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Ground-state baryon magnetic moments

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We discuss small contributions to baryon magnetic moments which provide corrections to the simplest nonrelativistic-quark-model estimates. In particular, we consider configuration mixing of ^2S_M states into the ground states ^2S_S, relativistic corrections to the additivity of quark moments, and isospin-violating effects. Although the "corrected" baryon magnetic moments are in better agreement with measurement, the agreement is not perfect; the remaining discrepancies are similar in size to those encountered in nuclear physics.

I. INTRODUCTION

One of the very early successes of the SU(6) symmetry scheme[1] was the calculation of baryon magnetic moments. In the symmetric quark model,[1,2] the same good results were obtained when one assumed that the quark baryon wave function was symmetric under permutations as appropriate to the 56-plet of SU(6). The predicted magnetic moments of baryons were found to be in approximate agreement with initial experiments (which were not very precise), especially if one allowed the quark magnetic moments to be free parameters.[3] The recent precise determinations of the Λ, Ξ^0, and Σ^+ magnetic moments differ (by perhaps as much as 20%) from these predictions. We discuss here two kinds of previously neglected effects which address the question of the origin of these deviations. One of these effects consists of relativistic corrections to the quark magnetic moments; the other is configuration and isospin mixing. The existence of these effects should be quite model independent. We find, however, that the precise numerical values of the magnetic moments are very sensitive functions of many details of ground-state baryons, details which are not well established. *As a result, this note should not be taken as an attempt at a very accurate description, but rather as an illustration of physical ideas which so far have been neglected when discussing magnetic moments.* We do not even know whether the effects discussed here are the only important ones neglected, and the spirit of our calculations is to illustrate that the accuracy of naive calculations may be as good as one can realistically expect. We therefore choose for convenience to estimate numerically the various effects we consider in a hybrid model: We use an oscillator model for mixing and a bag model for relativistic corrections.

Before actually turning to details, we note a related physical example which may be useful as a guide. In the simplest nuclear models[4] with a symmetric S-wave orbital wave function, the magnetic moments of the nuclei of tritium and ^{3}He were predicted to be equal to the magnetic moments of the odd nucleon they contain. This is easy to understand from the Pauli principle, which requires the two identical nucleons to pair off their spins to spin zero. Experiment is in rough agreement, with the measured moment of 2.98 nuclear magnetons (nm) for tritium close to the moment of the proton, $+2.79$ nm, while for ^{3}He -2.13 is close to -1.91 nm. The discrepancy is about 10% in both cases (or about 0.2 nm), and is believed to be due to strong-interaction (meson-exchange) effects.[5] Turning to the quark model (where we cannot compare free-quark moments with bound-quark moments), there is no reason to suppose that the magnetic moment of a given quark bound in one baryon will be identical to the moment of the same quark bound in a different baryon to any greater precision than is the case with nuclear moments. One particular effect of this type which changes quark moments comes through relativistic corrections and will be discussed below. However, strong interactions may also play a role, and one should, in view of the above example, presumably also expect corrections at the level of ± 0.2 nm from such effects in baryons.

II. CONFIGURATION MIXING

It has been argued elsewhere[6] that many violations of SU(6) selection rules have their origin in configuration mixing in the ground-state baryon wave functions, with the main impurities being ^2S_M states (or $[70, 0^+]$ states in SU(6) notation). These impurities, which are a consequence of color hyperfine interactions, give rise to the charge radius of the neutron and to various forbidden reaction amplitudes. In principle, the same interactions also admit the ^4D configurations in these states, but our calculations indicated that these impurities have very small amplitudes and we will neglect them henceforth. In the approximation that ground-state baryons are linear combinations of only ^2S_S and ^2S_M components, their magnetic moments can be expressed as follows:

$$3p = (4u - d)\cos^2\phi_8^N + (2u + d)\sin^2\phi_8^N, \tag{1a}$$

$$3n = (4d - u)\cos^2\phi_8^N + (2d + u)\sin^2\phi_8^N, \tag{1b}$$

$$3\Lambda = 3s\cos^2\phi_8^\Lambda\cos^2\phi_1^\Lambda + (u + d + s)(\cos^2\phi_1^\Lambda\sin^2\phi_8^\Lambda + \sin^2\phi_1^\Lambda) - 2(2s - d - u)\sin\phi_8^\Lambda\cos\phi_1^\Lambda\sin\phi_1^\Lambda, \tag{1c}$$

$$3\Sigma^+ = (4u - s)\cos^2\phi_8^\Sigma\cos^2\phi_{10}^\Sigma + (2u + s)(\sin^2\phi_8^\Sigma\cos^2\phi_{10}^\Sigma + \sin^2\phi_{10}^\Sigma) + 2\sqrt{2}(u - s)\sin\phi_8^\Sigma\cos\phi_{10}^\Sigma\sin\phi_{10}^\Sigma, \tag{1d}$$

$$3\Sigma^0 = (2u + 2d - s)\cos^2\phi_8^\Sigma\cos^2\phi_{10}^\Sigma + (u + d + s)(\sin^2\phi_8^\Sigma\cos^2\phi_{10}^\Sigma + \sin^2\phi_{10}^\Sigma) + \sqrt{2}(u + d - 2s)\sin\phi_8^\Sigma\cos\phi_{10}^\Sigma\sin\phi_{10}^\Sigma, \tag{1e}$$

$$3\Sigma^- = (4d - s)\cos^2\phi_8^\Sigma\cos^2\phi_{10}^\Sigma + (2d + s)(\sin^2\phi_8^\Sigma\cos^2\phi_8^\Sigma + \sin^2\phi_{10}^\Sigma) + 2\sqrt{2}(d - s)\sin\phi_8^\Sigma\cos\phi_{10}^\Sigma\sin\phi_{10}^\Sigma, \tag{1f}$$

$$3\Xi^0 = (4s - u)\cos^2\phi_8^\Xi\cos^2\phi_{10}^\Xi + (2s + u)(\sin^2\phi_{10}^\Xi\cos^2\phi_{10}^\Xi + \sin^2\phi_{10}^\Xi) + \cdots, \tag{1g}$$

$$3\Xi^- = (4s - d)\cos^2\phi_8^\Xi\cos^2\phi_{10}^\Xi + (2s + d)(\sin^2\phi_8^\Xi\cos^2\phi_{10}^\Xi + \sin^2\phi_{10}^\Xi) + \cdots, \tag{1h}$$

$$3\langle\Sigma^0|\Lambda\rangle = (\sqrt{3})(d - u)[\cos\phi_8^\Sigma\cos\phi_8^\Lambda\cos\phi_{10}^\Sigma\cos\phi_1^\Lambda] + \cdots. \tag{1i}$$

In these equations the particle symbol stands for its magnetic moment, except in the last equation (1i) which refers to the transition moment of the radiative decay from Σ^0 to Λ.

The structure of Eqs. (1) is rather simple. The first term is the contribution of the component belonging to a 56-plet. This is the only term which survives in the limit of no configuration mixing (all angles set equal to zero), and can be checked against earlier computations.[3] This term equals $\frac{4}{3}$ of the average magnetic moment of the "abundant" quarks minus $\frac{1}{3}$ the magnetic moment of the "rare" quark, except for the case of Λ where the "abundant" nonstrange quarks have their spins paired to zero. The second term is the contribution of the 70-plet components, which have a common magnetic moment equal to the average magnetic moment of all the quarks in the baryon. Finally, the third term (when present) is an interference contribution of different 70-plet contributions. The interference is numerically non-negligible only in the case of the Λ, where the angles are sufficiently large.

The parameters ϕ_8, ϕ_{10}, and ϕ_1 are mixing angles which perhaps need some additional explanation. The Λ can mix with two states belonging to a 70-plet, one of which is a member of an octet [under SU(3)], and the other a unitary singlet. Therefore, we need two mixing angles ϕ_8^Λ and ϕ_1^Λ to describe the linear combination. When computing the magnetic moment of the Λ there is no interference term between the ground-state 56 component and the 70-plet components since their orbital wave functions are orthogonal, but there is an interference between the singlet and octet components which have the same orbital wave functions. The Σ and Ξ particles have unitary octet and decuplet components as possible 70-plet impurities. Thus, only the nucleons have only octet mixing. It is important to note that the angles ϕ_8 and ϕ_{10} appropriate to different particles are not necessarily identical. These angles can be computed from ordinary perturbation theory, as explained in Ref. 6 for the specific case of the proton and neutron (for which $\sin\phi_8^N = -0.27$). The proton and neutron have the same value for ϕ_8 in the approximation that the up and down quarks are degenerate in mass. If this assumption is good, and if quarks have Dirac moments only, the "isospin" relations should hold:

$$u = -2d, \tag{2a}$$

$$\phi_8^p = \phi_8^n, \tag{2b}$$

$$\phi_8^{\Sigma^+} = \phi_8^{\Sigma^0} = \phi_8^{\Sigma^-}, \text{ etc.} \tag{2c}$$

We shall assume that these relations are valid. From (1a) and (1b) we then find that we have added a little "extra" error to the well known ratio (p/n):

$$(p/n) = -\tfrac{3}{2} - \tfrac{1}{2}\tan^2\phi_8^N = -1.54. \tag{3}$$

The deviation of this ratio from experiment (−1.46) is about 5%, and we shall be content to accept discrepancies of this size. While we assume that the mixing angles of particles in the same isospin multiplet are the same, in other words that SU(2) is good, we shall not assume that SU(3) is valid. In the limit of SU(3) (flavor) symmetry the angles ϕ_8 would be identical in all Eqs.

(1). Similarly in this limit the angles ϕ_{10} are all identical, but different from the angle ϕ_8. We do not assume that SU(3) is valid since, for example, splittings between states of different strangeness are not negligible when compared to the excitation energy of the positive-parity 70-plet; therefore SU(3) symmetry should not be a good approximation here. We have accordingly computed the mixing angles ϕ_8, ϕ_1, and ϕ_{10} arising from color hyperfine interactions using harmonic-oscillator approximations very similar to those described in Ref. 6; the results are given in Table I. As can be noted, there are indeed large SU(3)-breaking effects in these angles: Roughly speaking, they decrease with strangeness.

III. RELATIVISTIC CORRECTIONS TO ADDITIVITY

In nonrelativistic models the magnetic moment of a given constituent is independent of the composite system into which it is bound. This is experimentally the case, to a high degree of accuracy, with the magnetic moments of electrons bound in atoms. In fact, the first data on the electron's anomalous moment came from measurements of atomic magnetic moments.

In relativistic models the contribution of a given constituent to the magnetic moment of a bound state does depend on the bound state. This fact, while not emphasized in the literature, is apparent from existing bag-model calculations.[7] In this model the contribution of a given (massless) quark in a fixed mode to its hadron's magnetic moment is proportional to the radius R of the bag. In a potential model the hadron radius would decrease by 4% and 13% per additional strange quark in a harmonic and Coulomb potential, respectively. Since the true potential should lie somewhere in between, in Table II we have shown, *as an illustration*, the variation of the quark moments when they are scaled by 10% per strange quark from values chosen empirically for the down quark in the nucleon and the strange quark in the Λ.

There is another variation of this type in bound systems. The magnetic moment of a given quark in an excited mode does not equal the magnetic moment of the same quark in the ground-state mode. Therefore, strictly speaking, for a given hadron in one of the equations (1), we should use different values for each of the symbols u, d, and s: one for the ground-state 56 contribution and the other for each of the excited 70-plets. For simplicity we ignore this effect here, offering it only as a further illustration of deviations from the naive model. Note, however, that in our model[6] the radius of the Δ is about 35% larger than that of the nucleon from hyperfine interactions; this could help to increase the chronically low $\Delta \to N\gamma$ transition moment.

TABLE I. Mixing angles for baryons. The angles ϕ are computed in perturbation theory; see Ref. 6.

Baryon	$\sin\phi_8$	$\sin\phi_1$	$\sin\phi_{10}$
p, n	-0.27		
$\Sigma^+, \Sigma^0, \Sigma^-$	-0.16		$+0.015$
Λ^0	-0.20	$+0.05$	
Ξ^0, Ξ^-	-0.16		$+0.009$

TABLE II. Variation of quark magnetic moments in baryons. As described in Sec. III, we assume here $d(B) = -1.00[1 - \frac{1}{10} n_s(B)]$, where $n_s(B)$ is the number of strange quarks in the baryon B, and $s(B) = -0.58[1 - \frac{1}{10} \times (n_s(B) - 1)]$.

Baryon (B)	$d(B)$ (nm)	$s(B)$ (nm)
N	-1.00	
Σ	-0.90	-0.58
Λ	-0.90	-0.58
Ξ	-0.80	-0.52

IV. ISOSPIN VIOLATION

We have assumed, so far, that isospin is a perfect symmetry. In this approximation, as already noted, it is consistent to assume that the magnetic moments of the up and down quarks are in the ratio of their electric charges and that the mixing angles ϕ do not differ for different members of an isospin multiplet. In the same approximation Λ and Σ^0 do not mix. As an illustration of the effects of isospin breaking we shall only consider here, rather arbitrarily, the effect of $\Lambda - \Sigma^0$ mixing. We retain this particular effect since it gives a rather large contribution to the Λ magnetic moment. For the corrected magnetic moment Λ' of the Λ hyperon we find

$$\Lambda' = \cos^2\alpha\,\Lambda + 2\sin\alpha\cos\alpha\langle\Lambda|\Sigma^0\rangle ,$$

where Λ and $\langle\Lambda|\Sigma^0\rangle$ are as in Eqs. (1), and where the mixing angle $\alpha \simeq 0.0135$ rad has been determined elsewhere.[8] The correction to the Λ moment is about -0.05 nm and is the only substantial correction to ground-state moments due to isospin mixing. Comparable effects can arise, however, from intrinsic differences between the up and down quark masses, which would be expected to lead to deviations of the order of 2% in Eqs. (2).

TABLE III. Comparison with experimental magnetic moments.

Moment	"Corrected" theory[b] (nm)	SU(6) theory (nm)	Experiment (nm)	Difference (nm) "corrected" − experiment
p	+2.85	+2.82	+2.79 (Ref. 9)	+0.06
n	−1.85	−1.88	−1.91 (Ref. 9)	+0.06
Λ	−0.61	−0.94	-0.61 ± 0.01 (Ref. 10)	0.00 ± 0.01
Σ^+	+2.54	+2.82	$+2.33\pm0.13$ (Ref. 11)	$+0.21\pm0.13$
Σ^{0}[a]	+0.77	+0.94		
Σ^-	−1.00	−0.94	-1.48 ± 0.37 (Ref. 9)	$+0.48\pm0.37$
Ξ^0	−1.20	−1.88	-1.20 ± 0.06 (Ref. 12)	0.00 ± 0.06
Ξ^-	−0.43	−0.94	-1.85 ± 0.75 (Ref. 9)	$+1.42\pm0.75$
$\langle\Lambda\mid\Sigma^0\rangle$	−1.51	−1.63	-1.82 ± 0.22 (Ref. 9)	$+0.31\pm0.22$

[a] We realize that an experimental determination is not imminent.
[b] The theory estimates are obtained as described in the first paragraph of Sec. V.

V. A NUMERICAL COMPARISON TO EXPERIMENT AND CONCLUSIONS

We have tried to stress that we have no illusions that the effects we have considered here are either a complete or accurate account of the deviations one might have from the naive nonrelativistic model for magnetic moments. Nevertheless, to complete our illustration we will now tabulate the results of those effects which we have considered. We take the values of the mixing angles as given in Table I and the quark magnetic moments as detailed in Table II and compute the baryon magnetic moments using formulas (1). In addition, in the specific case of the Λ we take into account the isospin mixing discussed in the previous Section. The resulting magnetic moments are given in Table III which also shows the experimental values of these moments, with their errors, and the discrepancy between theory and experiment.

A glance at Table III shows that (leaving aside the Σ^- and Ξ^- magnetic moments which are not yet well determined) the agreement is reasonable with the discrepancies being of the order of 0.1 nm. However, these discrepancies are probably beyond the errors of measurement, especially in the case of the proton and the neutron, and should be attributed to theory. These errors are similar to the ones found in the nuclei ^{3}H and ^{3}He, and it is probable that their sources are related. We should warn again that there is nothing sacred about the numbers we have computed, this being especially the case with the quark magnetic moments of Table II; our prescription for scaling the moments is, after all, very much *ad hoc*.

Our main conclusion therefore is that the fine details of baryon magnetic moments depend on too many unknowns to be settled at this time. Nevertheless, we find some consolation in the fact that the main features of these moments have been predicted by the simplest quark models,[13] and that the observed deviations can be accommodated—though not reliably calculated—within the model.

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