Comments on quantum-mechanical interference due to the Earth's rotation

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We present an elementary derivation of the quantum-mechanical phase shift induced by the Earth's rotation.

In a remarkable experiment using a neutron interferometer, Werner, Staudenmann, and Colella[1] detected a quantum-mechanical interference effect due to the Earth's rotation. As a beam of thermal neutrons is split into two alternative paths and then recombined as in Fig. 1, the observed phase difference in the interference region (D of Fig. 1) is in good agreement with the theoretical prediction

$$\phi_{ACD} - \phi_{ABD} = (2m/\hbar) \int \vec{\omega} \cdot d\vec{\sigma}\,, \tag{1}$$

where the surface integral is over the oriented area of the parallelogram $ACDB$ and $\vec{\omega}$ stands for the angular-velocity vector of the rotating Earth.

Equation (1), or expressions equivalent to it, has been derived by various authors in a number of different ways—optical analogy,[2] general-relativity considerations,[3,4] the WKB approximation,[1] and the Doppler effect of moving media.[5] We wish to present yet another derivation which, because of its simplicity, may be of some interest.

In a rotating frame of reference the effect of rotation is simulated by the Coriolis force

$$\vec{F}_{\text{Cor}} = 2m\vec{v} \times \vec{\omega}\,. \tag{2}$$

This "fictitious" velocity-dependent force is analogous in form to the Lorentz force on a charged particle in the presence of a magnetic field

$$\vec{F}_{\text{Lor}} = (e/c)\ \vec{v} \times \vec{B} = (e/c)\vec{v} \times (\vec{\nabla} \times \vec{A})\,, \tag{3}$$

where $\vec{A}$ stands for the vector potential. Because the quantum mechanics of a charged particle in a magnetic field is well known, we can immediately solve our neutron-interferometer problem just by replacing the magnetic field by the angular-velocity vector as follows:

$$(e/c)\vec{B} \to 2m\vec{\omega}\,. \tag{4}$$

The applicability of this analogy is also apparent if we recall that the canonical momentum in a rotating frame is given by

$$\vec{p} = m\vec{v} + m\vec{\omega} \times \vec{x}\,, \tag{5}$$

to be compared with the well-known expression for the canonical momentum of a charged particle

$$\vec{p} = m\vec{v} + (e/c)\ \vec{A}\,, \tag{6}$$

where $\vec{A}$, for a spatially uniform $\vec{B}$, is just

$$\vec{A} = \tfrac{1}{2}\vec{B} \times \vec{x}\,. \tag{7}$$

When a beam of charged particles in the presence of $\vec{A}$ is split into two alternative paths and then recombined, the resulting phase difference in the interference region is known to be proportional to the magnetic flux enclosed by the two paths[6]:

$$\phi_{ACD} - \phi_{ABD} = (e/\hbar c) \oint \vec{A} \cdot d\vec{s} = (e/\hbar c) \int \vec{B} \cdot d\vec{\sigma}\,. \tag{8}$$

The substitution (4) applied to (8) immediately leads to the desired result (1).[7] Notice that our derivation makes it clear that (1) is valid for a loop of arbitrary shape.

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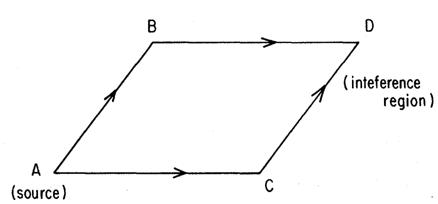

FIG. 1. Alternative paths in a neutron interferometer.

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[7]It ought to be mentioned that Stodolsky (Ref. 4) also comments on a comparison between a massive particle subjected to the Coriolis force and a charged particle subjected to the vector potential $\vec{A}$. It is, however, gratifying from a pedagogical point of view that one need not be familiar with the structure of metric tensor in a rotating frame to be able to arrive at the simple result (1). He correctly points out that the analogy between (1) and (8) holds only in the context of nonrelativistic quantum mechanics.