Lifetimes of charmed mesons

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We discuss a model of charmed-meson decay in which free-quark pairs rather than free quarks play a central role. This modification, while preserving the simplicity of the earlier model, has the advantage that it can incorporate some recent experimental developments which are difficult to understand in the earlier model.

Recent experiments indicate some unexpected features in the decays of charmed mesons. Among these are the preliminary results from the emulsion experiment[1]

$$\tau(D^{\pm})\ (=8.3\times10^{-13}\,\text{sec}) \gg \tau(D^0)\ (=0.66\times10^{-13}\,\text{sec})\,, \tag{1}$$

and those from DELCO (Ref. 2)

$$B(D^+\to e\nu X)\ (=23\pm6\%) \gg B(D^0\to e\nu X)\ (<4\%)\,. \tag{2}$$

Since the inclusive semileptonic rates for D^+ and D^0 are expected to be equal from rather general arguments, the results (1) and (2), in fact, corroborate each other. Another recent preliminary result is the measurement[3]

$$B(D^0\to\bar{K}^0\pi^0)=(2.0\pm0.9)\%\,, \tag{3}$$

which makes it comparable to the other known branching ratios of $D\to K\pi$, viz.,[3]

$$B(D^0\to K^-\pi^+)=(2.5\pm0.4)\%\,, \tag{4}$$

$$B(D^+\to\bar{K}^0\pi^+)=(1.9\pm0.3)\%\,. \tag{5}$$

If future experiments support the results (1)–(3), our understanding of charmed-meson decays would require considerable modification. This is because on the basis of the usual picture of free-charmed-quark decay, one expects $\tau(D^{\pm})=\tau(D^0)$ and[4,5]

$$B(D^0\to\bar{K}^0\pi^0)\ll B(D^0\to K^-\pi^+)\,.$$

In this note, we propose a simple modification of the usual picture, based on the idea of color confinement. We show that our model is consistent with (1)–(3).

In the usual model,[4] the effective nonleptonic weak Hamiltonian is given by

$$H_w=\frac{G_F\cos^2\theta_C}{\sqrt{2}}\left[\frac{f_++f_-}{2}\,(\bar{u}d)(\bar{s}c)+\frac{f_+-f_-}{2}\,(\bar{s}d)(\bar{u}c)\right]$$
$$+\,\text{Cabibbo-suppressed terms}\,, \tag{6}$$

where the coefficients $f_\pm$ contain the renormalization effects of hard-gluon exchange, which have been calculated from quantum chromodynamics (QCD). The final state of the quarks $q\bar{q}q\bar{q}$ must evidently be a color singlet. However, this can arise either if each $q\bar{q}$ pair is itself a singlet or if each is an octet, so that together the two pairs form an overall singlet. The inclusive nonleptonic (NL) decay is usually obtained by adding these two contributions incoherently, which gives $\Gamma_{NL}(D^0,D^+,F^+)=(2f_+^2+f_-^2)\Gamma_0$, where

$$\Gamma_0=\frac{G_F^2M_c^5}{192\pi^3} \tag{7}$$

represents the rate[6] for the free decay $c\to q\bar{q}q$.

Since all the observed hadrons are color singlets, it is clear that a color-octet $q\bar{q}$ pair cannot manifest itself in terms of ordinary hadrons without exchanging color with the other $q\bar{q}$ color octet. Such an interaction is not taken into account in the usual analysis. Moreover, if this effect were considered, the color–octet-octet state develops an overlap with the singlet-singlet state, leading to interference effects which have been omitted. We conclude that the simple picture[4,5] leading to equal lifetimes for charmed mesons is inadequate.

To remedy this, we have to effectively consider only those final states where each $q\bar{q}$ is a confined color singlet. Since the phenomenon of confinement is not well understood from first principles, for further progress, we proceed phenomenologically.

Very recently, Koide[7] has discussed a model with the same basic idea as outlined below. However, our work differs from his in some important respects. He concerns himself with inclusive and semi-inclusive processes only and does not discuss the two-body decays (3)–(5). Furthermore he uses a four-parameter fit compared to our two-parameter fit. Our parametrization and analysis are simpler and involve the exclusive

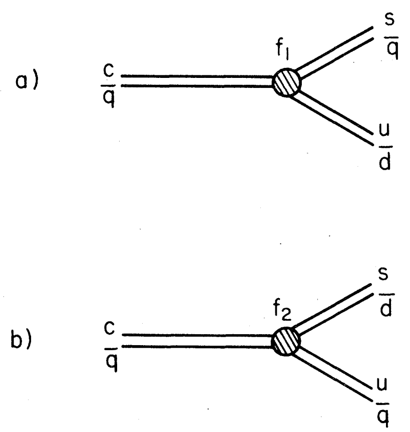

FIG. 1. The amplitudes in the nonleptonic decay of the charmed mesons. The quark-antiquark pairs are each in a color-singlet state.

modes directly.

We write the effective nonleptonic weak Hamiltonian in a form similar to Eq. (6), where the quark currents[8] now represent confined color-singlet $q\bar{q}$ pairs. Furthermore, the known coefficients $(f_+ \pm f_-)/2$ should be replaced by unknown parameters, which we call f_1 and f_2. The nonleptonic amplitudes can be written as

$$A([c\bar{q}] \to [s\bar{q}][u\bar{d}]) \propto f_1 \,, \tag{8}$$

$$A([c\bar{q}] \to [u\bar{q}][s\bar{d}]) \propto f_2 \,. \tag{9}$$

These amplitudes are represented diagramatically, in Fig. 1. Note that the SU(4) structure of the nonleptonic Hamiltonian in the present case is the same as before. Assuming, as usual, 20-plet dominance over the 84-plet, then suggests $f_2/f_1 < 0$. Except for this constraint, we shall treat f_1 and f_2 as free parameters in our analysis.

The inclusive total decay rates for charmed mesons can now be calculated easily. We obtain

$$\Gamma(D^0) = (f_1^2 + f_2^2 + 2)\Gamma_0 \,, \tag{10}$$

$$\Gamma(D^+) = [(f_1 + f_2)^2 \cos^2\theta_C + (f_1^2 + f_2^2)\sin^2\theta_C + 2]\Gamma_0 \,, \tag{11}$$

$$\Gamma(F^+) = [(f_1^2 + f_2^2)\cos^2\theta_C + (f_1 + f_2)^2 \sin^2\theta_C + 2]\Gamma_0 \,. \tag{12}$$

In Eqs. (10)–(12), the scale for the inclusive nonleptonic contribution has been taken to be the free-c-quark decay rate (7). This can always be done by a suitable redefinition of f_1 and f_2. The inclusive semileptonic decays are taken to be described in the usual way.[4,5] Here the outgoing $q\bar{q}$ pair can only be in a color-singlet state. The crucial difference among the three rates arises as follows. Consider, for example, the Cabibbo-dominant modes of D^0 and D^+. The final states $[s\bar{u}][u\bar{d}]$ and $[u\bar{u}][s\bar{d}]$ in D^0 are distinct, so the two contributions add incoherently. For D^+ decay, by contrast, the final states are indistinguishable; hence, a coherent addition of the amplitudes is called for. Eliminating f_1 and f_2 from Eqs. (10)–(12), we obtain a sum rule[9]

$$(\cos^2\theta_C - \sin^2\theta_C)\Gamma(D^0) = \cos^2\theta_C \Gamma(F^+) - \sin^2\theta_C \Gamma(D^+) \,. \tag{13}$$

An experimental test of this relation has to await a more accurate determination of the lifetimes.

We now turn our attention to the exclusive hadronic decays. For the Cabibbo-dominant two-body decays of D mesons, the amplitudes can be written as[10]

$$A(D^0 \to K^-\pi^+) \propto f_1 \,, \tag{14}$$

$$A(D^0 \to \bar{K}^0\pi^0) \propto f_2/\sqrt{2} \,, \tag{15}$$

$$A(D^+ \to \bar{K}^0\pi^+) \propto (f_1 + f_2) \,, \tag{16}$$

so that

$$A(D^0 \to K^-\pi^+) + \sqrt{2} A(D^0 \to \bar{K}^0\pi^0) = A(D^+ \to \bar{K}^0\pi^+) \,. \tag{17}$$

Equation (17) is, in fact, independent of our model and follows directly from isospin considerations, if we recall that the $\Delta C = \Delta S = 1$ piece of the nonleptonic Hamiltonian transforms as $I = 1$.

Without any specific assumption on the relative phases of the amplitudes, the sum rule (17) leads to a triangular relation for the decay rates. In terms of the branching ratios, one readily obtains

$$Z_- \leq \frac{\Gamma(D^+)}{\Gamma(D^0)} \leq Z_+ \,, \tag{18}$$

where

$$Z_\pm = [B^{1/2}(K^-\pi^+) \pm \sqrt{2} B^{1/2}(\bar{K}^0\pi^0)]^2 / B(\bar{K}^0\pi^+) \,. \tag{19}$$

For the experimental results (3)–(5), the bounds on $\Gamma(D^+)/\Gamma(D^0)$ cover too wide a range to be of much practical use.

In our model, where the amplitudes (14)–(16) are relatively real and $f_2/f_1 < 0$, Eq. (17) leads to the equality

$$\Gamma(D^+)/\Gamma(D^0) \equiv Z = Z_- \,. \tag{20}$$

It should be noted that relation (20) also obtains in the usual model, where because of the equality of lifetimes $Z = 1$. Because of the experimental uncertainties we have chosen to represent Eq. (20) in the manner shown in Fig. 2. The box encloses one standard deviation from the reported central values. For $B(D^+ \to \bar{K}^0\pi^+)$ we have used the central value from Eq. (5) and curves of constant Z have been displayed for this number. Any future change in this number can easily be accommodated by an

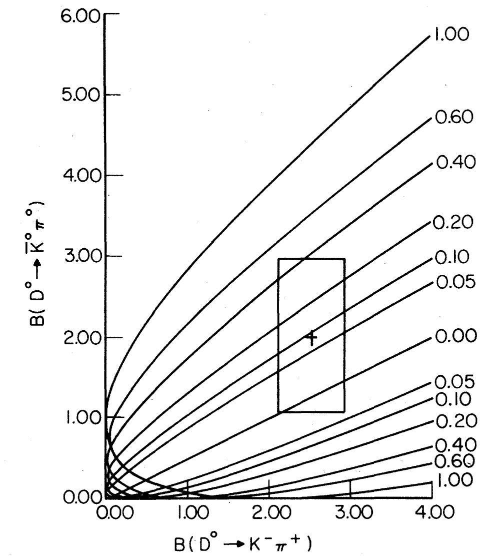

FIG. 2. Curves of constant Z [$= \Gamma(D^+)/\Gamma(D^0)$].

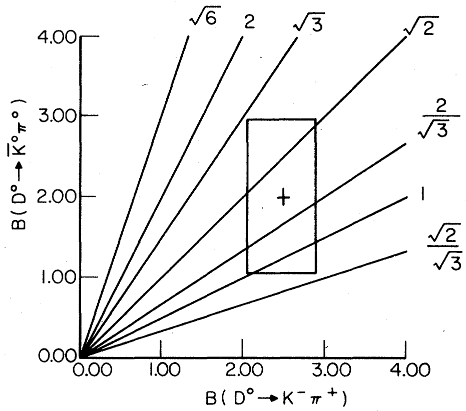

FIG. 3. Curves of constant $|R|$ ($= |f_2/f_1|$).

appropriate multiplicative change in the labeling of the Z values of the curves. We see that whereas $Z=1$ implies a negligible value of $B(D^0 \to \bar{K}^0\pi^0)$, the central values of (3) and (4) pick out a value of Z close to 0.1. *Thus our model shows that the large branching ratio (3) is quite consistent with the small ratio of the lifetimes of D^0 and D^+ given by (1).*

Another relation can be obtained from Eqs. (14) and (15):

$$B(\bar{K}^0\pi^0)/B(K^-\pi^+) = \tfrac{1}{2}R^2\,, \tag{21}$$

where

$$R = \frac{f_2}{f_1}\ (<0)\,. \tag{22}$$

Figure 3 represents a plot of R against the branching ratios (4) and (5).

We have seen that using (3)–(5) as input, (actually only *two ratios* among these three branching fractions are used as input) we can obtain the results for $Z=\Gamma(D^+)/\Gamma(D^0)$ and $R=f_2/f_1$. With these values of Z and R and using the ratio of Eqs. (10) and (11), we can then solve for the individual parameters f_1 and f_2. For purposes of illustration, if we take the values $Z=0.14$ and $R=-1.33$, we get $|f_1|=3.35$, $|f_2|=4.47$. Substituting these values in Eqs. (10)–(12) and using Eq. (7) with[11] $M_c = 1.66$ GeV, we obtain

$$\begin{aligned}\tau(D^0) &= 0.70\times10^{-13}\ \text{sec}\,,\\ \tau(D^+) &= 4.9\times10^{-13}\ \text{sec}\,,\\ \tau(F^+) &= 0.72\times10^{-13}\ \text{sec}\,.\end{aligned} \tag{23}$$

Also the semileptonic branching ratios are given by

$$\begin{aligned}B(D^0\to e\nu X) &= \Gamma_0/\Gamma(D^0) \simeq 3\%\,,\\ B(D^+\to e\nu X) &= \Gamma_0/\Gamma(D^+) \simeq 21\%\,.\end{aligned} \tag{24}$$

Our results (23) are different from Koide's.[7] In particular, he finds $\tau(D^0)=2\tau(F^+)$. This difference arises due to his parametrization which makes the contribution of the annihilation diagrams very large. Mathur and Rizzo[12] have considered the contribution of annihilation diagrams in a massive-quark model. Their calculations do not substantiate Koide's fit for the ratio of the contributions of annihilation diagrams to decay diagrams.

Recent emulsion data[13] seem to indicate $\tau(F^+) \simeq 3\tau(D^0)$. This is based on two F^+ events and hence is statistically poor. But in addition, the present model[14] as well as Koide's, shows that the Cabibbo-"suppressed" decay mode $D^+\to\bar{K}^0K^+$ may not be suppressed and its branching ratio may in fact be comparable to the dominant modes. Hence without a precise momentum measurement, the detection of kaons in the final state alone is not sufficient to decide between a D^+ and an F^+ decay.

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[7]Y. Koide, Phys. Rev. D 20, 1739 (1979). Our attention was drawn to this paper after the present work was completed.

[8]These quark currents need not have a $V-A$ structure. In fact for the present discussion their space-time structure is not important.

[9]Equation (13) is consistent with equal lifetimes for the three mesons, but only if either f_1 or f_2 vanishes. Also since $\tan^2\theta_C < 1$, Eq. (13) implies $\Gamma(F^+) > \tan^2\theta_C \Gamma(D^+)$.

[10]The constant of proportionality, which is the same in each case, contains a piece that describes the probability of one $q\bar{q}$ pair to form a K and another a π. (See last entry of Ref. 5 above).

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