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Quark "fragmentation" in a field-theoretic model of composite hadrons and e^+e^- annihilation

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We consider here a dynamical mechanism for the "fragmentation" of quarks with a field-theoretic model of composite hadrons with quarks as constituents, and with a phenomenological interaction of quarks mediated by vector mesons. In the present model the bounded nature of the transverse momentum of quark jets arises from the momentum localization of the wave functions of the hadrons. In the lowest-order contribution the kinematics becomes such that for the quark-antiquark pair, there is hardly any change in the four-momentum of one, whereas the four-momentum of the other gets shared between a quark and a meson with limited transverse momentum corresponding to the quark "fragmentation." As an example, the harmonic-oscillator wave functions for the mesons are used for the *calculation* of the quark-fragmentation functions which has been used throughout. Different signals for quark jets and for e^+e^- annihilation are then examined. The model automatically includes a transverse-momentum distribution which is discussed. This transverse momentum depends on the scaling variable giving rise to an energy-dependent sea-gull effect. It appears that the harmonic-oscillator wave functions, though adequate for the gross features of quark-fragmentation processes with practically no free parameter, as expected, fail regarding some details. Also, the mechanism for the recombination of the final quark and antiquark after repeated fragmentation becomes relevant for the analysis of data at low energies, an illustration of which has been given. The possibility that the above phenomenological interaction may be that of quantum chromodynamics is noted.

I. INTRODUCTION

Jets of hadrons had been anticipated in e^+e^- annihilation as well as in deep-inelastic lepton-hadron scattering for a long time.[1,2] For e^+e^- annihilation these have now been observed and analyzed.[3] Here the rigorous framework of quantum chromodynamics does not usually permit us to calculate the cross sections for the exclusive and semi-inclusive processes, although it describes many beautiful results which are experimentally verified. Based on quark-parton ideas,[4-8] our discussions here will be in the context of the quark-fragmentation model[5-7] for the generation of jets of hadrons.

In this model other authors assume specific forms for primordial quark-fragmentation functions $f_Q^M(z)$ which describe the sharing of the longitudinal momentum of the quark Q by the meson M and the residual quark Q', along with an arbitrary finite transverse-momentum distribution.[5-7] In the present analysis we give a *prescription* for the calculation of the primordial quark-fragmentation functions in terms of the wave functions of the mesons on some physical grounds. The merits of this prescription are that for the fragmentation process it automatically yields (i) the boundedness of the transverse momentum, (ii) the explicit distribution of the transverse momentum, and (iii) the quantitative nature of the primordial fragmentation function. As mentioned above, these three features, which we derive from our prescription, are essentially taken as input assumptions in the work of previous authors. Our subsequent analysis is based on the quark-fragmentation model where our calculated quark-fragmentation functions have been used. Thus with a single assumption, many results can be coordinated and compared with experiments.

The present discussions are organized as follows. In Sec. II we consider the process

$$Q_1+\tilde{Q}_2\to M+Q_3+\tilde{Q}_2 \qquad (1.1)$$

to obtain the advertised prescription for the derivation of the primordial quark-fragmentation functions. On physical grounds, this primordial fragmentation function is prescribed as

$$f_{Q_1}^M(z)=\frac{d\sigma}{dz}(Q_1+\tilde{Q}_2\to M+Q_3+\tilde{Q}_2)/\sigma_t\,, \qquad (1.2)$$

where z is the fraction of longitudinal momentum for the meson M. The cross sections in (1.2) are derived with a composite model of hadrons used earlier[9] and the lowest-order interaction of quarks with vector mesons as an illustration for the possible dynamical origin for the quark-fragmentation functions. Here we take the same harmonic-oscillator wave functions for the mesons as were used earlier.[9,10] Such wave functions are known to be fairly reasonable and have been used by many authors.[11-14] In Sec. III we consider the development of quark jets to hadrons using the fragmentation functions as estimated in Sec. II. An apparently large SU(3) violation obtained in Sec. II appears to be vindicated here from experimental results. In Sec. IV we consider semi-inclusive and some exclusive channels for e^+e^- annihilation to hadrons in the quark-fragmentation model again using the quantitative estimate of Sec. II for the fragmentation functions. In Sec. V we obtain the transverse-momentum distribution of hadrons in quark jets using the transverse-momentum distribution as derived through the prescription of Sec. II. The familiar energy-dependent sea-gull effect is noticed. In Sec. VI we discuss the results. We also illustrate here how we may generalize the model to include the recombination of the residual quark and antiquark after repeated fragmentation of the initial quark-antiquark pair for e^+e^- annihilation.

Our objective here has been to include the effect of the meson wave function for the quark-fragmentation model, give a possible dynamical prescription for the derivation of the same, and test the consequences, while correlating elastic[9,10] and fragmentation[5-7] processes. The broad agreements of many results as calculated here with the gross approximation of harmonic-oscillator wave functions seem to indicate some validity for this prescription with a vector-current interaction for the quarks.

We shall end this section by giving a motivation for the calculations of Sec. II. For e^+e^- annihilation to hadrons, in the context of the quark-fragmentation model,[5-7] the hadron jets will be formed by successive "fragmentation" of the quark and the antiquark. Since at each stage color-singlet mesons are produced, at any intermediate stage during this fragmentation process, there will be a quark and an antiquark in the color-singlet state. If adequate energy is available, this quark or antiquark will continue "fragmenting," yielding a fresh meson and again a quark-antiquark pair. With this heuristic background we consider the inelastic "scattering" process $Q_1+\tilde{Q}_2 \to M+Q_3+\tilde{Q}_2$.

II. QUARK-ANTIQUARK SCATTERING WITH MESON PRODUCTION AND PRIMORDIAL QUARK-FRAGMENTATION FUNCTIONS

We are interested in calculating the primordial quark-fragmentation function $f(z)$ where z is the light-cone fraction of the quark momentum carried by the meson corresponding to the quark-cascade jet-production model.[5-7] From the wave function of the meson we intuitively expect that the transverse momentum of the meson relative to the quark will be small. We also expect that the fragmentation function should be universal; it should not depend on the origin or the history of the state of the quark, whether it be produced in deep-inelastic lepton-hadron scattering, or large-$p_\perp$ reactions, or in e^+e^- annihilations. We would like to build a model which incorporates the above effects.

As an example of such a model, we consider the reaction $Q_1+\tilde{Q}_2 \to M+Q_3+\tilde{Q}_2$ mediated by the exchange of a vector boson whose coupling to the quark is g. We take the interaction Hamiltonian as[9]

$$\mathcal{H}_I(x)=g\bar{\psi}_Q^g(x)\gamma^\mu\lambda_i\psi_Q^g(x)V_\mu^i(x), \tag{2.1}$$

where summation over repeated indices is understood, λ_i are the color Gell-Mann matrices, and Q is the flavor index. In the above we have taken the vector mesons to be color octets to examine the possibility that these may be gluons. In the present context, this is not really material, and (2.1) is a phenomenological vector-meson coupling which is meant to be illustrative.[15] We shall see later that assuming that (2.1) is the interaction Hamiltonian of quantum chromodynamics is beset with difficulties which are not possible to resolve, in spite of some attractive features. We shall note finally that the fragmentation $Q_1 \to M + Q_3$ essentially "decouples" from the scattering process $Q_1+\tilde{Q}_2\to M+Q_3+\tilde{Q}_2$. In (2.1) the superscript g stands for generalized quark operators[9] which may describe quarks as constituents of any hadron in any frame of reference, or as we shall presently see, "free" quarks.[16] In fact, we may note that[9]

$$\psi_Q^g(x)=Q^g(x)+\tilde{Q}^g(x)$$

which separates the quark annihilation and the antiquark creation operators. When $\psi_Q^g(x)$ contracts with a constituent quark operator we replace $Q^g(x)$ by $Q^{L(p)}(x)$ where p is the four-momentum of the hadron of which the quark Q is a constituent.[9,10] However, here when we deal with free quarks, we shall replace $Q^g(x)$ by $Q^D(x)$ the free Dirac-operator, which is conventional.

The pseudoscalar meson state with four-momentum p' is described as[9]

$$|M(p')\rangle=(1/\sqrt{6})(p'^0/m)^{1/2}\int\delta^3(k_1+k_2)d\vec{k}_1d\vec{k}_2\tilde{u}_M(\vec{k}_1)\tilde{Q}_{1I}^{\dagger}(L(p')k_1)^{\dagger}\tilde{Q}_{3I}^{\dagger}(L(p')k_2)|\,\mathrm{vac}\rangle, \tag{2.2}$$

where $\tilde{u}_M(\vec{k}_1)$ is the wave function of the meson at rest. The dynamics for the above scattering process (1.1) in the lowest order is given by Fig. 1 where the crosses indicate interaction vertices *in quark space* and the open circle indicates the matrix element for the change of state for "spectator" quarks for the same space-time vertex x at which the quark pair is created. Translational invariance at this vertex will be applied to the above system.[9,10] The effective component of the S matrix here with a vector-meson contraction is given as

$$S_2=\tfrac{1}{2}(-ig)^2 \int \dot{V}^i_\mu(x_1)\dot{V}^j_\nu(x_2)d^4x_1d^4x_2 : \bar{\psi}^g_Q(x_1)\gamma^\mu\lambda_i\psi^g_Q(x_1)\bar{\psi}^g_{Q'}(x_2)\gamma^\nu\lambda_j\psi^g_{Q'}(x_2):$$

$$\equiv(-ig)^2 \int \dot{V}^i_\mu(x)\dot{V}^j_\nu(y)d^4x\,d^4y : \bar{Q}^D_3(x)\gamma^\mu\lambda_i\tilde{Q}^{L(p')}_3(x)\bar{\tilde{Q}}^D_2(y)\gamma^\nu\lambda_j\tilde{Q}^D_2(y):. \tag{2.3}$$

In Fig. 1 it is automatically understood that the circle is associated with the quark-interaction vertex x regarding translational invariance. There will be no transition of a free quark to a bound quark (or vice versa) without one of the constituent quarks interacting.[17] This assumption is already understood in (2.3). Explicitly with appropriate momenta and spin and color indices we obtain that

$$\tfrac{1}{3}\langle M(p')Q^{i'}_{3r_1'}(q_1')\tilde{Q}^{i'}_{2r_2'}(q_2')\,|\,S_2\,|\,Q^i_{1r_1}(q_1)\tilde{Q}^i_{2r_2}(q_2)\rangle=\delta^4(P_f-P_i)M_{fi}\,. \tag{2.4}$$

In (2.4) we have taken that the initial quark-antiquark system is in the color-singlet state, and thus also the final quark-antiquark system will be in the color-singlet state which yields $(1/\sqrt{3})^2$. From (2.3) we now have in (2.4), with μ as the mass of the vector meson, and neglecting a contribution which will ultimately yield zero,

$$M_{fi}=ig^2(2\pi)^4\frac{1}{(q_2-q_2')^2-\mu^2}\,\frac{1}{3}\,\langle M(p')Q^{i'}_{3r_1'}(q_1')\,|\,\bar{Q}^D_3(0)\gamma^\mu\lambda_j\tilde{Q}^{L(p')}_3(0)\,|\,Q^i_{1r_1}(q_1)\rangle$$
$$\times\langle\tilde{Q}^{i'}_{2r_2'}(q_2')\,|\,\bar{\tilde{Q}}^D_2(0)\gamma_\mu\lambda_j\tilde{Q}^D_2(0)\,|\,\tilde{Q}^i_{2r_2}(q_2)\rangle\,. \tag{2.5}$$

In the above, with translational invariance[9] the field operators occur *at the space-time origin* for the color currents in quark space. We recall that for the constituent operator in (2.5), we have[9]

$$\tilde{Q}^{L(p')}(0)=(2\pi)^{-3/2}(p'^0/m)^{1/2}\times\int d^3k\,S(L(p'))v(\vec{k})\tilde{Q}_I(L(p')k)\,,$$

where

$$v(\vec{k})=\begin{pmatrix} g\vec{\sigma}\cdot\vec{k} \\ f(\vec{k}^2)\end{pmatrix}$$

and $\tilde{Q}_I(L(p')k)$ is the two-component creation operator for the antiquark. For the sake of simplicity in the subsequent approximation we shall take in the above $g=0$ and thus $f=1$, which is equivalent to retaining the "large" component in the quark model[9] as is conventionally taken. The effect of Lorentz boosting however remains in this approximation.

The matrix element in Fig. 1 as given by (2.5) is one of the obvious available processes for the hadronization of the quark-antiquark pair, and we shall build our model with this. We shall see that this process will exactly correspond to the model of quark fragmentation.[5-7] We shall take the state $|M(p')\rangle$ as in (2.2) and shall directly evaluate the matrix element (2.5).

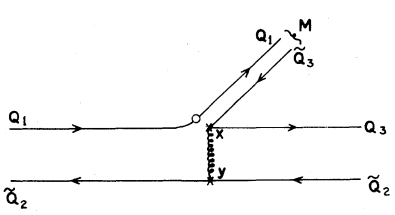


FIG. 1. The matrix element for the process $\sigma(Q_1\tilde{Q}_2\to MQ_3\tilde{Q}_2)$ with a gluon exchange, a spectator quark, and a quark pair creation.

We may now note that the quark Q_1 in Fig. 1 is initially a free quark and is finally a constituent of the meson M and we need the matrix element for such spectator quark states in (2.5). Proceeding as in Ref. 9 we assume the conventional anticommutation in the rest frame of the hadron given as

$$[Q^i_{Ir_1}(k_1),\,Q^{Dj}_{Is}(q)^\dagger]_+=\delta_{ij}\delta_{r_1s}\delta^3(k_1-q)\,, \tag{2.6}$$

where, on the left-hand side of (2.6), the first and the second two-component quark operators are, respectively, for the bound and the free quarks. Under a Lorentz transformation, we know the transformation properties for the bound quarks[9] and the free quarks.[18] Hence with a Lorentz transformation (2.6) yields that[16]

$$[Q^i_{Ir_1}(L'k_1),\,Q^{Dj}_{Is}(q)^\dagger]_+=\delta_{ij}D_{r_1s}(q,L'^{-1})\times[(m/p'^0)(L'^{-1}q)^0/q^0]^{1/2}\times\delta^3(k_1-L'^{-1}q)\,, \tag{2.7}$$

where m is the mass of the hadron of four-momentum p' of which the quark is a constituent and D_{r_1s} is the Wigner rotation matrix[18] arising from the transformation property of the free quark operator.

Evaluation of the matrix element in (2.5) is now straightforward, and we obtain, with m_1, m_2, and m_3 as the masses of the quarks Q_1, Q_2, and Q_3 and m, the mass of the meson,

$$M_{fi} = ig^2(2\pi)^4 \frac{1}{\sqrt{6}} \times \frac{1}{3} (\lambda_j)_{i'i}(\lambda_j)_{ii'} \frac{1}{(q_2 - q_2')^2 - \mu^2} \left[\frac{m^2}{q_2^0 q_2'^0} \frac{m}{p'^0} \frac{(L'^{-1}q_1)^0}{q_1^0} \frac{m_3}{q_1'^0} \right]^{1/2}$$
$$\times \bar{u}^D_{r_1'}(q_1')\gamma^\mu v^{L'}_{s_1}(\vec{k}_2)(v^\dagger_{I s_1} u_{I s_2}) D_{s_2 r_1}(q_1, L'^{-1})(2\pi)^{-6} \bar{v}^D_{r_2}(q_2)\gamma_\mu v^D_{r_2'}(q_2')\tilde{u}_M(\vec{k}_1), \quad (2.8)$$

where from (2.7) we may substitute

$$\vec{k}_{1i} = -\vec{k}_{2i} = (L'^{-1}q)_i .$$

Also, u^D and v^D are Dirac spinors[18] and[9]

$$v^{L'}_{s_1}(\vec{k}_2) = S(L(p'))v(\vec{k}_2)v_{I s_1} .$$

We shall now examine the kinematics of the above matrix element and show that it corresponds to quark fragmentation. We take the c.m. frame of reference and substitute $|\vec{p}'| = z|\vec{q}_1|$, $|\vec{q}_2'| = x_2|\vec{q}_1|$, $\vec{q}_1 \cdot \vec{p}' = z|\vec{q}_1|^2 \cos\theta$, and $\vec{q}_2 \cdot \vec{q}_2' = x_2|\vec{q}_1|^2 \cos\theta_2$. We note that,

$$\vec{k}_1^{\,2} = (p' \cdot q_1)^2/m^2 - m_1^2 . \quad (2.9)$$

Further, $p' \cdot q_1 = \lambda + f_1$, where

$$\lambda = z|\vec{q}_1|^2(1 - \cos\theta) \quad (2.10a)$$

and

$$f_1 = p'^0 q_1^0 - |\vec{p}'|\,|\vec{q}_1| . \quad (2.10b)$$

We now note that in the high-energy limit f_1 scales, and we get

$$f_1 \simeq \tfrac{1}{2}(m_1^2 z + m^2/z) . \quad (2.11)$$

Also, we may always assume that ultimately $\tilde{u}_M(\vec{k}_1)$ decreases as $\vec{k}_1^{\,2}$ increases. Hence the contribution from (2.8) will be suppressed unless $|\vec{q}_1|^2(1 - \cos\theta)$ remains bounded. This implies that in the high-energy limit effectively the angle θ must remain small for the matrix element (2.8) to be significant. *Thus the meson M effectively will be in the same direction as the quark Q_1* and, although the energy of the meson has so far no constraint, *its transverse momentum relative to the quark Q_1 will remain bounded.* Hence the jet structure of the mesons arising from the quark Q_1 as above may be anticipated. This becomes a consequence of the bound-state nature of the meson M with the quark Q_1 being a spectator quark[9] with the corresponding matrix element being given by equation (2.7).

We next note that in (2.8)

$$-t \equiv -(q_2 - q_2')^2 = 2x_2|\vec{q}_1|^2(1 - \cos\theta_2) + f_2 , \quad (2.12)$$

where

$$f_2 = 2(q_2^0 q_2'^0 - |\vec{q}_2|\,|\vec{q}_2'| - m_2^2) . \quad (2.13)$$

When we take the high-energy limit, we get

$$f_2 \simeq m_2^2(1 - x_2)^2/x_2 . \quad (2.14)$$

We note that the total cross section for the process $Q_1 + \tilde{Q}_2 \to M + Q_3 + \tilde{Q}_2$ is given by

$$\sigma = (4\pi^2/v_{\rm rel}) \int \delta^4(P_f - P_i)|M_{fi}|^2 d\vec{p}' d\vec{q}_2' d\vec{q}_1' , \quad (2.15)$$

where appropriate averaging and summing over spin and color indices is understood. In (2.15) we trivially perform the integration over $\vec{q}_1'$, and then estimate the integral

$$\int \delta(q_1^0 + q_2^0 - p'^0 - q_1'^0 - q_2'^0)[1/(t - \mu^2)]^2 d\vec{q}_2' . \quad (2.16)$$

We shall evaluate (2.16) under the approximation that μ is negligible corresponding to massless gluons. The results are broadly similar even when this assumption is not made, but μ enters as an unknown parameter. From (2.12) and (2.14) we note that in (2.16), while integrating over the angles, there will be significant contribution to this integral *only* when $1 - \cos\theta_2$ is bounded like $O(1/s)$. When we further take into account the energy δ function in (2.16), we shall really see that $1 - x_2$ effectively will also be small which will further restrict the bound on $1 - \cos\theta_2$ mentioned above. We shall demonstrate the consistency of this result as follows. We shall *substitute* $x_2 = 1 - h$, where we shall assume that in the high-energy limit h is small and only retains the leading contribution in this. Using momentum conservation, we then obtain after some minor algebra that

$$q_1'^0 = q_3^0 + \frac{\lambda}{q_3^0} - \frac{(1 - z)|\vec{q}_1|^2}{q_3^0} h , \quad (2.17)$$

where we have substituted $q_3^0 = (m_3^2 + (1 - z)^2|\vec{q}_1|^2)^{1/2}$. In deriving (2.17) we have assumed that effectively $h = O(1/s)$ and thus $1 - \cos\theta_2 = O(1/s^2)$. For the leading contribution in (2.16) we note that with (2.12)

$$\int (1/t^2)\sin\theta_2 d\theta_2 d\phi_2 = \int [2x_2|\vec{q}_1|^2(1 - \cos\theta_2) + f_2]^{-2} \sin\theta_2 d\theta_2 d\phi_2 = \frac{\pi}{x_2|\vec{q}_1|^2} \frac{1}{f_2} . \quad (2.18)$$

For the energy δ function we obtain

$$\delta(q_1^0 + q_2^0 - p'^0 - q_1'^0 - q_2'^0) = \delta\left(\sqrt{s} - p'^0 - q_3^0 - \frac{\lambda}{q_3^0} + \frac{(1 - z)|\vec{q}_1|^2}{q_3^0} h - q_2^0 + h\frac{|\vec{q}_1|^2}{q_2^0}\right) = (1/D)\delta(h - h_0) . \quad (2.19)$$

In (2.19) we have

$$D=\frac{|\vec{q}_1|^2}{q_2^0}+\frac{(1-z)|\vec{q}_1|^2}{q_3^0}\,, \tag{2.20}$$

which in the high-energy limit becomes

$$D\simeq 2|\vec{q}_1|\,. \tag{2.21}$$

Further, in (2.19) we have also substituted

$$h_0=(p'^0+q_3^0+\lambda/q_3^0+q_2^0-\sqrt{s})/D\,. \tag{2.22}$$

The high-energy limit of the above equation becomes

$$h_0\simeq\left(\frac{m^2}{z}+\frac{m_3{}^2}{(1-z)}-m_1{}^2+\frac{2\lambda}{(1-z)}\right)\Big/(4|\vec{q}_1|^2)\,, \tag{2.23}$$

which reflects the fact that in the high-energy limit h is bounded like $O(1/s)$ as was assumed. We now evaluate the integral (2.16), and obtain

$$\int\delta(\sqrt{s}-p'^0-q_1'^0-q_2'^0)(1/t^2)d\vec{q}_2{}'=\frac{\pi|\vec{q}_1|}{m_2{}^2h_0{}^2D} \tag{2.24}$$

$$=\frac{\pi}{2m_2{}^2h_0{}^2}\,, \tag{2.25}$$

where in the last equation we have taken the high-energy limit and thus (2.23) can be used.

We now consider evaluation of the integral (2.15). We have noted that $1-\cos\theta$ is effectively small, but have not used it. We have next noted that $1-x_2=O(1/s)$ which is equivalent to Eqs. (2.19) and (2.23). Hence, while using (2.24), for the other terms of (2.15) we shall substitute $x_2=1$. We next note from (2.14) and the derivation of (2.18) that effectively in (2.15) $1-\cos\theta_2=O(1/s^2)$, where the fact that h is small is used. Hence, while using (2.24), in the other terms of (2.15) we shall also substitute $\cos\theta_2=1$. This leads to the conclusion that for the matrix element considered the quark $\tilde{Q}_2$ loses momentum of the order of $O(1/s)$ and almost remains unchanged in direction. *Thus kinematically the quark $\tilde{Q}_2$ does not play any role.* In the high-energy limit, we may thus kinematically consider this process as equivalent to the "fragmentation" of the quark Q_1 to the mesons M and the quark Q_3, with a sharing of the longitudinal momentum between them and with a bounded transverse momentum, which thus anticipates quark-fragmentation models.[5-7]

In (2.8), summations over the color index are trivial. For the summations and averaging over the spin indices in (2.15) we get[9,18] with (2.8),

$$\begin{aligned}\frac{1}{4}\sum|\bar{u}^{D}_{r_1'}(q_1')\gamma^\mu v^{L'}_{s_1}(\vec{k}_2)(v^\dagger_{Is_1}u_{Is_2})D_{s_2r_1}(q_1,L'^{-1})\bar{v}^{D}_{r_2}(q_2)\gamma_\mu v^{D}_{r_2'}(q_2')|^2\\ =[1/(mm_3m_2{}^2)][2(q_2^0q_3^0+\lambda+(1-z)|\vec{q}_1|^2)(p'^0q_2^0-\lambda+z|\vec{q}_1|^2)-m_2{}^2(p'^0q_3^0+\lambda-z(1-z)|\vec{q}_1|^2-m_3m)]\\ \equiv[1/(mm_3m_2{}^2)]T(z) \qquad (2.26)\\ \simeq 4|\vec{q}_1|^4z(1-z)/(mm_3m_2{}^2)\,, \qquad (2.27)\end{aligned}$$

where in the last equation we have taken the high-energy limit and (2.26) gives the value of the function $T(z)$ in the general case. In (2.15) with (2.8), for the kinematic terms we also have

$$\frac{1}{v_{\rm rel}}\frac{m_2{}^2}{q_2^0q_2'^0}\frac{m}{p'^0}\frac{(L'^{-1}q_1)^0}{q_1^0}\frac{m_3}{q'^0}=\frac{m_2{}^2m_3(\lambda+f_1)}{\sqrt{s}|q_1|q_2^0q_3^0p'^0}\,. \tag{2.28}$$

Thus from (2.8) and (2.15), performing the integration over $\vec{q}_1{}'$ through the momentum δ function, the integration over $\vec{q}_2{}'$ by using (2.24), and substituting that effectively $d\vec{p}'=2\pi z dz d\lambda|\vec{q}_1|$ we obtain that

$$\frac{d\sigma}{dz}(Q_1\tilde{Q}_2\to MQ_3\tilde{Q}_2)=\frac{\alpha_s{}^2\pi^2\frac{256}{27}\times 4|\vec{q}_1|zF(z)}{mm_2{}^2D\sqrt{s}q_2^0q_3^0p'^0}\,, \tag{2.29}$$

where we have substituted $\alpha_s=g^2/(4\pi)$ and that

$$F(z)=\int d\lambda|\tilde{u}_M(\vec{k}_1)|^2(\lambda+f_1)T(z)/h_0{}^2\,. \tag{2.30}$$

With (2.23) and (2.27), we obtain in the high-energy limit that

$$F(z)=16z(1-z)^3|q_1|^8\int\frac{(\lambda+f_1)|\tilde{u}_M(\vec{k}_1)|^2d\lambda}{[\lambda+\psi(z)]^2}\,, \tag{2.31}$$

where we may use (2.11), and have substituted

$$\psi(z)=[(1-z)/2][m^2/z+m_3{}^2/(1-z)-m_1{}^2]\,. \tag{2.32}$$

We then obtain from (2.29) in the high-energy limit that

$$\frac{d\sigma}{dz}(Q_1\tilde{Q}_2\to MQ_3\tilde{Q}_2)=\alpha_s{}^2\pi^2\tfrac{256}{27}z(1-z)^2\frac{s^2}{mm_2{}^2}\times\int\frac{(\lambda+f_1)|\tilde{u}_M(\vec{k}_1)|^2d\lambda}{[\lambda+\psi(z)]^2}\,. \tag{2.33}$$

We now compute

$$\frac{1}{\sigma}\frac{d\sigma}{dz} = \frac{z(1-z)^2 \int (\lambda+f_1)(\lambda+\psi)^{-2} |\tilde{u}_M(\vec{k}_1)|^2 d\lambda}{\int dz\, z(1-z)^2 \int (\lambda+f_1)(\lambda+\psi)^{-2} |\tilde{u}_M(\vec{k}_1)|^2 d\lambda}, \tag{2.34}$$

where the coupling strength g cancels. In the high-energy limit, we may also see that the right-hand side of (2.34) is independent of the energy as well as of the antiquark $\tilde{Q}_2$. With this universality in mind, we take the present model for the primordial quark-fragmentation function corresponding to $Q_1 \to M + Q_3$ by substituting

$$f^M_{Q_1}(z) = \frac{(d\sigma/dz)(Q_1\tilde{Q}_2 \to MQ_3\tilde{Q}_2)}{\sigma_t(Q_1\tilde{Q}_2 \to MQ_3\tilde{Q}_2)}. \tag{2.35}$$

We note that the primordial quark-fragmentation function (2.35) is normalized to unity. Parallel to (2.34), we also take that the probability that the quark Q_1 fragments to the meson M is given by

$$p(Q_1, M) = \sigma_t(Q_1\tilde{Q}_2 \to MQ_3\tilde{Q}_2)/\sigma_t(Q_1\tilde{Q}_2). \tag{2.36}$$

Conventionally[7] $p(Q_1, M) f^M_{Q_1}(z)$ is written as the primordial quark-fragmentation function.

To illustrate the corresponding results, we plot in Fig. 2 $f^{\pi^+}_{\mathcal{P}}(z)$ and $f^{K^+}_{\mathcal{P}}(z)$ as derived from (2.33) and (2.35). In these calculations we have approximated the wave functions by[9-14]

$$|\tilde{u}_M(\vec{k}_1)|^2 = (R_M{}^2/\pi)^{3/2} \exp(-R_M{}^2 \vec{k}_1{}^2) \tag{2.37}$$

as mentioned earlier. Further, the present calculations indicate a strong SU(3) violation with $\mathcal{P}$ quark fragmenting to a K^+ being highly suppressed as compared to a π^+ production. This had been earlier anticipated from experimental analysis,[19] and will be discussed later. We have also plotted at adequately high energies the fragmentation functions $f^K_\lambda(z)$ and $f^D_c(z)$ in Figs. 3 and 4. In Fig. 4 the leading-particle behavior may be noted, which could be wrong,[20] but the experimental data have too large errors for a positive conclusion.

We note that the vector-exchange model above appears to be useful for generating fragmentation functions which are process independent, have appropriate scaling behavior, and have finite transverse momenta. The color degree of freedom was not very material. We now consider whether these vector mesons could be gluons. In such a case due to asymptotic freedom[21] one will be tempted to think that the lowest-order perturbation expansion corresponding to the four-point quark function may have reasonable validity. However, when one calculates the cross sections, these appear to be enormous and possibly unphysical. We may reinterpret this cross section indicating multiple scattering[22] with something like a mean free path. However, the corresponding calculations involve soft gluons and make perturbative quantum chromodynamics unreliable, since for soft-gluon processes the summation over higher orders, as for example while summing over leading logarithms,[23] can completely change the picture. In the context of quantum chromodynamics we regard our calculations in the present section as inadequate, in contrast to earlier observations.[22]

We thus note that the question of whether the interaction of quantum chromodynamics can give rise to quark fragmentation as above cannot be

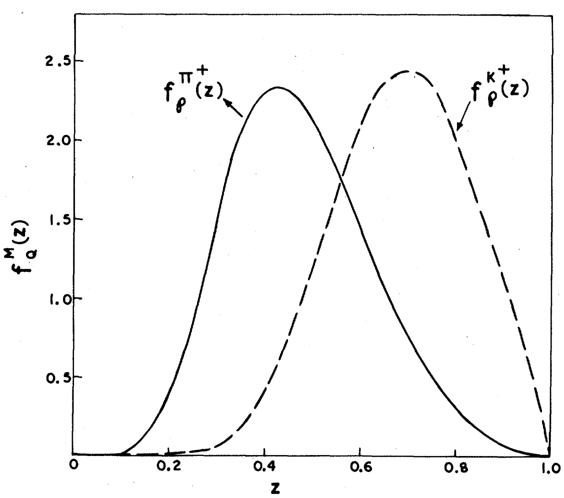


FIG. 2. The calculated primordial quark-fragmentation functions $F^{\pi^+}_{\mathcal{P}}(z)$ (solid lines) and $F^{K^+}_{\mathcal{P}}(z)$ (dashed line). In both cases $\tilde{\mathcal{P}}$ is the "companion," $\sqrt{s} = 4$ GeV; R^2_π and R^2_K are taken as in diffractive photoproduction (Ref. 10).

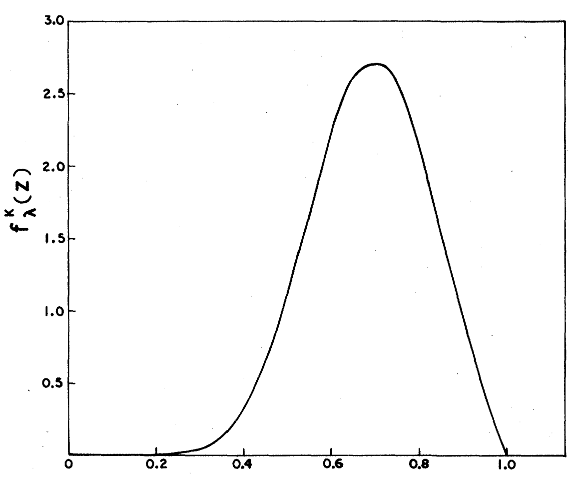


FIG. 3. The primordial quark-fragmentation function $f^K_\lambda(z)$ with $\tilde{\lambda}$ as companion. Here $\sqrt{s} = 4$ GeV and $R_K{}^2 = 9$ GeV^{-2}.

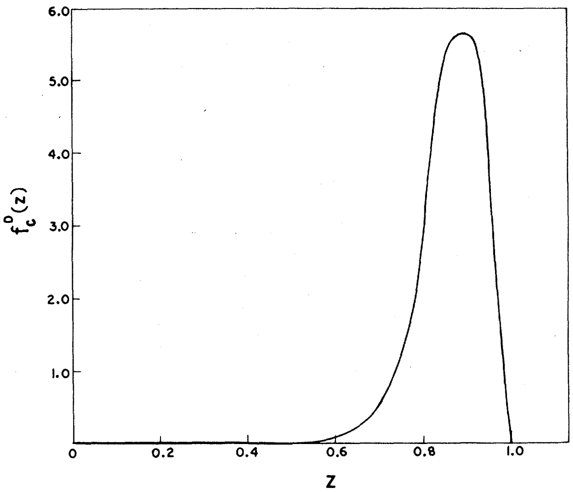


FIG. 4. The primordial quark-fragmentation function $f_c^{D^+}(z)$ with $\bar{c}$ as companion. Here $\sqrt{s}=7$ GeV and $R_D^2 = 6\ \mathrm{GeV}^{-2}$ as appears to be indicated from photoproduction analysis (for example see Ref. 10). The qualitative nature of the result is not very sensitive to this R_D^2.

answered unless a more careful analysis is carried out than what we have done here. However, we note here that the fragmentation functions in (2.34) generated here are partly the effect of dynamics as well as partly the effects of the wave functions, as also may be otherwise seen from Figs. 2–4. Hence in a qualitative manner we are able to retain an effect of the wave functions for the quark-cascade-jet model when we use the expressions (2.34) for the same although we may be comparatively ignorant about the nature of dynamics. We shall use these *calculated* primordial quark-fragmentation functions in the subsequent sections as phenomenological inputs.

In the present model for quark fragmentation, in (2.35) when the quark Q_1 is fragmenting, we shall call the quark $\tilde{Q}_2$ as the "companion" in the presence of which the fragmentation takes place. As noted earlier, in the high-energy limit the process is independent of the companion, which gives rise to the universality of the quark jets.[6,24] From the nature of the interaction, it is also clear that if the companion is a quark instead of an antiquark, the same conclusion as above will continue to hold true. However, it is possible to imagine that in certain situations or at moderate energies there will be a companion dependence. In such a situation, when we are able to *calculate* the primordial quark-fragmentation functions, this companion dependence can be included in case it becomes necessary, and this is a positive advantage in attempting to give a "microscopic model" for the primordial quark fragmentation. For example, the companion effects as well as the energy dependence will be very much there for the hadronization of heavy quarks even at moderate energies, which can be included in (2.35). However, these calculations involve many flavors and we have not carried these out here in the absence of experimental information regarding the corresponding process.

We note that at higher energies gluon brehmsstrahlung as well as creation of quark-antiquark pairs will become relevant[25] which will give rise to scaling violations. Further, we have only considered the incoherent production of quark-antiquark pairs, whereas coherent production of hadrons will also be relevant at moderate energies. Also, we have taken the harmonic-oscillator wave functions for the mesons, which are known to be incorrect at space origin from the known coupling of vector mesons to the e^+e^- channel. Hence we expect that the primordial fragmentation functions we have calculated can only possibly have illustrative validity. With this in mind we shall calculate the hadronization process for quarks to jets in the next section. We find that there are gross features which agree with experiments in spite of the above approximations and in spite of almost total absence of free parameters.

III. QUARK JETS

With the primordial quark-fragmentation functions[6,7] already calculated, we can apply the quark-fragmentation model to derive the nature of individual quark jets. As mentioned earlier, we shall assume that the fragmentation of the quark stops[7] when it has a momentum μ, which is the only parameter in our description after we have chosen the wave functions. However, the abrupt stopping of fragmentation is a crude assumption and the results which are sensitive to this parameter are likely to be unreliable. This parametrization is useful since in the high-energy limit the results are independent of it and at finite energies to some extent the parameter fixes the scale for approach to the high-energy limit.

We shall now illustrate the present method by explicitly calculating the quark jets due to nonstrange light quarks. Although we shall follow the quark-fragmentation model, we shall normalize our probabilities in a slightly different manner as we shall mention below, by explicitly considering all the multiplicities of the quark jets.

We have noted in Sec. II that the dominant signal for the fragmentation of the quark q is through pions, with the primordial fragmentation function $f_q^{\pi}(z) = f_{\mathcal{P}}^{\pi^+}(z)$. For the present, let us ignore all other modes of fragmentation. We then *assume* that at a given energy $\sqrt{s}$ the probability for the quark jet to have multiplicity n is equal to

$$p_n = C^{-1} \int f_q^\pi(z_1) f_q^\pi(z_2) \cdots f_q^\pi(z_n) dz_1 dz_2 \cdots dz_n \times \delta(y_1 y_2 \cdots y_n - 2\mu/\sqrt{s}) . \quad (3.1)$$

In the above, $y_i = 1 - z_i$ $(i = 1, 2, \cdots n)$ and we have used that in the high-energy limit the last quark has momentum $y_1 \cdots y_n \sqrt{s}/2$ which must be μ. The constant C in equation (3.1) is determined such that the total probability for the quark going to jets of all multiplicities is given by

$$\sum p_n = 1 . \quad (3.2)$$

With this we shall calculate the quark-fragmentation function $D_q^\pi(z)$ defined in a conventional manner,[6,7] but as we shall see, with a slightly different normalization. We may note that the multiplicity of the quark jet is given by

$$\langle n \rangle = \sum_n n p_n , \quad (3.3)$$

where the constant C has been determined from (3.2). Clearly this does not include the last quark, which combines with the last antiquark for the fragmentation of $\bar{q}$ to yield a meson. Thus the multiplicity for the $q\bar{q}$ system is given as $2\langle n \rangle + 1$. We may also calculate the standard deviation σ for the *quark jet* given by

$$\sigma^2 = \sum_n (n - \langle n \rangle)^2 p_n , \quad (3.4)$$

which is an experimentally measurable quantity.

Now let $F_q^\pi(n, k, z)$ be the fragmentation function for the exclusive process where the multiplicity is n and we are observing the kth-rank pion.[6,7] We then have

$$F_q^\pi(n, k, z) = C^{-1} \int f_q^\pi(z_1) dz_1 f_q^\pi(z_2) dz_2 \cdots f_q^\pi(z_n) dz_n \times \delta(y_1 \cdots y_n - 2\mu/\sqrt{s}) \times \delta(y_1 \cdots y_{k-1} z_k - z) , \quad (3.5)$$

where $y_0 = 1$ and as before $y_i = 1 - z_i$ for all i. Using mesons of all ranks and multiplicities, we now obtain that

$$D_q^\pi(z) = \sum_n \sum_{k=1}^n F_q^\pi(n, k, z) . \quad (3.6)$$

With the normalizations $\int_0^1 f_q^\pi(z) dz = 1$ and (3.2) which are already assumed, we then have[6,7]

$$\int D_q^\pi(z) dz = \langle n \rangle \quad (3.7)$$

and

$$\int z D_q^\pi(z) dz = 1 - 2\mu/\sqrt{s} . \quad (3.8)$$

We note the slight difference in normalization in (3.8), which merely reflects that the momentum of the last quark which does not fragment has not been included. Clearly this difference disappears in the high-energy limit. Except for this difference, it is the same as solving the integral equations of the quark-fragmentation model[6,7] by an infinite series which corresponds to different multiplicities. We have taken the equivalent assumption that the total probability of the quark fragmenting to mesons of all multiplicities adds to 1. We may also note the useful equation already used in (3.7), that

$$\int F_q^\pi(n, k, z) dz = p_n , \quad (3.9)$$

where we may see that the right-hand side is independent of k.

We may now consider some explicit signal of quark jets corresponding to light quarks.

To be specific, let us consider the $\mathcal{P}$-quark jet. A simple combinatorial analysis yields that the probability that the kth rank meson is a π^+, π^0, or π^- is given by $\frac{1}{3} + (-1)^{k-1}/3^k$, $\frac{1}{3}$, or $\frac{1}{3} - (-1)^{k-1}/3^k$, respectively. This yields that

$$D_{\mathcal{P}}^{\pi^+}(z) = \tfrac{1}{3} D_q^\pi(z) + \sum_n \sum_{k=1}^n \frac{(-1)^{k-1}}{3^k} F_q^\pi(n, k, z) , \quad (3.10)$$

$$D_{\mathcal{P}}^{\pi^0}(z) = \tfrac{1}{3} D_q^\pi(z) , \quad (3.11)$$

and

$$D_{\mathcal{P}}^{\pi^-}(z) = \tfrac{1}{3} D_q^\pi(z) - \sum_n \sum_{k=1}^n \frac{(-1)^{k-1}}{3^k} F_q^\pi(n, k, z) . \quad (3.12)$$

In Fig. 5 we have plotted the above fragmentation functions corresponding to $\sqrt{s} = 4$ GeV using Sec. II and taking that $\mu = 0.5$ GeV, which has really been adjusted to fit Fig. 7 for $s(d\sigma/dz)$ at $\sqrt{s} = 3$ GeV. The disagreement with the experimental points may be noted which reveals that the harmonic-oscillator wave function we have taken probably cannot give this much detail. We may however, note that for the fragmentation functions, *no* parameter has been adjusted. The disagreement at large z is due to the cutoff μ taken, and is not significant. For small z, say $z \lesssim 0.2$, it appears from calculations (not there in Fig. 5) that $D_{\mathcal{P}}^{\pi^-}(z)$ may be *higher* than $D_{\mathcal{P}}^{\pi^+}(z)$ by about 30%. It may be useful to see if this signal is present. We note that here for pions the multiplicity is 2.2 with a standard deviation of 0.66.

We have already noted in Sec. II that the K signal from the $\mathcal{P}$ quark appears to be abnormally low as compared to all conventional models,[26] and hence we shall explicitly calculate the fragmentation function $D_{\mathcal{P}}^K(z)$. For this purpose we retain all the modes of fragmentation given in Sec. II of the $\mathcal{P}$ quark, and substitute with $p_M = p(\mathcal{P}, M)$,

$$f_{\mathcal{P}}^A(z) = \sum_M p_M f_{\mathcal{P}}^M(z)\,, \qquad (3.13)$$

where (2.35) and (2.36) are used. From Sec. II we note that once the K meson and the λ quark are produced, the strange quark will preferentially yield another K meson and will go over to a light nonstrange quark. In fact we have

$$D_q^K(z) = C^{-1} \sum_n \sum_k \int f_q^A(z_1)dz_1 \cdots p_k f_q^K(z_k)dz_k f_\lambda^K(z_{k+1})dz_{k+1} \cdots f_q^A(z_n)dz_n \delta(y_1 \cdots y_n - 2\mu/\sqrt{s})$$
$$\times [\delta(y_1 \cdots y_{k-1} z_k - z) + \delta(y_1 \cdots y_k z_{k+1} - z)]\,. \qquad (3.14)$$

In (3.14) we may find the normalization constant C again, or even take it as the same as in (3.1), since the K signal is small and will not alter the value of this constant. The fragmentation function $D_{\mathcal{P}}^K(z)$ at $\sqrt{s} = 3.5$ GeV is plotted in Fig. 6 along with the experimental points as extracted from data in Ref. 19. The agreement appears to be reasonable when we average the experimental data for $\mathcal{P}$ and $\mathfrak{N}$ quarks. The sudden fall of the fragmentation function for large z is the effect of the way fragmentation stops with the constant μ, and is unreliable. We note that the surprisingly large SU(3) violation appears to be reproduced in contrast to the earlier curves quoted in Fig. 6.

We next calculate the K signal from the $\lambda\bar{\lambda}$ quark pair. From Sec. II we conclude that primarily it will be a rank one meson. Thus we substitute

$$D_\lambda^K(z) = C^{-1} \sum_n \int f_\lambda^K(z_1)dz_1 f_q^A(z_2)dz_2 \cdots f_q^A(z_n)dz_n$$
$$\times \delta(y_1 \cdots y_n - 2\mu/\sqrt{s})\delta(z - z_1)\,, \qquad (3.15)$$

where, as before, the normalization constant C is to be determined by taking the total probability as one. We plot $D_\lambda^K(z)$ at $\sqrt{s} = 3.5$ GeV in Fig. 6.

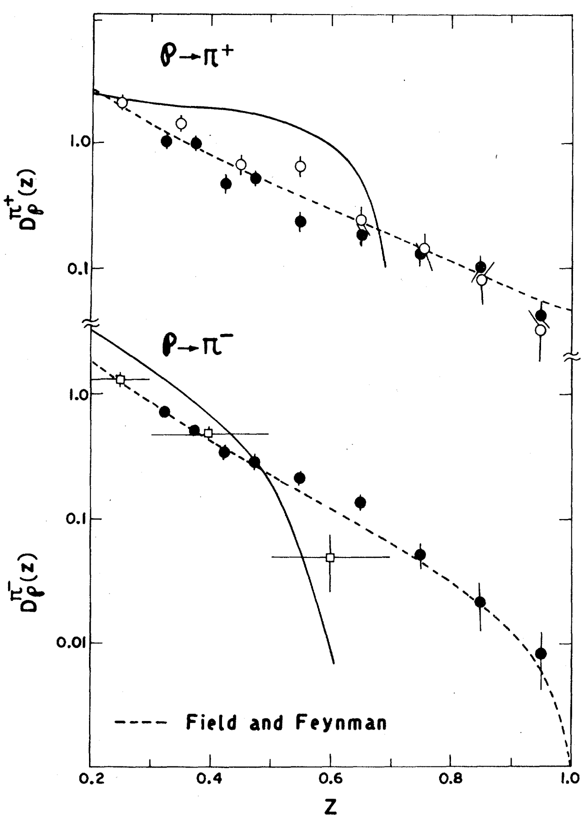


FIG. 5. The quark-fragmentation functions $D_{\mathcal{P}}^{\pi^+}(z)$ and $D_{\mathcal{P}}^{\pi^-}(z)$ at $\sqrt{s} = 4$ GeV with $\mu = 0.5$ GeV. The experimental points are from Drews *et al.* (Ref. 24).

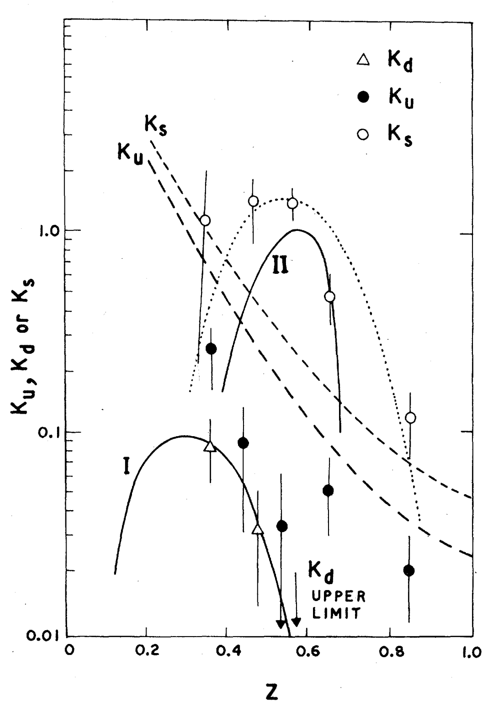


FIG. 6. $D_{\mathcal{P}}^{K^+}(z) = D_{\mathfrak{N}}^{K^0}(z)$ (solid line I) and $D_\lambda^{K^0}(z)$ (solid line II) are the respective calculated quark-fragmentation functions at $\sqrt{s} = 3.5$ GeV. The dotted line is $D_\lambda^{K^0}(z)$ as calculated in Sec. VI. The data are taken from Cohen *et al.* (Ref. 19). The dashed lines are as obtained by Field and Feynman.

We again note that the agreement with Ref. 19 is reasonable, but let us see that really this agreement is misleading.

In Eq. (3.5) when $n=1$, the fragmentation function is a δ function, and has been omitted in all our considerations except while normalizing the probabilities where the δ function gets integrated. In all the earlier cases the probability of $n=1$ was less than 10% and thus we could ignore the signal except that it will alter the large z behavior. But in the calculation of $D_\lambda^K(z)$ above, probability appears to be about 70% and in which way the δ function will get modified is a low-energy effect and is not known. Thus the result critically depends on the mechanism for the stopping of the fragmentation process and as noted the agreement above is not reliable. For this we have suggested an alternative procedure in Sec. VI. In order to see the effect of the δ function distribution, which we have excluded in Fig. 6 for $n=1$ in Eq. (3.14), we shall examine this effect now. When this is included we get from (3.14) that

$$D_q^K(z)=D_q^K(z)\big|_{n\geq 2}+C^{-1}p_K f_q^K(z_1)\delta(z-z_1)\,.$$

We then obtain

$$\int_{0.3}^1 zD_q^K(z)dz\approx 0.022\,, \tag{3.16}$$

which may be compared with the experimental value[24] of 0.028 ± 0.006.

In a similar way we calculate that

$$\int_{0.3}^1 zD_q^\pi(z)dz\approx 0.46\,, \tag{3.17}$$

which disagrees with the experimental result of 0.25 ± 0.01 in Ref. 24, which indicates that our "wave function" may not be adequate regarding all the details, and yields higher pion production for larger z.

While considering the jets due to charmed quarks, for available energies we also find the strong dependence of the results on the recombination mechanism of the residual quark and antiquark, which is here parametrized as a sudden stoppage of fragmentation. For this reason we have not calculated here the nature of such jets which will be unreliable. However, it is obvious from Sec. II and Fig. 4 that there will be a strong leading particle behavior, about which the conclusion does not appear to be final[27] although the recent data on e^+e^- annihilation indicate that this behavior may be really absent.[20] In any case we know from charmonium spectroscopy[28] that here the pure harmonic-oscillator wave function which we have taken cannot be a good approximation. We ultimately expect that such experiments and analysis may help us to determine the wave functions. With the present level of experimental accuracy for obtaining the primary signal of the mesons,[20] we have not reached such a stage. At high energies the higher-order effects are also to be included in the theoretical calculations[25] which have not been included here.

IV. e^+e^- ANNIHILATION TO HADRONS

We shall consider here some specific effects regarding e^+e^- annihilation to hadrons. However, we may note that while trying to compare the results at low energies, the recombination mechanism of the residual final quark and antiquark becomes relevant, while for center-of-mass energies of 4 GeV and higher, the signal of D, D^*, F, and F^* become quite significant. These charmed mesons are not seen directly and except for the D meson the decay modes are not very well known.[29] Thus the signal of charmed mesons in yielding secondary π, K, and ρ mesons does not permit a clean prediction for the observations beyond the charm threshold. Hence we shall first study the hadrons in e^+e^- annihilation when the effect of the charmed meson production is not very significant.

We have already considered quark-fragmentation functions yielding K signals in Sec. III, Fig. 6 where the experimental extrapolation of data[19] includes information from e^+e^- annihilations.[30] Considering pions as the main signal, below charm threshold, we have

$$s(d\sigma/dz)(e^+e^-\to\pi X)=4\pi\alpha^2[\tfrac{5}{9}D_q^\pi(z)+\tfrac{1}{9}D_\lambda^\pi(z)]\,. \tag{4.1}$$

In the above, from Sec. II it is clear that $D_\lambda^\pi(z)$ will be given by an equation parallel to (3.15) taking mesons of rank higher than one. The results are plotted in Fig. 7 at $\sqrt{s}=3$ GeV against the experimental points.[30,31] The only free parameter here is $\mu=0.5$ GeV, which in fact has been adjusted for this curve and the same value is taken in all the other problems. Figure 7 represents an *energy-dependent* cross section and not the scaling limit, and the appropriate low-z behavior of the cross section may be noted. The bad high-z behavior of this is linked with the sudden stoppage of fragmentation, and is, as stated earlier, unreliable. In fact, a δ function for high z in the cross section for multiplicity $n=1$ of the jet in (3.5) has been omitted, which will get distributed when we are able to make this hypothesis better.

We shall next obtain some exclusive pion signals of low multiplicity in e^+e^- annihilation. For this purpose we can use (3.1) for small n, but to be able to do so, we need the constant C. We now

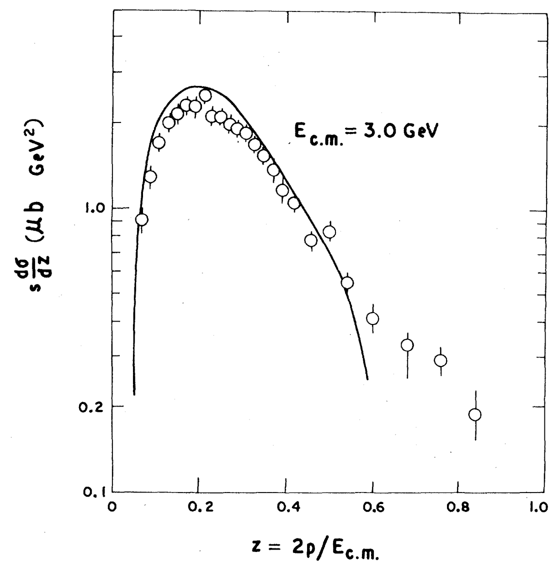


FIG. 7. $s(d\sigma/dz)(e^+e^- \to \pi X)$ at $\sqrt{s}=3$ GeV with $\mu = 0.5$ GeV. The experimental points are from Ref. 31.

substitute $\lambda = 2\mu/\sqrt{s}$ and from (3.2) note that

$$C(\lambda) = \sum_n \int f_q^\pi(z_1)dz_1 \cdots f_q^\pi(z_n)dz_n \delta(y_1 \cdots y_n - \lambda) .$$

We next put $g(y) = f_q^\pi(1-y)$. Then that $C(\lambda)$ satisfies the integral equation[6,7]

$$C(\lambda) = g(\lambda) + \int_\lambda^1 (dy/y)g(y)C(\lambda/y) . \tag{4.2}$$

For high energies or small λ, $C(\lambda)$ is determined by considering the moments of the respective functions. In fact, with

$$\tilde{C}(r) = \int_0^1 y^r C(y)\, dy$$

and

$$\tilde{g}(r) = \int_0^1 y^r g(y) dy ,$$

we obtain that

$$\tilde{C}(r) = \tilde{g}(r)/[1 - \tilde{g}(r)] . \tag{4.3}$$

This yields that for small $\lambda = 2\mu/\sqrt{s}$, we have[32]

$$C(\lambda) = R\sqrt{s}/2\mu , \tag{4.4}$$

where

$$R^{-1} = -[d\tilde{g}(r)/dr]_{r=0}$$
$$= \int_0^1 \ln(1/y) g(y) dy . \tag{4.5}$$

We shall now consider the total cross section for production of four-charged pions during e^+e^- annihilation, which we take as (16/41) times the signal for production of four pions. From Sec. II we know that the exclusive signal of pions is likely to come from $\mathcal{P}$ and $\mathfrak{N}$ quarks. We thus obtain by considering both the quark and antiquark that

$$\sigma_t(e^+e^- \to 4 \text{ charged pions}) = 4\pi\alpha^2 \tfrac{5}{9} \times \tfrac{16}{41}(2p_1p_2)/s , \tag{4.6}$$

where p_1 and p_2 are given by (3.1), using (4.4). We have plotted this cross section against experimental points[33] in Fig. 8(a), where the qualitative agreement appears to be reasonable, although the

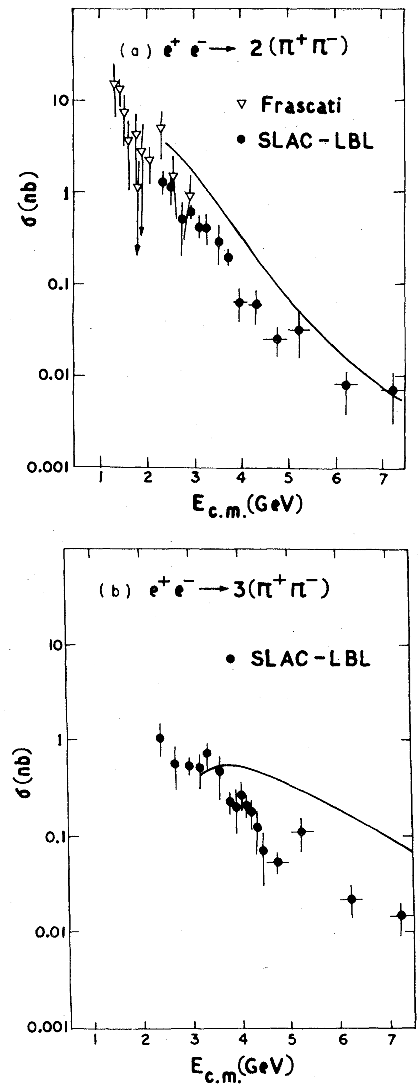


FIG. 8. (a) Total cross sections for exclusive four-charged-pion production plotted against c.m. energy. The experimental points are taken from Ref. 33. (b) Total cross sections for exclusive six-charged-pion production plotted against the c.m. energy. The experimental points are taken from Ref. 33.

theoretical curve is always on the higher side. With a similar analysis, we next note that

$$\sigma_t(e^+e^- \to 6 \text{ charged pions})$$
$$= 4\pi\alpha^2 \tfrac{5}{9} \times \tfrac{64}{365} (2p_1p_4 + 2p_2p_3)/s\,. \quad (4.7)$$

We have plotted the result in Fig. 8(b), where the disagreement with the experiments appears to be pronounced, indicating probably that our theoretical estimates are wrong. However, a confirmation of the above experiments should be desirable, since it is inherently difficult to look for the exclusive channels. We note that our theoretical estimates are also for the minor signals during quark fragmentation with low multiplicities, and thus need not be very reliable either, but such minor but clean signals will give useful information regarding details of dynamics. We may in particular remark that *through virtual gluons*, $q\bar{q}$ quark-antiquark pair may yield sizable signal of K mesons or other heavy mesons at high energies, processes which are omitted in our estimates, and will decrease the estimates for (4.6) and (4.7).

Regarding the equations (4.6) and (4.7), we may note that the probability factor within the parentheses in these equations is really the coefficient of x^{N-1} in the expansion of $(p_1x + p_2x^2 + \cdots)^2$, where we naturally have $N \geq 3$.

We may note that the comparatively large signal of K mesons[31,34] does not seem to come in the present model. We have seen earlier in Fig. 6 that this is not inconsistent with some data as examined in Ref. 19, where it is seen that for such processes there may be even 87% SU(3) violation with $K_d/(D_{\mathfrak{N}}^{*+} + D_{\mathfrak{N}}^{*-}) \simeq 0.13 \pm 0.03$, which is comparable to what we get and is in contrast to the assumptions of some other standard models.[26] We expect that the K mesons at higher energies may come from $c\bar{c}$ jets with the decay of the charmed mesons. Also at higher energies the gluon brehmsstrahlung and the quark-antiquark pair creation terms may give significant contribution[25] decreasing the magnitude of SU(3) violation we have mentioned above and in better agreement with that of the standard models.[26] Since the K mesons may be the products of decays, the situation needs further theoretical[35] and experimental analysis regarding the deciphering of such signals.

It appears from Sec. II that $c\bar{c}$ jets are likely to give rise to two charmed mesons, the remaining being mostly pions. However, the kinematic dependence of these heavy mesons becomes too restrictive[20] to get the z dependence of *both* the charmed mesons. It is possible that the leading particle behavior for the fragmentation $c \to D^+ + \mathfrak{N}$ will make the second D meson too slow. However, there does not appear to be any clean way of taking into account such kinematics and we have not been able to calculate this. We may suspect that this second D meson may be observed in Ref. 20 with the leading-particle behavior[27] being there for the first fragmentation, but both the theoretical and the experimental situation appears to us as unclear.

V. TRANSVERSE MOMENTUM OF QUARK JETS

We know that the transverse momentum of the jets is an important concept even for the definition of the jet[36] in the context of higher-order effects in quantum chromodynamics,[37] which we are not retaining here, but which can probably be included within the present model.[25] The transverse-momentum distribution is also included in the primordial quark-fragmentation functions in an *ad hoc* manner[38] for the description of this aspect of the quark jets. However, we may presently see that when we calculate the quark-fragmentation functions, the transverse-momentum distribution also gets prescribed. The present calculations are based on the wave functions of the hadrons and besides this do not include any other nonperturbative aspect of quantum chromodynamics.[37] Hence, besides the calculable quantum-chromodynamic effects which are omitted[37] even the nonperturbative part may have limited validity.[39] At the outset it is impossible to know how adequate the wave functions will be for the calculation of the transverse momenta. The present model with the harmonic-oscillator wave functions at least serves the purpose of a toy-model calculation as a first step and gives some results which appear to be relevant.

Let p_t be the transverse momentum of the meson M in the process (1.1). We then have in the high-energy limit from (2.10a) that

$$\lambda = p_t^2/(2z)\,, \quad (5.1)$$

where we have used that θ remains small. Hence, by (2.33) we get,

$$\frac{d\sigma}{dz\,dp_t^2}(Q_1\tilde{Q}_2 \to MQ_3\tilde{Q}_2)$$
$$= \alpha_s^2\pi^2 \tfrac{256}{27} \frac{(1-z)^2}{2} \frac{s^2}{mm_2^2} \frac{(\lambda + f_1)|\tilde{u}_M(\vec{k}_1)|^2}{[\lambda + \psi(z)]^2}\,. \quad (5.2)$$

Hence from Sec. II we obtain that the primordial quark-fragmentation function $f_{Q_1}^M(z, p_t^2)$ including the distribution of the squares of transverse momenta becomes

$$f_{Q_1}^M(z, p_t^2)$$
$$= \frac{d\sigma}{dz\,dp_t^2}(Q_1\tilde{Q}_2 \to MQ_3\tilde{Q}_2)/\sigma_t(Q_1\tilde{Q}_2 \to MQ_3\tilde{Q}_2)\,. \quad (5.3)$$

In the above and subsequently we always assume that the transverse momenta are small in comparison with the longitudinal momenta. We now substitute

$$g_{t2}(z)=\int p_t{}^2 f^M_{Q_1}(z,p_t{}^2)dp_t{}^2\,, \tag{5.4}$$

such that $g_{t2}(z)dz$ is the expectation value of the square of the transverse momentum when the longitudinal momentum lies within the appropriate momentum fraction.

Corresponding to (3.5), we shall now calculate the expectation value of the transverse momentum of the kth rank meson when the total multiplicity of the quark jet is n. We note that for the primordial quark fragmentation as in Sec. II the transverse momenta of the fragmenting quark and the meson balance. Hence, for the process corresponding to (3.5), if we substitute that $\vec{p}_i$ is the *transverse momentum* of the *meson* for the ith fragmentation, then the *total transverse momentum* of the kth-rank meson becomes

$$\vec{p}_{tr}=\sum_{i=1}^{k-1}(-\vec{p}_i)+\vec{p}_k\,. \tag{5.5}$$

We may now note that from axial symmetry when we try to obtain the expectation value of the square of (5.5), we shall effectively have

$$p_{tr}{}^2\equiv\sum_{i=1}^{k}\vec{p}_i{}^2\,. \tag{5.6}$$

Hence, if $F_{t2}(n,k,z)dz$ is the expectation value of the square of the transverse momentum of the kth-rank meson when the total multiplicity is n and when the longitudinal momentum of the meson lies within the appropriate momentum fraction, then, parallel to (3.5) and with (5.4) and (5.6) we obtain that

$$F_{t2}(n,k,z)=C^{-1}\int[g_{t2}(z_1)dz_1 f^\pi_q(z_2)dz_2\cdots f^\pi_q(z_n)dz_n+f^\pi_q(z_1)dz_1 g_{t2}(z_2)dz_2\cdots f^\pi_q(z_n)dz_n$$
$$+\cdots+f^\pi_q(z_1)dz_1\cdots g_{t2}(z_k)dz_k\cdots]\delta(y_1\cdots y_n-2\mu/\sqrt{s})\delta(y_1\cdots y_{k-1}z_k-z)\,. \tag{5.7}$$

We may now add over all multiplicities and also mesons of all ranks. If $G_{t2}(z)dz$ is the sum of the squares of the transverse momenta of the mesons produced with appropriate longitudinal-momentum fraction, we then obtain parallel to (3.6) that

$$G_{t2}(z)=\sum_{n,k}F_{t2}(n,k,z)\,. \tag{5.8}$$

Clearly

$$p_t{}^2=\int g_{t2}(z)dz \tag{5.9}$$

is the expectation value of the square of the transverse momentum during the primordial fragmentation, and that

$$P_t{}^2=\int G_{t2}(z)dz \tag{5.10}$$

is the expectation value of the sum of the squares of the transverse momenta of the mesons produced as a result of repeated fragmentation with any multiplicity, i.e., of the quark jet. From (5.10), knowing the multiplicity in (3.3), we can obtain the appropriate mean of the square of the transverse momentum.

We now note that sphericity is defined as

$$S=\tfrac{3}{2}P_t{}^2/(P_L{}^2+P_t{}^2)\,, \tag{5.11}$$

where (5.10) is to be used, and for the sum of the squares of longitudinal momenta we have, retaining only pion signals,

$$P_L{}^2=(s/4)\int z^2 D^\pi_q(z)dz\,. \tag{5.12}$$

As mentioned earlier we shall now derive the estimates of the transverse momenta from the above results for the quark-fragmentation functions corresponding to the harmonic-oscillator wave functions of Sec. II. With $\sqrt{s}=3$ GeV and as before only considering pion signals, we obtain that for the primordial quark-fragmentation functions the transverse momentum as in (5.9) is given as

$$p_t=0.086\text{ GeV}\,. \tag{5.13}$$

We note that for (5.9) and with the pions, already the scaling limit is reached at the above energies, and thus (5.13) will be energy independent. We next use (5.10) and obtain that at $\sqrt{s}=3$ GeV, the root-mean-square average transverse momentum is given as

$$(P_t{}^2/\langle n\rangle)^{1/2}=0.14\text{ GeV}\,. \tag{5.14}$$

Hence by (5.11) we obtain that the sphericity is

$$S=0.094\,. \tag{5.15}$$

In the above, as before, we have taken[9] $R_\pi{}^2=15$ GeV^{-2}. If we take $R_\pi{}^2=10$ GeV^{-2}, we get instead of (5.15)

$$S=0.15\,. \tag{5.16}$$

In contrast to both the above results, we know that experimentally we have[31] at this energy $S \simeq 0.35$. The small value of the sphericity as calculated here may be attributed to the small p_t in (5.13). However, we note that the quantum chromodynamic corrections will enhance the transverse momentum.[37] Further, the coherent production of heavy hadronic resonances as well as the production of heavy hadrons above the charm threshold at higher energies will tend to give rise to a more spherical distribution. All these effects, not included in (5.11) and (5.15), will tend to increase the sphericity and the correction here will depend on the energy. To illustrate this aspect let us imagine that in (5.11) there is another overlapping *spherical* distribution of the momentum which is a fraction α of the known momentum signal. We next estimate α such that the sphericity is 0.35, and obtain that $\alpha = 0.15$ becomes adequate, indicating that quite often a small admixture of other effects which tend to make the distribution more spherical will yield the observed value of the sphericity. Besides, we have a feeling that the detailed nonperturbative forces of confinement may give additional broadening at low energies which need not be covered adequately by the wave function. It is our motivation also to know what effects are not covered by the wave functions. We note that as per our calculations even in the present crude model that more than half the nonperturbative effect may already be taken into account when we include the wave function.

In the present model the primordial fragmentation function $f_Q^M(z, p_t^2)$ is *not* factorizable in z and p_t^2 contrary to many conventional assumptions.[38] With this in mind we examine the z dependence of the transverse momenta. We substitute

$$f_{t2}(z) = g_{t2}(z)/f^M_{Q_1}(z) \tag{5.17}$$

and

$$F_{t2}(z) = G_{t2}(z)/D_q^\pi(z)\,. \tag{5.18}$$

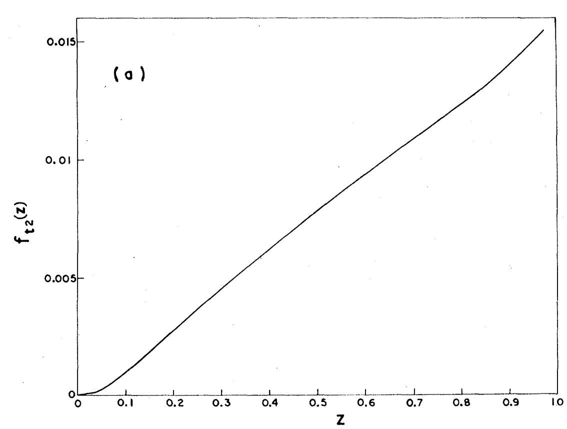
$f_{t2}(z)$ and $F_{t2}(z)$, respectively, yield the distribution functions for the squares of transverse momenta during the primordial fragmentation and the final fragmentation. We have plotted these functions for the pions at $\sqrt{s} = 3$ GeV in Figs. 9(a) and 9(b), respectively. The linear rise of $f_{t2}(z)$ in Fig. 9(a) may be noted; however, this feature is completely submerged in Fig. 9(b) for $F_{t2}(z)$, which appears to exhibit a familiar sea-gull form.[40] We have also plotted $F_{t2}(z)$ in Fig. 9(b) at $\sqrt{s} = 5$ GeV, which shows the energy dependence of this sea-gull effect. Although the qualitative form of the curves in Fig. 9(b) is pleasing, the magnitude of transverse momentum is smaller than expected, as we also noted earlier.

We may also consider some other broad signals regarding the transverse momenta. With pion signals, one may think that at high energies we shall *approximately* have with (5.13)

$$\langle P_t \rangle_{\rm av} = 0.086 \times \langle n \rangle\,, \tag{5.19}$$

where the average multiplicity is given as[7]

$$\langle n \rangle = R \ln[(\sqrt{s}/2)/\mu]\,, \tag{5.20}$$



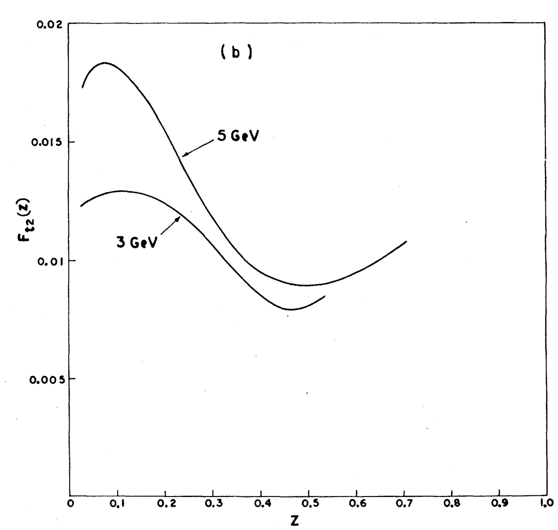


FIG. 9. (a) The distribution of the square of the primordial transverse momentum of the pions plotted against z at $\sqrt{s} = 3$ GeV. (b) The distribution of the sum of the squares of the transverse momenta of the pions plotted against z at $\sqrt{s} = 3$ GeV and 5 GeV, respectively.

with[7]

$$R^{-1}=\int_0^1 \ln(1/x)f_q^\pi(1-x)dx\,. \qquad (5.21)$$

We may note that at high energies this is slightly less than the values quoted in recent experiments[41] at DESY, which is expected, since we are omitting the signal of charmed mesons. We note that for the estimate (5.19) we have not used any free parameter.

VI. DISCUSSIONS: RECOMBINATION OF RESIDUAL QUARK-ANTIQUARK PAIR

We shall first note the assumptions which have gone into the present model. We describe mesons as bound states of quark-antiquark in a field-theoretic model of hadrons with Lorentz boosting[9] which has been found useful earlier[9,10] and the present model is a continuation of the same. As earlier[16] and as described in Sec. II, we have incorporated free quarks in the same model, which we expect may be a good description because of asymptotic freedom[21]; in any case the success of quark parton models seems to indicate that this is likely to be a valid approximation. We have next considered the interaction Hamiltonian with a vector-meson exchange in the context of Ref. 9, and find that in the lowest order, for processes like (1.1) the kinematics is the same as that of quark-fragmentation models where also the fragmentation process effectively decouples from the scattering process. We interpret this with an identification which yields a prescription for the quark-fragmentation functions in terms of the wave functions of the meson and the dynamics of interaction, here taken as due to vector-meson exchange parallel to quantum chromodynamics. When the wave functions are known or assumed, this fragmentation function *does not* have a single free parameter when the mass of the vector meson can be neglected. For application of the quark-fragmentation model at finite energies, we postulate a momentum μ with which we assume that the fragmentation of the quark stops.[7] This is the *only* free parameter in our model after we choose the harmonic-oscillator wave functions for an average description of the mesons. This parameter sets the scale at which the high-energy limit is reached. We calculate some quark-fragmentation functions and the cross sections for e^+e^- annihilation to hadrons and compare these with the experimental results. Also, in the present model automatically the quark-fragmentation function has a nonfactorizable transverse-momentum distribution which appears to yield the energy-dependent sea-gull effect regarding the dependence of the transverse momentum on the scaling variable. Although there is broad agreement of many results, some of the details are different, which as expected indicates that the harmonic-oscillator wave functions can only have limited validity. Some of the results are also inappropriate in this model since the quarks are assumed to stop fragmenting suddenly, which constitutes an obvious limitation of the model. We shall here illustrate in a simple manner how possibly this limitation may be avoided, and how it may change the results.

For this purpose we simultaneously consider both the quark and the antiquark yielding mesons. In particular we shall consider the signal of λ and $\bar\lambda$ yielding a K meson, which has been examined for a single quark by (3.15) at $\sqrt{s}=3.5$ GeV. It was calculated earlier and plotted in Fig. 6 and the result was dominated by the cutoff μ and was thus unreliable. Let λ and $\bar\lambda$ quarks "fragment" to m and n mesons, respectively, with a recombination of the final quark and antiquark yielding multiplicity $N=m+n+1$. The nature of the fragmentation functions in Sec. II yields that the final quark-antiquark pair will combine to yield a pion, where we assume that $m,n\geqslant 1$. We next postulate that the probability for the recombination of the final quark and antiquark is given by the longitudinal momenta of the same in terms of a *recombination function* $R(k_{1z},k_{2z})$. Hence with the results of Sec. II, we take that the probability for the quark giving rise to the K meson (which will be of rank one) with momentum fraction z is given as

$$\begin{aligned}F(m,z;n)=\int & f_\lambda^K(z)f_q^\pi(z_2)dz_2\cdots f_q^\pi(z_m)dz_m\\ &\times f_{\bar\lambda}^K(z_1')dz_1'f_q^\pi(z_2')dz_2'\cdots f_q^\pi(z_n')dz_n'\\ &\times R(y_1y_2\cdots y_m\sqrt{s}/2,\,y_1'\cdots y_n'\sqrt{s}/2)\,.\end{aligned} \qquad (6.1)$$

We can define in a similar fashion $F(m;n,z)$ for the corresponding probability for the antiquark yielding the K meson. We note that the recombination function $R(k_{1z},k_{2z})$ is normalized with the constraint

$$\sum_{m,n}\int F(m,z;n)dz=1\,, \qquad (6.2)$$

which merely ensures that the total probability for the quark-antiquark pair to go over to hadrons is unity and using from Sec. II that the quark λ yields only one K meson.

We now easily obtain that

$$D_\lambda^K(z)=\sum_{m,n}F(m,z;n)\,, \qquad (6.3)$$

where the normalization (6.2) may be emphasized.

To illustrate the results, we take explicitly

$$R(k_{1z}, k_{2z}) \propto k_{1z}{}^3 k_{2z}{}^3 , \qquad (6.4)$$

and then calculate $D_\lambda^K(z)$, which is plotted in Fig. 6 along with the earlier curve. The present curve agrees with the experimental points, which is not surprising since we have chosen the recombination function (6.4) accordingly. To give it a heuristic meaning, since the earlier estimates showed that low multiplicity is preferred, we thought that the recombination function may be merely proportional to the phase space of the quark and antiquark. However, dynamically how this function should behave at high energies and whether the form (6.1) is the correct way of taking into account this effect needs further investigations. We may note that the recombination function of Das and Hwa proposed in a similar context is quite different.[42] There the whole model consists of recombination, whereas here we are taking the quark fragmentation initially and are using the idea of recombination at the end only. The actual dynamics may as well be a more complicated combination of both.

We had noted in Sec. II that if the intermediary vector mesons are taken as gluons, then the cross sections for the hadronization process (1.1) become extremely large, which, within a distance of 1 fm may be regarded as indicative of multiple scattering with the *saturation* of probability of scattering in quark space being operative. Hence we may expect that the unitarity bound for hadronic scattering is likely to be also saturated.[43] However, such a question of principle needs further careful analysis,[44] since we may recognize that the quarks are "free" well within the domain of color confinement, but ultimately hadronic states are created, which raises an unusual dynamical situation regarding the definition of asymptotic states.

With the above interpretation of multiple scattering, we feel that Sec. II in the context of quantum chromodynamics is tantalizing in the sense that for quark interactions, even a volume of 1 fm^3 can be possibly regarded as an "infinite" volume in which free quarks and hadrons may be created. Clearly in this volume there can be gluon brehmsstrahlung as well as creation of quark-antiquark pairs.[25] In addition to the mechanism we have considered, this quark-antiquark "gas" may lead to the validity of other models[42] considered for high-energy collision of hadrons. It may possibly lead even to the hydrodynamic models[45] where the "wall" for the gas may consist of color-confinement forces, and this wall may be transparent to the color-neutral hadrons.

The above speculations apart, which in the context of discussions of Sec. II may not be valid, the present model permits us to see the quark-fragmentation models[5-7] in a new angle with the possibility to calculate the quark-fragmentation functions in terms of the wave functions of the hadrons in the lowest order with a vector intermediary as an illustrative dynamical process along with a nonfactorizable transverse-momentum distribution dependent on the wave functions of the mesons. It is disappointing that we cannot relate the process to unambiguous perturbative predictions of quantum chromodynamics. However, it is good to note that such processes may be partly tractable with the wave function describing a major part of nonperturbative effects with a possible unity of description of hadronic dynamics of coherent and incoherent processes.

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