3x\, q_0^\dagger(x)q_0(x) = 3m, \qquad (4.2)$$

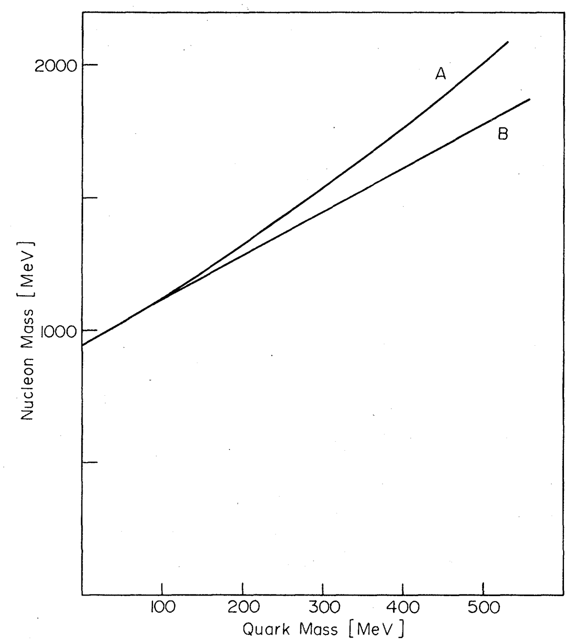

FIG. 2. A: Internal quark contribution to the mass of a nucleon composed of three quarks of mass m including first-order gluon interactions; B: linear approximation to this curve.

since as $m \to \infty$ the lower components in the quark wave functions and gluon corrections become negligible. $1/2(x_0 - 1)$ is a significant parameter in the bag model which measures the square of the upper components of the quark's Dirac wave function minus the square of their lower components. It is (to zeroth order in α_s) the parameter Weinberg[13] calls Z_m. For free quarks as $m \to 0$ so does $\int \bar{q}q$. Z_m is nonzero in the chiral limit of the bag model because quarks are confined. From Fig. 2 it is evident that $E_0(m)$ is well approximated by the linear term even up to quark masses of 500 MeV. This is the justification for replacing $E_0(m)$ by the linearized form $H_{\rm SB}^{\rm quark}$ in Eqs. (3.6) and (3.9).

The second term in Eq. (3.9) is new. It can, in principle, contain an SU(3)-flavor $\underline{27}$ piece and will be seen shortly not to be linear in meson squared masses except for very small μ^2. To work with $H_{\rm SB}^{\rm meson}$ it is convenient to write it in terms of π, K, and η contributions separately. As shown in Fig. 1, the emission of meson b involves a vertex with quarks i and j. By integrating Eqs. (3.3), the contribution of meson b to $H_{\rm SB}^{\rm meson}$ can be calculated

$$H_{\rm SB}^b = -\frac{1}{48\pi}\frac{1}{f^2R_H^3}\left[\frac{1+\mu_bR_H}{1+\mu_bR_H+\frac{1}{2}\mu_b^2R_H^2}\sum_{ijkl}\zeta_i\zeta_j\zeta_k\zeta_l(\lambda^{\dagger b}\vec{\sigma})_{ij}\mathcal{P}_8(\lambda^b\vec{\sigma})_{kl} - (\zeta_0)^4\sum_{ijkl}(\lambda^{\dagger b}\vec{\sigma})_{ij}\mathcal{P}_8(\lambda^b\vec{\sigma})_{kl}\right], \qquad (4.3)$$

where μ^b is the mass of meson b. ζ_j is a bag-model number depending on the mass of quark j:

$$\zeta_j \equiv \frac{x_j}{[2\alpha_j(\alpha_j - 1) + \xi_j]^{1/2}}, \tag{4.4}$$

where $\xi_j = m_j R$, $x_j = x(\xi_j)$ is a bag eigenvalue determined by the transcendental equation

$$\tan x = \frac{x}{1 - \xi - \alpha}, \tag{4.5}$$

and $\alpha \equiv (\xi^2 + x^2)^{1/2}$. ζ_0 is ζ evaluated at $m = 0$, where $x = x_0 = 2.043$. The second term in Eq. (4.3) is merely the subtraction of the meson energy shift in the chiral limit.

H_{SB}^b saturates rapidly with increasing meson mass. This is illustrated in Fig. 3. The contribution to a nucleon's mass from hypothetical kaons of mass μ is plotted versus the mass of the strange quark. μ^2 and m_s are assumed to be related by the GMOR-GW analysis

$$\mu^2 \propto (m + m_s) \tag{4.6}$$

with $m = 15$ MeV and the constant of proportionality chosen so that a quark of mass 350 MeV generates a pseudoscalar meson of mass 490 MeV. (These are roughly the numbers which are obtained at the conclusion of this paper for $\overline{m}$ and m_s.) If the matrix elements of H_{SB}^{meson} were linear in $\mu^2 \propto m_s$, as is usually assumed, then the contribution of mesons of mass 500 MeV ($\approx \mu_K$) would be roughly 12 times the contribution of mesons of mass 140 MeV. Instead, the ratio is about 4. This is to be expected. As explained in the Introduction, one expects very heavy mesons (such as the D and B families) to contribute negligibly to ordinary baryon masses. The difference between the contribution of the very massive meson (which is zero) and its contribution in the chiral limit (which is not) goes to a constant. The scale for this saturation is set by R_H which is $O(1/\mu_\pi)$. So H_{SB}^{meson} is highly nonlinear in μ^2 in a region where H_{SB}^{quark} is quite nicely linear in the corresponding $\overline{m}$.

The expectation values of H_{SB}^b for $b = \pi$, K, and η can be evaluated within the nucleon octet. The only new input required is the F/D ratio for the

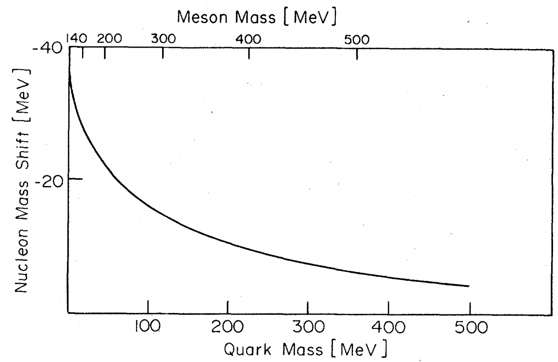

FIG. 3. The contribution to the mass of hypothetical nucleon from external hypothetical "kaons" of mass μ composed of one quark of mass 15 MeV and another of mass m. The quark and meson mass scales are normalized so that when $m = 350$ MeV, $\mu = 490$ MeV.

operator $\lambda\vec{\sigma}$ in the octet. For this we use $F/D = \frac{2}{3}$ which obtains in all SU(6)-like quark models (including the bag model) and agrees well with experimental determinations of octet axial-vector charges. Given $F/D = \frac{2}{3}$, the expectation values of $-48\pi f^2 R_H{}^3 H_{SB}^b$ are listed in Table I, where

$$\beta_b \equiv (1 + \mu_b R)/(1 + \mu_b R + \tfrac{1}{2}\mu_b{}^2 R^2),$$

$R_H = 5$ GeV^{-1} (the result of lowest-order bag calculation[24]), and $\overline{\zeta}$ and ζ_S are $\zeta(\xi)$ evaluated at $\xi = \overline{m} R_H$ and $\xi = m_s R_H$, respectively. The η contributions are more complicated than the π and K because the η (assumed to be a pure octet) is a linear superposition of quark pairs of different masses.

The elements of Table I are functions of the quark masses, as are the matrix elements of H_{SB}^{quark}. Given the experimental values of the octet baryon masses and of $\Sigma_{\pi N}$, it is possible to extract $\overline{m}$ and m_s. Since the F/D ratio of octet baryon masses is not predicted in this model, the two parameters $\overline{m}$ and m_s must be fit to two octet mass differences (the third mass difference simply fixes the F/D ratio of H_{SB}^{quark}) and $\Sigma_{\pi N}$. The remaining constraint is the Gell-Mann–Okubo relation. Below I will show (1) that the quark masses extracted from octet mass difference and

TABLE I. Expectation values of $-48\pi f^2 R_H{}^3 H_{SB}^b$ for $b = \pi$, K, and η.

	π	K	η
Nucleon	$25(\overline{\zeta}^4\beta_\pi - \zeta_0{}^4)$	$10(\overline{\zeta}^2\zeta_s{}^2\beta_K - \zeta_0{}^4)$	$\overline{\zeta}^4\beta_\eta - \zeta_0{}^4$
Σ	$\frac{44}{3}(\overline{\zeta}^4\beta_\pi - \zeta_0{}^4)$	$\frac{52}{3}(\overline{\zeta}^2\zeta_s{}^2\beta_K - \zeta_0{}^4)$	$\frac{4}{9}[(\zeta_s{}^2 + 2\overline{\zeta}^2)^2\beta_\eta - 9\zeta_0{}^4]$
Λ	$12(\overline{\zeta}^4\beta_\pi - \zeta_0{}^4)$	$20(\overline{\zeta}^2\zeta_s{}^2\beta_K - \zeta_0{}^4)$	$4(\zeta_s{}^4\beta_\eta - \zeta_0{}^4)$
Ξ	$(\overline{\zeta}^4\beta_\pi - \zeta_0{}^4)$	$26(\overline{\zeta}^2\zeta_s{}^2\beta_K - \zeta_0{}^4)$	$\frac{1}{9}[(8\zeta_s{}^2 + \overline{\zeta}^2)^2\beta_\eta - 81\zeta_0{}^4]$

TABLE II. Expectation values of H_{SB}^{b} (in MeV) for $b = \pi$, K, and η with $\overline{m} = 15$ MeV, $m_s = 325$ MeV, and $R = 5\ \text{GeV}^{-1}$.

	π	K	η
Nucleon	27	37	3
Σ	16	64	14
Λ	13	74	18
Ξ	1	97	38

$\Sigma_{\pi N}$ do yield a Gell-Mann–Okubo relation and (2) are consistent with the Gell-Mann–Oakes–Renner analysis of meson masses. The most straightforward way to approach this calculation is to assume values of $\overline{m}$ and m_s in the matrix elements of Table I, then recalculate the values of $\overline{m}$ and m_s from the physical baryon masses and the quark-model matrix elements of $H_{\text{SB}}^{\text{quark}}$. The process converges rapidly because the elements of Table I are numerically small compared to the matrix elements of $H_{\text{SB}}^{\text{quark}}$. The resulting values are close to $\overline{m} = 15$ MeV and $m_s = 325$ MeV. I shall take these as input and verify that they emerge at the end. Given $\overline{m}$, m_s, and the physical masses of the π, K, and η, the numerical values for the matrix elements of H_{SB}^{b} are given in Table II. From Table II it is clear that the mass shifts induced by chiral-symmetry violation in the boson field outside the bag are small. Only 67 MeV of the N-Ξ mass difference (≈ 380 MeV) comes from this effect. The rest, of course, comes directly from the quark-mass shifts inside hadrons.

The first element of the first column of Table II is the contribution of the external bosons to the σ term:

$$\Delta\Sigma_{\pi N} \cong 27\ \text{MeV},$$

since $\Sigma_{\pi N}$ is the shift in the nucleon mass when chiral SU(2) is broken. This is of the right sign and magnitude to make up the discrepancy noted in Sec. II. Before going into this further it is necessary to study the SU(3)-flavor transformation properties of $H_{\text{SB}}^{\text{meson}}$. In particular, it is necessary to check that it does not generate a large $\underline{27}$ component which would ruin the Gell-Mann–Okubo relation. Denoting the mass shifts of Table II as δM_N, δM_Σ, δM_Λ, and δM_Ξ, the $\underline{1}$, $\underline{8}$, and $\underline{27}$ pieces in $H_{\text{SB}}^{\text{meson}}$ are given by[29]

$$\begin{aligned} h_1 &= \tfrac{1}{8}(2\delta M_N + 2\delta M_\Xi + \delta M_\Lambda + 3\delta M_\Sigma) = 99\ \text{MeV},\\ h_{8f} &= \delta M_N - \delta M_\Xi = -67\ \text{MeV},\\ h_{8d} &= \frac{1}{\sqrt{5}}(3\delta M_\Sigma - \delta M_\Lambda - \delta M_\Xi - \delta M_N) = -12\ \text{MeV},\\ h_{27} &= -\frac{9}{8\sqrt{5}}(3\delta M_\Lambda + \delta M_\Sigma - 2\delta M_N - 2\delta M_\Xi) = -3\ \text{MeV}. \end{aligned} \tag{4.7}$$

The largest contribution is the singlet, and the $\underline{27}$ is entirely negligible. The effect of this is to increase $\Sigma_{\pi N}$ while leaving the usual analysis of baryon mass differences intact. The small size of the $\underline{27}$ piece in $H_{\text{SB}}^{\text{meson}}$ is not an accidental consequence of choices of parameters. From Table I one can construct the $\underline{27}$ piece for unspecified $\tilde{\zeta}$ and ζ_s,

$$h_{27}(\tilde{\zeta}, \zeta_s) = -\frac{9}{8\sqrt{5}}\left\{-\tfrac{4}{3}(\tilde{\zeta}^4\beta_\pi - \zeta_0^4) + \tfrac{16}{3}(\tilde{\zeta}^2\zeta_s^2\beta_K - \zeta_0^4) - 4\left[\tfrac{1}{9}(\tilde{\zeta}^2 + 2\zeta_s^2)^2\beta_\eta - \zeta_0^4\right]\right\}, \tag{4.8}$$

and see that the contributions of the π and K mesons to h_{27} are much smaller than their contributions to individual octet baryon masses. The η has low statistical weight—one compared to 3 and 4 for the pion and kaon—so the fact that its contribution to h_{27} is comparable to its contribution to individual masses is of little consequence. Therefore, the 27-plet piece of $H_{\text{SB}}^{\text{meson}}$ is naturally very small and the success of the Gell-Mann–Okubo relation is *not* evidence for the validity of the linear approximation. Note that in the linear limit, in which all coefficients in Table 1 (including μ_b^2) are expanded only to lowest order in quark masses, h_{27} is identically zero as required since in this limit H_{SB} is an octet operator.

To complete the analysis, subtract the external-meson-generated mass shifts from the experimental nucleon masses yielding effective baryon masses (M^*) which can then be used to determine quark masses. The effective masses are

$$\begin{aligned} M_N^* &= 873\ \text{MeV},\\ M_\Sigma^* &= 1098\ \text{MeV},\\ M_\Lambda^* &= 1011\ \text{MeV},\\ M_\Xi^* &= 1181\ \text{MeV}, \end{aligned}$$

and the effective σ term is the "experimental" value of 51 ± 5 MeV minus the pion contribution from Table II:

$$\Sigma_{\pi N}^* \cong 24 \pm 5\ \text{MeV}.$$

Since we have seen (from Fig. 2) that $H_{\text{SB}}^{\text{quark}}$ is quite linear, we may decompose it in a form analogous to Eq. (2.8) and repeat the analysis of Sec. II, incorporating the quark-model assumption $\lambda = \sqrt{2}$ and find from the equation analogous to Eq. (2.13) that $m_s/\overline{m} \cong 24 \pm 4$. Here the quoted eror reflects the uncertainty in the experimental determination of $\Sigma_{\pi N}$ while the approximate equality reflects the reliability of the theoretical analysis. Since $m_s = 325$ MeV and $\overline{m} = 15$ MeV were assumed as input, the calculation is consistent. The large value of the experimentally measured σ term is compatible with a large ratio of $m_s/\bar{m}$ and the standard quark-model assumption that the nucleon contains few if any $s\bar{s}$ pairs. Indeed, the source of the new found agreement is that the environs of the nucleon are anomalously depleted in $s\bar{s}$ pairs in the form of kaon and η-meson pairs due to a natural failure of linearity in the hybrid chiral model.

It remains to estimate the quark masses themselves. This is easily done from Eqs. (2.3) and (4.1) with $\Sigma_{\pi N}$ replaced by $\Sigma^*_{\pi N}$:

$$\Sigma^*_{\pi N} = \langle E_0(\bar{m})\rangle - \langle E_0(0)\rangle \,, \tag{4.9}$$

whence $\bar{m} \cong 14 \pm 3$ MeV, while $m_s \cong 336 \pm 70$ MeV. The reader will notice that when all the dust has settled, these estimates of quark masses are roughly twice as large as commonly quoted estimates.[17] This is not some subtlety associated with the σ term—indeed $\Sigma_{\pi N}$ is considered an outstanding problem in Ref. 17—instead, the factor of 2 arises from the integral of $\bar{q}q$ in the bag model. For nonrelativistic quarks $\bar{q}q = q^\dagger q$ and $\int d^3x q^\dagger q = n$ versus $n/2(x_0 - 1) = 0.48n$ [gluon corrections modify 0.48 to 0.55, see Eq. (4.1) *et seq.*] in the bag model, where n is the quark number operator. The reduction from unity measures the small components of the quark wave functions and is, I believe, a necessary part of any estimate of quark masses measured in the baryon sector. The exact numerical values obtained for $\bar{m}$ and m_s should not be taken too seriously, for the theoretical uncertainties in this analysis are not small. The value of m_s in the bag model[24] ($m_s = 280$ MeV) differs from the estimate given here because I have ignored nonlinearities in $E_0(m_s)$ (see Fig. 2) which were kept in Ref. 23. The important point is the qualitative one: The large value of $\Sigma_{\pi N}$ can be understood in a world where $m_s/\bar{m} \approx 25$ if the chiral-symmetry-violating effects of exterior Goldstone bosons are properly included in the quark-model analysis.

ACKNOWLEDGMENTS

I would like to thank N. Cottingham, H. Fritzsch, L. Maiani, and P. Minkowski for useful conversations, and CERN, where this work was begun, for hospitality and support. This work is supported in part through funds provided by the U. S. Department of Energy (DOE) under Contract No. EY-76-C-02-3069.

APPENDIX: THE SCALE DEPENDENCE OF LIGHT-QUARK MASSES

The connection between the quarks of quark models and the quarks in the QCD Lagrangian is by no means obvious. The notion of quark masses which depend on the scale at which they are observed is well defined in perturbation theory and has been analyzed using several versions of the renormalization group.[30,31] Any connection between these analyses and constituent quark models such as the bag model is largely guesswork. The idea that confinement and spontaneous symmetry breakdown are properties of the vacuum outside hadrons while QCD inside hadrons is largely perturbative leads one to attempt to extrapolate perturbative renormalization-group analysis down to typical hadronic mass scales. Accepting this, I will assume that there is a mass scale $(-\mu_0^2)$ such that the proton (for example) is, to a reasonable approximation, a state of three of the quanta of the quark field renormalized at $-\mu_0^2$. A model of baryons such as the HCB model is to be interpreted then as an effective Lagrangian for QCD renormalized at this mass scale.

The scale dependence of quark masses has been studied by many authors. The approach closest in spirit to quark models is that of Georgi and Politzer.[30,32] They define the quark mass $m(q^2)$ as the ratio of the components of 1 and of $\not{q}$ in the inverse quark propagator $S_F^{-1}(q^2)$ and find to lowest order

$$m_j(q^2) = \left(\frac{\alpha_s(q^2)}{\alpha_s(q_0^2)}\right)^d m_j(q_0^2)\,, \tag{A1}$$

where j labels flavor, $\alpha_s(q^2) \equiv g^2(q^2)/4\pi$ is the QCD running charge, $\alpha_s(q^2) \equiv 4\pi b_0 \ln q^2/\Lambda^2$, $b_0 = 11 - \frac{2}{3}N_f$, and $d \equiv 4/b_0$. If (contrary to my assumption) chiral symmetry were spontaneously broken inside hadrons, Eq. (A1) would acquire an additional term proportional to $\langle \Omega | \bar{q}_j q_j | \Omega\rangle/q^2$.

As it stands, Eq. (A1) predicts quark-mass ratios to be independent of q^2. The quark-mass parameters fit from spectroscopy must stand in the same ratios as the quark masses obtained from meson masses via PCAC and current algebra.[5,6] This picture is clearly inconsistent with the old nonrelativistic quark model with its canonical values of $\bar{m}$ (300 MeV) and m_s (500 MeV) since the GMOR-WS analysis requires $m_s/\bar{m} \approx 25$. If chiral symmetry is not spontaneously broken inside hadrons and if the corrections to Eq. (A1) discussed below are as small as expected, then it should be possible to formulate a successful constituent quark model with $m_s/\bar{m} \approx 25$. The conclusion of the present paper is that the HCB is one such model. In bag models, up, down, and strange quarks carry *energies* of roughly 300 and 500 MeV, respectively, but most of this energy persists in the chiral limit.

Corrections of order m_j^3/q^2 as well as terms higher order in g^2 have been omitted from Eq. (A1). The order m_j^3/q^2 corrections could induce a scale dependence for quark-mass ratios. The corrections can be calculated from the work of Georgi and Politzer.[29] The ratio of the $O(m_j^3/q^2)$ correction to the leading term is less than

$$\frac{\alpha_s(\mu_0^2)m_s^2(\mu_0^2)}{\pi\mu_0^2}c_j\,,$$

where m_s is the strange-quark mass ($\bar{m}$ is negligible) and c_j is $\approx\frac{64}{27}$ for the s quark and $\frac{10}{27}$ for the u and d quarks. The correction is small if it makes sense to do perturbation theory in α_s at the scale μ_0^2 which is the assumption on which the entire analysis is predicated.

---

[1]C. Callan, R. Dashen, and D. J. Gross, Phys. Rev. D 19, 1826 (1979).

[2]R. L. Jaffe, in *Pointlike Structures Inside and Outside Hadrons*, *Proceedings of the* 1979 *Erice Summer School*, edited by A. Zichichi (Plenum, New York, to be published); MIT Report No. MIT-CTP-814 (unpublished).

[3]G.E. Brown and M. Rho, Phys. Lett. 82B, 177 (1979); G. E. Brown, M. Rho, and V. Vento, Phys. Lett. (to be published).

[4]M. V. Barnhill, W. K. Cheng, and A. Halprin, Phys. Rev. D 20, 727 (1979); M. Barnhill and H. Halprin, *ibid.* 21, 1916 (1980).

[5]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[6]S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968).

[7]R. L. Jaffe, Oxford University Report No. 15/79, 1979 (unpublished).

[8]I thank P. Langacker for pointing out inconsistencies in Ref. 7.

[9]S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964); S. Coleman and H. J. Schnitzer, *ibid.* 136, B223 (1964).

[10]W. N. Cuttingham, Ann. Phys. (N.Y.) 25, 424 (1963).

[11]S. Weinberg, Phys. Rev. Lett. 17, 616 (1966).

[12]T. P. Cheng and R. Dashen, Phys. Rev. Lett. 26, 594 (1971).

[13]A. Altarelli, N. Cabibbo, and L. Maiani, Nucl. Phys. B34, 621 (1971).

[14]L. S. Brown, W. J. Pardee, and R. D. Peccei, Phys. Rev. D 4, 2801 (1971).

[15]Several recent analyses and the value (s) of $\Sigma_{\pi N}$ obtained are (in MeV): M. G. Olsson, University of Wisconsin Report No. COO-881-84, 1979 (unpublished) (77 ± 9); D. C. Moir, R. J. Jacobs, and G. E. Hite, Nucl. Phys. B103, 477 (1976) (67 ± 8, 74 ± 7); Y. A. Chao, R. E. Cutkosky, R. L. Kelly, and J. W. Alcock, Phys. Lett. 57B, 150 (1975) (57 ± 12); W. Langbein, Nucl. Phys. B94, 519 (1975) (61 ± 16, 55 ± 13); H. Nielsen and G. O. Oades, *ibid.* B72, 310 (1974) (66 ± 9); A. A. Carter, D. V. Bugg, and J. R. Carter, Lett. Nuovo Cimento 8, 639 (1973) (86 ± 12).

[16]H. J. Pagels and W. J. Pardee, Phys. Rev. D 4, 3235 (1971).

[17]For an example, see S. Weinberg, in *Festschrift for I. I. Rabi*, edited by L. Motz (N. Y. Academy of Sciences, New York, 1977).

[18]T. P. Cheng, Phys. Rev. D 13, 2161 (1976).

[19]Estimates of $\langle\bar{s}\rangle/\langle q+\bar{q}\rangle$ [where $\langle\bar{s}\rangle\equiv\int_0^1 dx\,\bar{s}(x)$ and $\langle q+\bar{q}\rangle$ is the same summed over all flavors of quark and antiquark] range from 0.00 ± 0.03 (CERN-Dortmund-Heidelberg-Saclay collaboration) to 0.032 ±0.015 (Harvard-Pennsylvania-Wisconsin-Fermilab collaboration). See for a review, A. K. Mann, in Proceedings XIV Rencontre de Moriond, edited by J. Trần Thanh Vân (CNRS, Paris, 1979). These estimates of $\langle\bar{s}\rangle$ pertain to some (large) Q^2 where approximate scaling is observed. According to perturbative QCD, $\langle\bar{s}\rangle$ decreases with decreasing Q^2 (since $m_s\langle\bar{s}\rangle$ is a renormalization-group invariant and m_s grows with decreasing Q^2, cf. Ref. 30), in which case these estimates are upper limits on $\langle\bar{s}\rangle$ at the bag scale μ_0^2.

[20]A. Chodos and C. B. Thorn, Phys. Rev. D 12, 2733 (1975).

[21]T. Inoue and T. Maskawa, Prog. Theor. Phys. 54, 1833 (1975).

[22]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974).

[23]To the accuracy of these calculations it suffices to take $f_\pi = f_K = f_\eta$.

[24]T. A. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975).

[25]After a quark emits a pion at a nucleon's surface, the three-quark system has the quantum numbers of an N or Δ. Other baryon states do not occur because the field q_0 has been truncated and includes only the lowest cavity mode. Excitation of higher-lying baryon resonances involves momentum transfers substantially larger than $1/R_H$ and cannot be studied in this model. Explicit evaluation of the contribution of higher resonances to baryon mass shifts where it is possible (e.g., the Cottingham analysis of electromagnetic mass differences) shows them to be small.

[26]The mechanism of chiral-symmetry violation in the HCB model resembles the "Goldstone pair mechanism" of Li and Pagels (Ref. 27). Roughly speaking, the external boson contribution in the HCB model corresponds to the dispersion integral of Li and Pagels with a cutoff of momenta at $O(1/R_H)$, while the internal quark (tadpole) contribution corresponds to the high-momentum contribution to the dispersion integral. The method of Li and Pagels was applied to the σ term by Renner (Ref. 28).

[27]L.-F. Li and H. Pagels, Phys. Rev. Lett. 26, 1204 (1971); Phys. Rev. D 5, 1509 (1972).

[28]B. Renner, Phys. Lett. 40B, 473 (1972).

[29]J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).

[30]H. Georgi and H. D. Politzer, Phys. Rev. D 14, 1829 (1976).

[31]S. Weinberg, Phys. Rev. D 8, 3497 (1973); O. Nachtmann and W. Wetzel, Nucl. Phys. B146, 273 (1978); W. Konnetschny, *ibid.* B152, 153 (1979).

[32]H. D. Politzer, Nucl. Phys. B117, 397 (1977).