# Study of antiproton-proton annihilations using the topological-cross-section differences between $\bar{p}p$ and pp interactions at 48.9 GeV/c

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The multiplicity-cross-section differences  $\Delta \sigma_n$  between  $\bar{p}p$  and pp interactions are determined and are compared with measured annihilations and  $\Delta \sigma_n$  at other energies. To the extent that these cross-section differences measure the values of  $\sigma_n^A$ , the topological cross sections for annihilations, we present evidence for a decided break from a single-cluster-model prediction for the parameter  $f_2^{A^{--}}$ . Alternatively, a picture of precocious Koba-Nielsen-Olesen scaling in  $\bar{p}p$  annihilations leads to a reasonably good representation of  $f_2^{A^{--}}$  vs  $\langle n_-^A \rangle$  over the whole measured range. We find  $\langle n^A \rangle = 7.57 \pm 0.31$ ,  $D^A = 2.77 \pm 0.10$ ,  $f_2^{A^{--}} = -1.86 \pm 0.20$ , and  $\langle n^A \rangle / D = 2.73 \pm 0.15$ . Finally we observe a remarkable agreement with theoretical prediction for  $R_n^* = \Delta \sigma_n / \sigma_{n+2}(pp)$ , an experimental ratio based on a strict application of the counting rules for quark duality diagrams, and we thereby find evidence that in the topological-cross-section difference  $\Delta \sigma_n$  the small nonannihilation contribution becomes progressively more negligible as n increases.

#### I. INTRODUCTION

A large total-cross-section difference between antiproton-proton and proton-proton interactions,  $\Delta \sigma \equiv \sigma(\overline{p}p) - \sigma(pp)$ , which decreases rapidly with energy, is a well known experimental fact.<sup>1</sup> There have been two different theoretical approaches advanced to interpret this experimental observa $tion^{2,3}$ : (a) The magnitude and energy dependence of  $\Delta \sigma$  is a direct measure of the annihilation cross section which we denote by  $\sigma^A$ . (b) The magnitude and energy dependence of  $\Delta \sigma$  can be attributed to the contribution of both inelastic nonannihilation as well as annihilation processes present in the total  $\overline{p}p$  cross section. If (b) is the correct interpretation, then the physical content of  $\Delta \sigma$  is not clear. In the following we discuss the merits of both points of view and argue that although for low multiplicities  $\Delta \sigma_n$  may not completely give the topological annihilation cross section  $\sigma_n^A$ , nevertheless, for higher multiplicities  $(n \ge 4)$  the topological-cross-section differences are indeed dominated by annihilation processes. We define  $\Delta \sigma_n \equiv \sigma_n(\overline{p}p) - \sigma_n(pp)$ , where  $\sigma_n$  denotes the measured topological cross section with n observed charged particles.

#### II. EMPIRICAL ARGUMENTS TO RELATE $\Delta \sigma$ AND $\Delta \sigma_n$ TO ANNIHILATION PROCESSES

It has been noted that the total-cross-section difference  $\Delta\sigma$  at high energies connects and smoothly continues the power-law energy dependence of the  $\bar{p}p$  annihilation cross section<sup>4</sup> which is measured directly below about 10 GeV/c. While this suggests that the total-cross-section difference (or more correctly the difference in total *inelastic* cross sections) may well represent the annihilation process, it does not necessarily follow that each of the individual topologicalcross-section differences directly measures the corresponding annihilation topological cross section.

One clear example of a possible problem is the zero-prong (charge-annihilation) cross section  $\sigma_0(\bar{p}p) = \Delta \sigma_0$ , which has no counterpart in the pp interaction to cancel the meson-exchange processes which, in addition to annihilation, must certainly be present. Figure 2 of our preceding paper<sup>5</sup> shows that  $\sigma_0(\bar{p}p)$  drops rather sharply as a function of the beam momentum with a dependence  $s^{-1.45\pm0.14}$ . Thus,  $\Delta \sigma_0$  is relatively small when compared to the  $\Delta \sigma_n$  for  $n \ge 4$ , and we find its effect on the multiplicity moments to be negligible.

A second example of a possible problem occurs in the two-prong events, where the value of  $\Delta\sigma_2$ (see Table I) is zero within errors  $(0.10 \pm 0.28$  mb). For this partial cross section, we extrapolate the energy dependence  $\sigma_2^A$  (mb) = 1540 s<sup>-2.46</sup> (where s is in GeV<sup>2</sup>) based<sup>6</sup> on measurements at energies up to 9 GeV to obtain  $\sigma_2^A = 0.022$  mb, which is consistent with the measured  $\Delta\sigma_2$  at 50 GeV/c. Again, the moments of the annihilation multiplicity distribution are not very sensitive to this topological cross section since the bulk of the measured annihilation cross section lies at higher charged multiplicities, even at the lower energies where it is measured directly.

For multiplicities  $n \ge 4$ , there are four observations which suggest that the identification  $\Delta \sigma_n$  $= \sigma_n^A$  is a good approximation. Firstly the highenergy  $\Delta \sigma_n$  values smoothly continue those of  $\sigma_n^A$ measured at lower energies,<sup>4</sup> and similar behavior is seen in the moments of these distribu-

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TABLE I. Topological cross sections for 48.9-GeV/c  $\bar{pp}$ , parametrized pp, and the topological-cross-section difference  $(\Delta \sigma_n)$  between  $\bar{pp}$  and pp as a function of the charged-particle multiplicity.

n	σ <sub>n</sub> ( <i>pp</i> ) (mb)	$\sigma_n(pp)^a$ (mb)	$\sigma_n(\bar{p}p - pp)$ (mb)
0	$0.149 \pm 0.039$		$0.149 \pm 0.039^{b}$
2(inel)	$5.69 \pm 0.22$	$5.585 \pm 0.178$	0.10 ±0.28 <sup>b</sup>
4	$10.34 \pm 0.20$	$9.094 \pm 0.196$	$1.25 \pm 0.28$
6	9.27 ±0.19	$8.410 \pm 0.115$	$0.86 \pm 0.22$
8	$6.42 \pm 0.16$	$4.848 \pm 0.112$	$1.57 \pm 0.20$
10	$2.85 \pm 0.11$	$1.912 \pm 0.099$	$0.94 \pm 0.15$
12	$0.994 \pm 0.067$	$\textbf{0.566} \pm \textbf{0.048}$	$0.43 \pm 0.08$
14	$0.288 \pm 0.036$	$0.138 \pm 0.018$	$0.15 \pm 0.04$
16	$0.042 \pm 0.017$	$0.031 \pm 0.006$	$0.012 \pm 0.018$
18	$0.009 \pm 0.007$	$\textbf{0.007} \pm \textbf{0.002}$	$0.002 \pm 0.007$

<sup>a</sup> The overall normalization error has been minimized by setting  $\sigma_{\text{inel}} = \sigma_{\text{tot}} - \sigma_{\text{el}}$  with  $\sigma_{\text{tot}}$  from Ref. 30 and  $\sigma_{\text{el}}$ from Ref. 31.

<sup>b</sup> Zero-prong and inelastic two-prong cross sections are replaced by 0 and 0.022 mb, respectively; see Ref. 6.

tions as discussed below. Secondly, the distribution of  $\Delta \sigma_n$  is found to peak at relatively large charged multiplicities, in agreement with the pattern found in directly measured annihilations. Thirdly, the  $\pi^{\dagger}p$  topological-cross-section differences<sup>4</sup>  $\sigma_n(\pi^{-}p) - \sigma_n(\pi^{+}p)$  are close to zero for  $n \ge 4$ . Although the meson-exchange contributions to the above differences are dominated by  $\rho$  exchange because of *G*-parity constraints, both  $\rho$ and  $\omega$  exchange can contribute to  $\Delta \sigma_n$ . However, by  $\omega - \rho$  universality,<sup>4</sup> it seems plausible that all meson-exchange contributions to  $\Delta \sigma_n$  must be very small for  $n \ge 4$ . Thus, the observed large values of  $\Delta \sigma_n$  in the higher multiplicities are presumably dominated by annihilation.

Finally, we note that if annihilation dominates the difference, then the quantities  $R_n^* = \sigma_n^A / \sigma_{n+2}(pp)$ can be written as

$$R_n^* = \frac{\Delta \sigma_n}{\sigma_{n+2}(pp)} \,. \tag{1}$$

Upon evaluating all possible quark-exchange diagrams for  $\sigma_{n+2}(pp)$  and annihilation diagrams for  $\sigma_n^A$  in the leading order<sup>3</sup> one obtains the general form for  $R_n^*$ ,

$$R_n^* = \beta^n s^{-\alpha} , \qquad (2)$$

where  $\beta$  and  $\alpha$  are independent of *n* and *s*. Since  $\beta$  is expected theoretically to be  $\frac{3}{2}$ ,  $R_n^*$  would increase as a function of *n* at fixed *s*. As it is discussed later, Eq. (2) gives an adequate representation of our data. We emphasize that the result expressed in Eq. (2) depends on simple counting rules used in quark-duality-diagram models<sup>7</sup> and

not on details of the Eylon-Harari model.<sup>3</sup> Henceforth, we identify  $\Delta \sigma_n$  with the *n*-prong annihilation topological cross section  $\sigma_n^A$  for  $n \ge 4$ .

In this paper we report on a study of the  $\Delta\sigma_n$ distribution at 48.9 GeV/c based on the present experiment and on a parametrization of existing pp data at several energies. The 48.9-GeV/c  $\bar{p}p$ data consisting of 10 000 events are presented in the preceding paper.<sup>5</sup> The existing 50-GeV/c pp topological cross sections, based on some 2 000 events,<sup>8</sup> would contribute excessively to the statistical errors on the computed cross-section differences. We have therefore parametrized the relatively abundant pp topological-crosssection data at a series of energies to obtain improved values at 48.9 GeV/c.

## III. PROTON-PROTON TOPOLOGICAL-CROSS-SECTION PARAMETRIZATION

Many authors have parametrized topological cross sections and their moments as a function of energy.<sup>9-15</sup> Fits of both logarithmic (~lns) and power-law (~s<sup> $\alpha$ </sup>) forms of energy dependence to existing data on  $\langle n \rangle$ , the average charged multiplicity, indicate that the energy dependence gradually changes from ~s<sup> $\alpha$ </sup> at low energy ( $\leq 10$  GeV) to ~lns at high energies ( $\geq 100$  GeV/c).<sup>16</sup> This transition is necessarily reflected in the topological cross sections, although how it affects each one separately is not clear.

Several studies of charged-multiplicity data<sup>12, 13, 17-22</sup> confirm that there are certain features of the data which are essentially energy independent at sufficiently high energy thus affording an energy-independent parametrization of the data. They are:

(1) Koba-Nielsen-Olesen (KNO) scaling<sup>23</sup> sets in precociously<sup>11,14</sup> ( $P_{LAB} < 25 \text{ GeV}/c$ ), that is, at energies below those where Feynman scaling of single-particle inclusive reactions is observed. This result is primarily responsible for the success of normal and quasinormal multiplicitydistribution parametrizations.<sup>12,13</sup>

(2) Global and local charge-conservation models strongly favor charged-pair production over independent single-particle emission, so the greatest success in parametrizing the multiplicity data is found where the variable used is  $n_{-}=n/2$ - 1 for pp interactions.<sup>13,19</sup>

(3) Modal multiplicity (m) is less dependent upon the poorly determined tails of the distribution than is the mean  $\langle n \rangle$ , so it is desirable to parametrize multiplicity in terms of m rather than  $\langle n \rangle$  (Refs. 21 and 13).

We have used a parametrization model, due to

,	D	n			

$\langle n \rangle = 7.57 \pm 0.31$	$D = (\langle n^2 \rangle - \langle n \rangle^2)^{1/2} = 2.77 \pm 0.10$	$f_2^{} = -1.86 \pm 0.20$
$\langle n \rangle / D = 2.73 \pm 0.15$	skewness = $\frac{\langle (n - \langle n \rangle)^3 \rangle}{D^3}$	$f_2^{cc} = \langle n(n-1) \rangle - \langle n \rangle^2$
$c_2 = \frac{\langle n^2 \rangle}{\langle n \rangle^2} = 1.134 \pm 0.015$	$= 0.32 \pm 0.14$	$= 0.12 \pm 0.62$

TABLE II. Moments of the difference multiplicity distribution  $\Delta \sigma_n (\bar{p}p - pp)$ .

Tomozawa,<sup>13</sup> which possesses all three features. The model predicts a KNO-type quasinormal multiplicity-scaling function, expanded about the modal multiplicity  $m_{-}$  of the negative-charged particles. We chose the form given below since it appears to work best at high multiplicity (form *B* in Ref. 13):

$$m_{-}\frac{\sigma_{n_{-}}}{\sigma_{\text{inel}}} = \frac{1}{\sqrt{2\pi}b} \exp\left[-\frac{1}{2d^{2}}\left(\frac{n_{-}}{m_{-}}-1\right)^{2} + \frac{a_{3}}{d^{3}}\left(\frac{n_{-}}{m_{-}}-1\right)^{3} + \cdots\right] .$$
 (3)

We have attempted to improve the fit reported in Ref. 13 by including recent 60 GeV/c pp topological cross sections<sup>24</sup> and by updating the other cross sections.<sup>8,25,30,31</sup> Using 50, 60, 69, 102, 205, and 300 GeV/c data, we obtain the following bestfit parameters (almost identical to those found in Ref. 13):  $b = 0.94 \pm 0.01$ ,  $d = 1.02 \pm 0.02$ ,  $a_3 = 0.040$  $\pm 0.003$ , and  $m_{-}(50) = 1.34 \pm 0.03$ . The parameters b, d, and  $a_3$  were found to be independent of energy and the best-fit modal multiplicities found for the other input energies are  $m_{-}(60) = 1.38$  $\pm 0.03, m_{-}(69) = 1.52 \pm 0.02, m_{-}(102) = 1.67 \pm 0.03,$  $m_{-}(205) = 2.11 \pm 0.03$ , and  $m_{-}(300) = 2.38 \pm 0.04$ . The  $\chi^2$  per degree of freedom for this fit is 114/51. We did not include data at 28.5 GeV/c (Ref. 14) because the resulting prediction at 50 GeV/c changes only slightly and the  $\chi^2$  per degree of freedom for that fit is 180/57. The results for 50-GeV/c pp are shown along with our 48.9- $GeV/c \bar{p}p$  data and the cross-section differences in Table I. Moments of the  $\Delta \sigma_n$  multiplicity distribution are given in Table II.

### IV. RESULTS AND DISCUSSION

Figure 1 shows the moments  $\langle n_{-} \rangle$  and  $f_{2}^{-}$  plotted as functions of energy and of  $\langle n_{-} \rangle$ , respectively. Our data points smoothly interpolate between the neighboring points at 32 and 100 GeV/c.<sup>26,6</sup> The value of  $f_{2}^{--} = -1.86 \pm 0.20$  for 50 GeV/c ( $\bar{p}p$ -pp) clearly agrees with the upward trend away<sup>6</sup> from the single-cluster-model line.<sup>27,28</sup> Our result is two standard deviations away from the single-cluster-model prediction (other parametrizations of pp topological cross sections yield even *larger* deviations from this linear prediction). Our result therefore adds credence to the notion that there is multiple-cluster formation in the  $\bar{p}p$  annihilation reaction for  $p_{\text{LAB}} \gtrsim 30 \text{ GeV}/c$ .

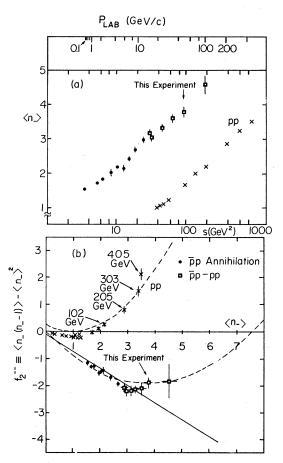


FIG. 1. (a) The average negative-particle multiplicity  $\langle n_{-} \rangle$  as a function of s for  $\overline{p}p$  annihilations (closed circles) and for the  $\overline{p}p-pp$  topological-cross-section differences (open squares). The × represents the pp data. (b) The second multiplicity moment  $f_{2}^{-} \equiv \langle n_{-}(n_{-}-1) \rangle - \langle n_{-} \rangle^{2}$  as a function of s. Symbols are the same as in (a). The straight line corresponds to the single-cluster model with  $f_{2}^{-} = -0.61 \langle n_{-} \rangle - 0.20$ . The dashed lines are given by Eq. (4) with  $\langle n \rangle / D = 1.99$  (see Ref. 32) for pp and  $\langle n \rangle / D = 2.73$  for annihilations and for the  $\overline{p}p-pp$  differences.

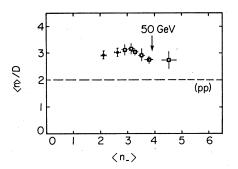


FIG. 2. The ratio  $\langle n \rangle / D$  as a function of  $\langle n_{-} \rangle$  for  $\overline{p}p$  annihilations (closed circles) and for the  $\overline{p}p-pp$  differences (open squares). The dashed horizontal line at a value of 2.0 represents the average value of  $\langle n \rangle / D$  for pp interactions with  $50 \leq p_{\text{LAB}} \leq 400 \text{ GeV}/c$ .

Figure 2 shows the energy dependence of the variable  $\langle n \rangle / D$ . As in Fig. 1, we have plotted both  $\overline{p}p$  annihilation data and high-energy  $\overline{p}p - pp$  difference data together. The nearly constant value of  $\langle n \rangle / D^{-2}$ .73 requires the upturn of  $f_2^{--}$  as a function of  $\langle n_{-} \rangle$ , as shown by the lower dashed curve in Fig. 1(b). This follows from the definition<sup>32</sup>

$$f_{2}^{--} \equiv \left( \langle n \rangle / D \right)^{-2} \langle n_{-} \rangle^{2} - \langle n_{-} \rangle .$$
<sup>(4)</sup>

The well-known constancy of  $\langle n \rangle / D \simeq 2$  for highenergy nonannihilation reactions (e.g., pp) (Ref. 5) yields an upturn in  $f_2^-$  at  $\langle n_- \rangle \simeq 1$ , whereas  $\langle n \rangle / D \simeq 2.73$  for  $\bar{p}p - pp$  yields the upturn at about  $\langle n_- \rangle \simeq 3$  (see Fig. 1). Whether the energy behavior of  $\langle n \rangle / D$  or  $f_2^-$  is more fundamental is a matter of speculation at this time. We note, however,

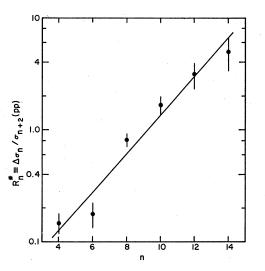


FIG. 3. The ratio  $R_n^* \equiv \Delta \sigma_n / \sigma_{n+2}(pp)$  as a function of *n*, the total number of charged particles. The straight line is a best fit to the form  $R_n^* = s^{-\alpha}\beta^n$  with  $\beta = 1.49 \pm 0.04$  and  $\alpha = 0.80 \pm 0.05$ .

that the independent fireball (cluster) model of Levy<sup>15</sup> may provide a physical interpretation for  $\langle n \rangle / D$ . In this model, multiparticle production proceeds by independent (Poisson) emission of identical clusters. Each cluster then decays into a fixed number of pions at a given energy. The assumption of independent emission combined with the definition of  $\langle n \rangle / D$  implies that  $\langle c \rangle$ , the average number of clusters emitted, is

$$\langle c \rangle = (\langle n \rangle / D)^2$$
. (5)

It is known that this model works very well in predicting topological cross sections of protonproton interactions.<sup>15</sup> In this case,  $\langle c \rangle$  approaches the constant value of  $\langle c \rangle = 4$ . If, however, we apply the formula to antiproton-proton annihilation reactions, then  $\langle c \rangle \simeq 7.5 \pm 0.8$ . We note that the topological-cross-section predictions of Levy's model using  $\langle c \rangle = 7.5$  are quite good for the 50-GeV/ $c \ \bar{p}p - pp$  data.

Finally, we have considered the duality-diagram model of Eylon and Harari (EH) (Ref. 3) and have fitted the form  $R_n^* = s^{-\alpha} \beta^n$  to our 50 GeV/c data, where  $R_n^*$  is defined in Eq. (1). Here, one expects that  $\beta = \frac{3}{2}$ , based on quark-duality diagram counting, and  $\alpha = 2\alpha_{M}(0)$  where  $\alpha_{M}(0)$  is the intercept of the leading Regge meson-exchange trajectory at t=0. Our results, shown in Fig. 3, are well fitted by this expression and we obtain fitting parameters  $\beta = 1.49 \pm 0.04$  and  $\alpha = 0.80$  $\pm 0.05$ , with a  $\chi^2$  per degree of freedom = 10.9/4, in remarkable agreement with theoretical expectations<sup>33</sup> and with the well-known value of the intercept of the  $\rho$ -meson trajectory. We emphasize that our  $R_n^*$  is distinct from the  $R_n = \Delta \sigma_n / \sigma_n (pp)$ given by EH, Eq. (21), and used by the authors of Ref. 6 in fitting their 100-GeV data and data at lower energies.<sup>33</sup> Fitting our data for  $R_n$ , as opposed to  $R_n^*$ , we obtain  $\beta = 1.30 \pm 0.04$  and  $\alpha = 0.75 \pm 0.06$ , very close to the values found at 100 GeV. The  $\chi^2$  per degree of freedom is 8.0/4. An  $R_n^*$  analysis of the 100-GeV data gives  $\beta = 1.35$  $\pm 0.04$  and  $\alpha = 0.71 \pm 0.06$  with a  $\chi^2/\text{NDF} = 3.49/5$ . We have also investigated the effect of including a  $\overline{p}p$  nonannihilation term in the theoretical expression for  $R_n^*$  and find in fitting that it contributes to the 4-, 6-, and  $\ge$  8-prong topologies, a fraction equal to 30%, 8%, and less than 2%, respectively. This strengthens our conviction that annihilation dominates the cross-section difference  $\Delta \sigma_n$  in the higher multiplicities.

## V. CONCLUSIONS

Using a smooth pp topological-cross-section parametrization model evaluated at 50 GeV/c and using our 48.9-GeV/c  $\bar{p}p$  data, the resulting dif-

ferences  $\Delta \sigma_n$  yield multiplicity moments in good agreement with the energy dependences indicated by other experiments. In particular,  $\langle n \rangle$  [Fig. 1(a)] rises steadily as a function of ln(s), at a constant difference of  $\sim 2$  units above the pp values; and  $f_2^{-}$  [Fig. 1(b)] is definitely turning upward away from a linear dependence on  $\langle n_{-} \rangle$ . The variable  $\langle n \rangle / D \simeq 2.7$  shows little change between 32 and 100 GeV. The constancy of  $\langle n \rangle / D$  implies a parabolic shape of  $f_2^{-}$  vs  $\langle n_- \rangle$  [Eq. (4)] and follows from KNO scaling (i.e.,  $c_2$  in Table II is approximately constant). The magnitude of  $\langle n \rangle / D$  is determined by the shape of the KNO-scaling curve.<sup>29,14</sup> In this picture, precocious KNO scaling leads to a reasonably good representation of the curve of  $f_2^{-1}$ vs  $\langle n_{-} \rangle$  over the whole measured range of  $\overline{p}p$  interactions. Alternatively, if one interprets these data in terms of a certain single-cluster model of

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- <sup>1</sup>A.S.Carroll, et al., Phys. Lett. <u>61B</u>, 303 (1976); C. Bricman et al., ibid. <u>75B</u>, 1 (1978).
- $^{2}$ J. J. J. Kokkedee, Nuovo Cimento <u>43A</u>, 919 (1966);
- J. Rosner, C. Rebbi, and R. Slansky, Phys. Rev. <u>188</u>, 2367 (1969); P. D. Ting, *ibid*. <u>181</u>, 1942 (1969);
  R. Henzi, A. Kotanski, D. Morgan, and L. Van Hove, Nucl. Phys. <u>B16</u>, 1 (1970); H. Goldberg, Phys. Rev. D <u>6</u>, 2542 (1972); R. E. Hendrick *et al.*, *ibid*. <u>11</u>, 536 (1975).
- <sup>3</sup>Y. Eylon and H. Harari, Nucl. Phys. <u>B80</u>, 349 (1974).
- <sup>4</sup>J. G. Rushbrooke and B. R. Webber, Phys. Rep. <u>44</u>, 1 (1978).
- <sup>5</sup>D. E. Zissa *et al.*, preceding paper, Phys. Rev. D <u>21</u>, 3059 (1980).
- <sup>6</sup>J. G. Rushbrooke *et al.*, Phys. Lett. <u>59B</u>, 303 (1975).
- <sup>7</sup>H. Lee, Phys. Rev. Lett. <u>30</u>, 719 (1973).
- <sup>8</sup>V. V. Ammosov et al., Phys. Lett. <u>42B</u>, 519 (1972).
- <sup>9</sup>P. Slattery, Phys. Rev. D 7, 2073 (1973).
- <sup>10</sup>V. Gupta, Lett. Nuovo Cimento <u>11</u>, 668 (1974).
- <sup>11</sup>K. von Holt *et al.*, Nucl. Phys. <u>B103</u>, 221 (1976).
- <sup>12</sup>E. de Wolf *et al.*, Nucl. Phys. <u>B87</u>, 325 (1975).
- <sup>13</sup>Y. Tomozawa, Phys. Rev. D <u>8</u>, 2138 (1973); <u>8</u>, 2319 (1973).
- <sup>14</sup>J. Frøyland and O. Skontorp, Nucl. Phys. <u>B68</u>, 93 (1974).
- <sup>15</sup>D. Levy, Nucl. Phys. <u>B59</u>, 583 (1973).
- <sup>16</sup>D. R. O. Morrison, CERN Report No. D. Ph. II/PHYS 72-19 (unpublished).
- <sup>17</sup>J. Whitmore, Phys. Rep. <u>10C</u>, 273 (1974).
- <sup>18</sup>J. Whitmore, Phys. Rep. <u>27C</u>, 187 (1976).
- <sup>19</sup>C. P. Wang, Phys. Rev. <u>180</u>, 1463 (1969).
- <sup>20</sup>W. R. Frazer *et al.*, Rev. Mod. Phys. <u>44</u>, 284 (1972).
- <sup>21</sup>B. R. Webber, Phys. Lett. <u>42B</u>, 69 (1972).
- <sup>22</sup>F. Grard et al., Nucl. Phys. <u>B57</u>, 407 (1973).
- <sup>23</sup>P. Suranyi, Phys. Lett. <u>36</u>, 47 (1971); Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972).
- <sup>24</sup>C. Bromberg *et al.*, Phys. Rev. D <u>15</u>, <u>64</u> (1977).

annihilations,<sup>27,28</sup> one must postulate the onset of multiple-cluster formation for energies above about 30 GeV.

Finally, we observe a remarkable agreement with theoretical predictions for  $R_n^* = \Delta \sigma_n / \sigma_{n+2}(pp)$ , an experimental ratio based on a strict application of the counting rules for quark duality diagrams,<sup>3,7</sup> and we thereby find evidence that in the topological-cross-section differences  $\Delta \sigma_n$  the small nonannihilation contribution becomes progressively more negligible as *n* increases.

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- <sup>25</sup>100 GeV/c: C. Bromberg et al., Phys. Rev. Lett. <u>31</u>, 1563 (1973); W. M. Morse et al., Phys. Rev. D <u>15</u>, 66 (1977); 205 GeV/c: S. Barish et al., ibid. <u>9</u>, 2689 (1974); 300 GeV/c: A. Firestone et al., ibid. <u>10</u>, 2080 (1974).
- <sup>26</sup>M. A. Jabiol et al., Nucl. Phys. <u>B127</u>, 365 (1977).
- <sup>27</sup>T. Fields *et al.*, CERN Report No. 74-18 (unpublished), p. 151.
- <sup>28</sup>H. T. Miettinen, CERN Report No. 74-18 (unpublished), p. 405.
- <sup>29</sup>G. Bozóki, E. Gombosi, M. Posch, and L. Vanicsek, Nuovo Cimento <u>64A</u>, 881 (1969).
- <sup>30</sup>A. S. Carroll *et al.*, Phys. Lett. <u>61B</u>, 303 (1976).
- <sup>31</sup>D. S. Ayres et al., Phys. Rev. D 15, 3105 (1977).
- <sup>32</sup>Note that this formula is only valid for Q = 0 reactions; for Q = 2 such as pp, the formula reads  $f_2^{-} = \langle \langle n \rangle / D \rangle^{-2} \langle n_- + 1 \rangle^2 - \langle n_- \rangle$ .
- <sup>33</sup>As EH (Ref. 3) state, the counting rules deal with produced mesons, which for annihilations means all finalstate particles, while for both pp and  $\overline{p}p$  nonannihilation reactions it means all final-state particles excluding the two original baryons. We have assumed that noriginal produced particles corresponds to n (or n+2) observed charged particles for annihilation (or nonannihilations), respectively. Under this assumption, the neglect of produced neutral pions is partly compensated for by the fact that significant numbers of pions of all charges come from higher meson states such as  $\rho$  and  $\omega$ . Ideally, resonance and neutral-particle production should be taken explicitly into account since the model refers to the total number of originally produced mesons. Our model also assumes two final-state protons per event, whereas data<sup>34</sup> indicate an average of ~1.5 protons per event in the final state of 50-GeV/c *pp* interactions. Owing to limitations of presently available experimental techniques, such a detailed analysis cannot be performed.
- <sup>34</sup>M. Antinucci *et al.*, Lett. Nuovo Cimento <u>6</u>, 121 (1973);
   P. Slattery, Phys. Rev. D <u>10</u>, 2304 (1974).