PHYSICAL REVIEW D VOLUME 21, NUMBER 1 1 JANUARY 1980

Mixing of gluon bound states and quark-antiquark states

Richard H. Capps
Physics Department, Purdue University, West Lafayette, Indiana 47907
(Received 26 June 1979)

The possibility is investigated that some physical mesons contain appreciable components both of $q\bar{q}$ (quark-antiquark) states and of gluon bound states. Experimental data suggest this type of mixing both for pseudoscalar mesons and scalar mesons. Since the interactions of gluon bound states with $q\bar{q}$ states are not known accurately theoretically, no definitive test of this type of mixing can be given at present. However, a consistency test for gluon-bound-state mixing is given and shown to be satisfied by a recent analysis of the pseudoscalar-meson interactions. An extension of this analysis suggests that the $\eta(549)$ has an appreciable gluon-bound-state component while the $\eta'(958)$ does not. The scalar-meson data are anomalous unless there is appreciable mixing of the lightest $q\bar{q}$ states with some other kind of state. The masses of the light scalar and pseudoscalar gluon bound states that are suggested by the data are in the regions expected. Some differences between the $q\bar{q}$ and $qq\bar{q}\bar{q}$ models of the scalar mesons are discussed. Better data concerning scalar mesons are needed, but some significant conclusions may be drawn from the limited data available at present.

I. INTRODUCTION

In the usual quark theory of hadrons, the quarks are held together by vector gluons, the gluons corresponding to the octet of the color-SU(3) group. The requirement that all observed hadrons are color singlets prevents the existence of an isolated gluon or quark; however, color-singlet gluon bound states should exist. The identification of physical gluon bound states is a key problem of hadron physics today, since such states would be the most direct confirmation possible of the existence of gluons. The gluon bound states are flavor singlets, and must be identified through their interactions with ordinary hadrons (quark composites) and photons.

The possibility that certain observed resonances are gluon bound states has been discussed in the literature; for example, Robson has suggested that the scalar meson $S^*(980)$ is such a state.[1] Unfortunately, it is difficult to distinguish between a gluon bound state and a flavor-singlet hadron made of quarks.

Recently, I have suggested that in some cases, the mixing between gluon bound states and quark composites may be large, so that some mesons may have quark-state and gluon-state components of comparable amplitudes.[2] It was shown in Ref. 2 that experimental data involving the $\eta(549\text{ MeV})$ and $\eta'(958)$ suggest that these two pseudoscalar mesons contain appreciable parts that interact weakly with mesons. These inert components may be gluon bound states. The purpose of this paper is to study the possibility that this type of strong gluon-state–quark-state mixing occurs for various meson multiplets, including other multiplets beside the pseudoscalar-meson multiplet. One of the main conclusions is that the scalar mesons $S^*(980)$ and $\epsilon(1300)$ may have appreciable gluon-state components. The most striking evidence concerning the scalar mesons is discussed in Sec. II B.

In the analysis of this paper the "signature" of a gluon bound state is a weak interaction with a meson-meson state. In the absence of a precise theory one cannot differentiate conclusively between gluon bound states and other hadron states that are inert in the same way. However, a consistency condition for the hypothesis that an inert state is a gluon bound state is introduced and discussed in Sec. III. This condition is applied to the pseudoscalar- and scalar-meson multiplets in Secs. IV A and IV B.

One of the fundamental predictions of quantum chromodynamics is that gluon bound states interact more strongly with light $q\bar{q}$ states that with heavy $q\bar{q}$ states. Consequently, we neglect states containing charmed or other heavy quarks.

II. THE GLUON-STATE MIXING PROBLEM

A. The various meson multiplets

In this section we discuss the general problem of quark-state–gluon-state mixing, as it applies to meson multiplets of various spins and parities. In any $q\bar{q}$, SU(3) meson nonet there are two isoscalar states, with the quark structures,

$$\psi_u = (u\bar{u} + d\bar{d})/\sqrt{2}\,, \tag{1a}$$

$$\psi_s = s\bar{s}\,, \tag{1b}$$

where u, d, and s denote the up, down, and strange quarks. For certain spin-parity combinations, the diagram of Fig. 1(a), connecting a two-gluon

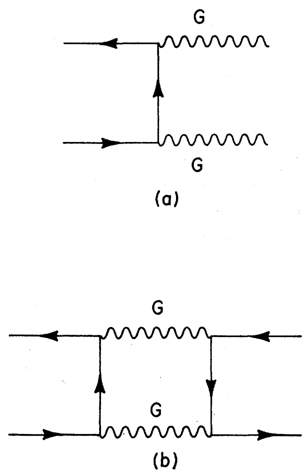

FIG. 1. (a) Coupling of a two-gluon state to a quark-antiquark state; (b) A "virtual annihilation" graph.

state to a $q\bar{q}$ state, need not be weak. In fact, it is commonly believed that the doubling of this diagram [Fig. 1(b)] is responsible for the observed strong mixing of the ψ_u and ψ_s states in the pseudoscalar meson η and η'. In almost all of this paper I make the assumption that the Okubo-Zweig-Iizuka (OZI) rule holds for the couplings of ψ_u and ψ_s states to other hadron states.[3] [At the end of Sec. II B a brief discussion is given of the possibility of fitting the $S^*(980)$, $\epsilon(1300)$, $\delta(980)$, and $\kappa(1400)$ into a scalar nonet if the OZI rule is discarded.]

If gluon bound states of the appropriate spin and parity exist in the same mass region as the $q\bar{q}$ states, then the interaction of Fig. 1(a) may lead to mixing of these gluon states with ψ_u and ψ_s. If this occurs, then the multiplet contains more than two isoscalar states. However, the presence of extra isoscalar states in a given mass region is not convincing evidence for a gluon bound state, since the extra states may belong to other quark-state multiplets whose other members are of very different mass. On the other hand, the apparent absence today of such extra scalar states is not strong evidence against gluon-state–quark-state mixing, since isoscalar states are usually hard to detect and identify.

The pseudoscalar (P) meson case is discussed in Ref. 2. It is postulated there that the physical particles $\eta(549)$, $\eta'(958)$, and $E(1416)$ are orthogonal combinations of the quark states ψ_u, ψ_s, and one gluon bound state ψ_0. I define the gluon-state mass M_{00} to be the expectation value of the mass matrix in the state ψ_0. In this model, M_{00} must be less than 1416 MeV; in Sec. IV of the present paper we estimate M_{00} to be around 1100 ± 100 MeV.

Appreciable gluon-state–quark-state mixing may occur in other meson states of spins and parities such that the two-gluon component of the gluon bound state exists. Two obvious possibilities are the j^P combinations 2^+ and 0^+. In these even-parity cases, the situation is opposite to that of the P mesons, in that the $q\bar{q}$ state cannot have zero orbital angular momentum, while the two-gluon state can have zero orbital angular momentum. Consequently, one expects that the masses of the lightest 2^+ and 0^+ gluon bound states to be less than 1100 MeV, perhaps less than the corresponding $q\bar{q}$ states. This estimate corresponds to one obtained from the bag model, for which the lightest gluon bound states of spin-parities 1^-, 2^+, and 0^+ are 1290, 960, and 960 MeV.[4]

We first consider the 2^+ meson possibility. Analyses of the PP decays of the 2^+ nonet show no evidence of inert components of the $f(1270)$ or $f'(1515)$.[5] Furthermore, the decays indicate that these two mesons correspond closely to the quark states ψ_u and ψ_s, respectively. We conclude that if a $j^P = 2^+$ gluon bound state exists, it does not couple strongly to the lightest quark states. A possible reason for this is that the f and f' are fairly massive, since the asymptotic-freedom argument of quantum chromodynamics (QCD) implies that the coupling of a gluon bound state to $q\bar{q}$ states is smaller for heavier $q\bar{q}$ states.

B. The scalar-meson multiplet

In this section, I consider the scalar multiplet. A nonet of 0^+ mesons has been identified with reasonable certainty, composed of the strange particle $\kappa(1400)$, the isovector meson $\delta(980)$, and the two isoscalar mesons $S^*(980)$ and $\epsilon(1300)$. The experimental data concerning scalar particles are sparse and do not exclude the existence of other isoscalar, 0^+ particles in the same mass range.[6]

It can be seen easily that if this nonet is a complete multiplet, it is unusual. In each of the vector and tensor nonets (for which there seems to not be strong mixing of any gluon states) the K^* mass is intermediate between those of the two isoscalar mesons. This is as expected in a simple quark model, since the two isoscalar states are mixtures of the light state ψ_u and the heavy state ψ_s, while the strange meson is composed of one light quark and one strange quark. In the P-meson nonet the K is lighter than the two isoscalar mesons, but heavier than the isovector meson. However, in the scalar-meson nonet discussed here, the κ is heavier than all the nonstrange mesons. The possibility that the two 0^+, $q\bar{q}$ states are mixed with a light gluon bound state suggests itself immediately.

An alternate way of stating the scalar multiplet anomaly is to use the Gell-Mann–Okubo mass formula.[7] If one assumes arbitrary octet-singlet mixing between the two isoscalar members of a meson nonet, the mass formula is an inequality. This inequality cannot be satisfied if the strange meson has the largest or smallest mass of all the particles in the nonet.[8] This conclusion does not depend on whether one uses masses or the squares of masses.

We now discuss the scalar-meson nonet quantitatively. I use the symbol F_{ri} to denote the amplitude of the coupling of the scalar meson r to the PP state i. If r decays into i, the decay partial width Γ_{ri} is given by the formula,

$$\Gamma_{ri}=F_{ri}{}^2k_{ri}\,, \tag{2}$$

where k_{ri} is the magnitude of the three-momentum of one of the P mesons in the rest system of the scalar meson. The F_{ri} are assumed to satisfy SU(3) symmetry. An overall coupling constant G^2 may be determined from either the $\kappa\to\pi K$ or $\delta\to K\overline{K}$ decays, i.e.,

$$F_{\kappa\pi K}{}^2=\tfrac{9}{10}\,G^2\,, \tag{3}$$

$$F_{\delta K\overline{K}}{}^2=\tfrac{3}{5}\,G^2\,. \tag{4}$$

If the OZI rule is assumed, the F for the $\delta\pi\eta$ is given by[9]

$$F_{\delta\pi\eta}{}^2=\tfrac{6}{5}\,G^2a_{\alpha u}{}^2\,, \tag{5}$$

where $a_{\alpha u}$ is the amplitude of the quark state ψ_u in the wave function for the η_α (549 MeV) meson.

I assume that there are n scalar gluon bound states that are mixed appreciably with the S^* and ϵ. (It is unlikely that n is greater than 2.) There are then $n+2$ physical, isoscalar, scalar mesons in the multiplet. The wave function χ_j for the meson j may be written in the form

$$\chi_j=A_{ju}\psi_u+A_{js}\psi_s+\sum_{i=1}^{n}A_{ji}\psi_{0i}\,, \tag{6}$$

where ψ_{0i} is the ith gluon bound state, and the A are coefficients of an orthogonal matrix of rank $n+2$. (When discussing the mixing of pseudoscalar gluon bound states, we will use a_{kl} rather than A_{kl} for the elements of the matrix.)

The phase relation between ψ_u and ψ_s is chosen so that the SU(3) singlet $q\overline{q}$ state is

$$\psi_{\rm sing}=(\tfrac{2}{3})^{1/2}\psi_u+(\tfrac{1}{3})^{1/2}\psi_s\,. \tag{7}$$

If the OZI rule and SU(3) are assumed, the coupling constants F_{ji} for the decay of the scalar meson j into $\pi\pi$ and $K\overline{K}$ states are given by[9]

$$F_{j\pi\pi}=(\tfrac{9}{5})^{1/2}GA_{ju}\,, \tag{8}$$

$$F_{jK\overline{K}}=(\tfrac{3}{5})^{1/2}G(A_{ju}+\sqrt{2}A_{js})\,. \tag{9}$$

Since the A matrix is orthogonal, the sums over scalar mesons of the coupling strengths F^2 satisfy the relations

$$\sum_j F_{j\pi\pi}{}^2=\sum_j F_{jK\overline{K}}{}^2=\tfrac{9}{5}\,G^2\,. \tag{10}$$

I will use the $\kappa\to\pi K$ decay and Eq. (3) to estimate G^2. If the κ mass and partial width are taken as 1430 ± 30 MeV and 225 ± 75 MeV, the result is[6]

$$G^2=0.403\pm0.13\,. \tag{11}$$

This number may be checked with the data on the $\delta\to\pi\eta$ and $\delta\to K\overline{K}$ decays. The experimental value 51 ± 4 MeV for the $\delta\to\pi\eta$ partial width,[10] together with the above G^2 and Eqs. (2) and (5), lead to the value

$$a_{\alpha u}{}^2=0.33\pm0.11\,, \tag{12}$$

where $a_{\alpha u}{}^2$ is the probability of the quark state ψ_u in the wave function of the $\eta_\alpha(549)$. This is consistent with the range 0.33–0.55 given for this parameter in Ref. 2. The $\delta\to K\overline{K}$ coupling is difficult to measure, because the $K\overline{K}$ threshold is in the middle of the δ resonance. However, the recent analysis of Ref. 10 is consistent with our G^2. This can be seen from the following argument. It is pointed out in Ref. 10 that the $K\overline{K}/\pi\eta$ coupling ratio of the δ is consistent with SU(3) and the assignment of a pure-octet quark state for the η_α. The $a_{\alpha u}{}^2$ for such an assignment is $\frac{1}{3}$, a value consistent with Eq. (12).

Since Eq. (11) is consistent with both the $\delta\to\pi\eta$ and $\delta\to K\overline{K}$ decays, we use this value of G^2 to determine the quark-state probabilities of the S^*. We denote the S^* and ϵ by S_α and S_B, respectively. If we take the S^* width to be 40 ± 10 MeV, and assume a 100% branching ratio for the $\pi\pi$ decay state, the value of $A_{\alpha u}{}^2$ from Eqs. (2) and (8) is small, i.e.,[6]

$$A_{\alpha u}{}^2=0.12\pm0.05\,. \tag{13}$$

We define $A_{\alpha u}$ to be positive.

It is more difficult to obtain $A_{\alpha s}$, since the S^* straddles the $K\overline{K}$ threshold. One analysis has given a value of 4 for the ratio $R=(F_{S^*K\overline{K}}/K_{S^*\pi\pi})^2$,[11] while Cason *et al.* have estimated that $R=\frac{2}{3}$.[12] Morgan has argued that interference of the S wave with the D wave corresponding to the f resonance in the $\pi\pi\to K\overline{K}$ amplitude favors a positive sign for $F_{S^*K\overline{K}}/F_{S^*\pi\pi}$.[13] If this sign is taken to be positive and Eqs. (8) and (9) are used, then $R=4$ leads to $A_{\alpha s}=0.60$ while $R=\frac{2}{3}$ leads to $A_{\alpha s}=0.10$. We shall assume only that

$$0.10<A_{\alpha s}<0.60\,. \tag{14}$$

Combination of Eqs. (13) and (14) indicates that $A_{\alpha u}{}^2+A_{\alpha s}{}^2$ is significantly less than one. This is evidence for a gluon-bound-state component in the S^*. However, this conclusion is not compelling at present, because of the difficulty in the measurement of the $S^*K\bar{K}$ coupling constant. On the other hand, even if the evidence concerning $A_{\alpha s}^2$ is ignored, the value $A_{\alpha u}^2 \sim 0.12$ is enough to show that the S^* is anomalous if the scalar mesons form only a nonet. In every other established nonet (the vector, tensor, and pseudoscalar nonets) the probability of the light-quark component ψ_u in the lighter isoscalar meson is large, larger than the corresponding probability for the heavier isoscalar meson.

Another argument favoring gluon-state mixing for scalar mesons may be obtained by considering only the ψ_u components of the S^* and ϵ. If there is no such mixing, then the G value of Eq. (11) and the sum rule of Eq. (10) imply that the $\pi\pi$ partial width of the ϵ is given by

$$\Gamma_{\epsilon\pi\pi} = (405 \pm 140)\ \text{MeV}. \qquad (15)$$

This partial width is not known accurately experimentally, but the total ϵ width appears to be less than 400 MeV,[6] while Pawlicki *et al.*[14] have given evidence that the ϵ couples strongly to the $K\bar{K}$ channel. Thus the $\Gamma_{\epsilon\pi\pi}$ value of Eq. (15) is unlikely, suggesting the admixture of another state with ψ_u and ψ_s.

The OZI rule was assumed in the preceding analysis. If this rule is relaxed, then the ratio of the couplings of the singlet and octet scalar states to PP states becomes a free parameter. If only the $\kappa \to \pi\kappa$, $\delta \to \pi\eta$, and $S^* \to \pi\pi$ partial widths are considered well enough known experimentally to be used in an analysis, then the assumption that the δ, κ, S^*, and ϵ form a nonet with SU(3)-symmetric couplings is consistent with the data; furthermore the $S^* \to \pi\pi$ width does not determine the probability $A_{\alpha u}^2$. The recent SU(3) analysis of the S^* and ϵ by Cason *et al.* would not be consistent if the κ were also included.[12] The PP couplings of the octet scalar states in the analysis of Ref. 12 would imply a $\kappa \to \pi K$ partial width of ~5 MeV, in contrast to the experimental value of ~225 MeV.[6]

Several authors have proposed that the scalar mesons discussed here are $qq\bar{q}\bar{q}$ states rather than $q\bar{q}$ states.[4,15] The four-quark model has an important feature in common with the two-quark model, i.e., the expected flavor multiplets do not include SU(3) singlets isolated in energy. The expected $qq\bar{q}\bar{q}$ multiplets are either SU(3) nonets or larger multiplets. (We continue to ignore mesons containing quarks heavier than the strange quark.) Therefore, the clearest possible evidence for gluon-bound-state mixing is the same in the two-quark and four-quark models—the presence of extra isoscalar, nonstrange mesons in the multiplets.

One obvious test of the four-quark model is the possible future identification of many predicted mesons of various spins and parities. However, even if we consider only the decays of members of the scalar nonet, there is an important difference between the two-quark and four-quark models. Some of the predictions of this paper depend on the use of the OZI rule. When combined with SU(3) symmetry, this rule implies specific ratios of the constants of interaction of the octet and singlet in a nonet.[16] The singlet-octet interaction ratios may be specified in a $qq\bar{q}\bar{q}$ model also, if some appropriate dynamical assumption is made. In general the ratios specified in a $qq\bar{q}\bar{q}$ model are not the same as those resulting from the OZI rule in the $q\bar{q}$ model. We illustrate this point by considering a scalar nonet that is formed from a diquark in the flavor-SU(3) representation $\underline{3}^*$ and an antidiquark in the representation $\underline{3}$. If the δ^+ meson is in such a nonet, its quark structure is $us\bar{d}\bar{s}$. In this model it is reasonable to assume that the $\delta^+ \to \pi^+\eta_\alpha$ decay proceeds entirely through the ψ_s component of the η_α, whereas in our two-quark model the decay goes through the ψ_u component. I will refer to this particular model (in which the $\delta \to \pi\eta_\alpha$ decay proceeds through ψ_s) as the "alternate model". In the alternate model Eq. (5) must be replaced by $F_{\delta\pi\eta}^2 = \frac{3}{5} G^2 a_{\alpha s}^2$, and $a_{\alpha s}^2 = 0.33 \pm 0.11$ [Eq. (12)] must be replaced by

$$a_{\alpha s}^2 = 0.66 \pm 0.22\,. \qquad (16)$$

I have pointed out earlier in this section that $a_{\alpha u}^2 \approx 0.33$ is consistent with the value obtained from Ref. 2. However, in one common picture the η_α is a nearly pure $q\bar{q}$ state that is close to an octet state. If this picture is correct both Eqs. (12) and (16) are satisfied, and the $\delta \to \pi\eta_\alpha$ decay does not distinguish between the $q\bar{q}$ and alternate models of the δ.

We conclude that in the future various PP partial widths of the scalar mesons involving either isoscalar, P mesons or isoscalar, scalar mesons may help distinguish between $q\bar{q}$ and $qq\bar{q}\bar{q}$ models of the scalar mesons.

III. CONSISTENCY RELATIONS AND EULER ANGLES

In this section we discuss two types of additional conditions that are useful in discussing strong mixing between inert states and quark states. The first type is useful for studying the likelihood that the inert states are gluon bound states. The second type occurs when there is only one mixed, inert state. These conditions are applied to the P mesons in Sec. IV A and to the scalar mesons in Sec. IV B.

The physical states are the eigenfunctions of the mass matrix (eigenfunctions of the Hamiltonian in the center-of-mass system). The elements M_{ij} of the mass matrix in the representation of ψ_u, ψ_s, and the gluon bound states may be obtained from the equation

$$M_{ij} = \sum_\nu A_{\nu i} A_{\nu j} m_\nu , \qquad (17)$$

where m_ν is the mass of the physical meson ν. (In the P-meson case one replaces the $A_{\nu k}$ by $a_{\nu k}$.) This relation is useful if enough is known about the A coefficients. For example, we hope to use it to test the argument (of Sec. II A) that the quark-state–gluon-state mass ratios M_{uu}/M_{00} and M_{ss}/M_{00} should be smaller for P mesons than for scalar mesons.

If gluon bound states are responsible for the ψ_u-ψ_s mixing, as in Fig. 1(b), then the mixing terms in the complete Hamiltonian are of the form H_{ui} and H_{si}, where i is a gluon bound state. The ψ_u-ψ_s mixing is of second order. One expects that the gluon states that are most important for the mixing are the lighter states that are included in the mass matrix. In this case M_{us} should be smaller than M_{u0} and M_{s0} for at least one gluon state 0. Furthermore, in QCD gluon states are expected to be coupled more strongly to light $q\bar{q}$ states than to heavier $q\bar{q}$ states. If this is combined with the preceding condition, the result is,

$$|M_{us}| < |M_{s0}| < |M_{u0}| , \qquad (18)$$

for some gluon state 0. This is a consistency test for the hypothesis that gluon-state mixing is the main cause of breaking of the OZI rule. If the mixing is small, the condition that $|M_{us}|$ is smaller than $|M_{s0}|$ and $|M_{u0}|$ leads to the condition proposed in Ref. 2 [Eq. (5) of Ref. 2]. However, the above formulation is to be preferred, because the mixing may not be small and because Eq. (18) above also contains the condition $|M_{s0}| < |M_{u0}|$.

Additional simplification results, if, for a given spin and parity, there is only one gluon bound state that is mixed with the quark states ψ_u and ψ_s. In this case the nine coefficients A_{ij} (or a_{ij}) depend on only three Euler angles, θ_1, θ_2, and θ_3. The transformation matrix may be written,

$$\begin{array}{c} \\ \alpha \\ \beta \\ \gamma \end{array} \begin{array}{c} \begin{array}{ccc} u & s & 0 \end{array} \\ \left[\begin{array}{ccc} C_1 & -S_1C_2 & S_1S_2 \\ S_1C_3 & -S_2S_3 + C_1C_2C_3 & -C_2S_3 - C_1S_2C_3 \\ S_1S_3 & S_2C_3 + C_1C_2S_3 & C_2C_3 - C_1S_2S_3 \end{array} \right] \end{array} , \qquad (19)$$

where C_i and S_i denote $\cos\theta_i$ and $\sin\theta_i$.

The only meaningful relative phase relating two rows or two columns is that relating ψ_u and ψ_s; this phase is defined by Eq. (7). The other signs may be chosen such that

$$(C_1, C_2, C_3, \text{ and } S_2) > 0 . \qquad (20)$$

IV. APPLICATION TO SPIN-ZERO MULTIPLETS

A. The pseudoscalar multiplet

In this section, I apply the results of Sec. III to the P-meson multiplet. The η_α and η_β are identified with the $\eta(549)$ and $\eta'(958)$, respectively; the possible identity of the η_γ will be discussed later. I use the Euler angles and phase conventions of Eqs. (19) and (20). In Ref. 2 various experimental data involving the η and η' were used to estimate $a_{\alpha u}$, $a_{\alpha s}$, $\alpha_{\beta u}$, and $\alpha_{\beta s}$; the results are summarized in Table II of Ref. 2. The results of this table require that the sign of the ratio $a_{\alpha s}/a_{\alpha u}$ is negative. Our phase conventions then imply that S_1 and $a_{\beta u}$ are positive. The principal results of this table may then be written,

$$0.57 < a_{\alpha u} < 0.74 , \quad a_{\alpha s} \sim -0.45 , \qquad (21)$$
$$0.5 < a_{\beta u}/a_{\alpha u} < 0.7 .$$

The approximate value $a_{\alpha s} \sim 0.45$ results from the width of the $\phi \to \eta_\alpha \gamma$ decay and consideration of the dip at about $t = -0.4$ (GeV/c)^2 in the $K^- p \to \eta_\alpha n$ cross sections.[2,17]

Any choice of the three input parameters in the ranges of Eq. (21) determines two possible solutions for the Euler angles, corresponding to the two possible signs of S_3. However, a positive choice of S_3 leads to a negative $a_{\beta s}/a_{\beta u}$ ratio, in contradiction to the results of Ref. 2. Thus, we take S_3 to be negative. The range of solutions for the range of parameters of Eq. (21) is shown in column 1 of Table I, while column 2 corresponds to the mean values of $a_{\alpha u}{}^2$ and $a_{\beta u}{}^2$ in this range. Column 3 corresponds to a larger $|a_{\alpha s}|$, i.e.,

TABLE I. Parameters resulting from specific choices of $a_{\alpha u}$, $a_{\beta u}$, and $a_{\alpha s}$. The M_{ij} values are in units of MeV/c^2.

Parameter	1	2	3
$a_{\alpha u}$	0.57–0.74	0.66	0.66
$a_{\beta u}/a_{\alpha u}$	0.50–0.71	0.61	0.61
$a_{\alpha s}$	−0.45	−0.45	−0.66
$a_{\beta s}$	0.85–0.89	0.89	0.71
$a_{\gamma 8}{}^2$	0.10–0.28	0.20	0.03
M_{us}	58–135	93	249
M_{u0}	(−317)–(−387)	−384	−306
M_{s0}	92–200	147	12
M_{uu}	811–1091	958	958
M_{ss}	875–910	882	805
M_{00}	953–1199	1083	1159

to $a_{\alpha s} = -a_{\alpha u}$. This column is shown because the value $a_{\alpha s} \approx -a_{\alpha u}$ is obtained in some analyses of the η-η' system that do not include the possibility of inert components.[18] This value of $a_{\alpha s}$ does not correspond to the analysis of Ref. 2.

The parameter $a_{\gamma 8}$ is a measure of the coupling of the n_γ to the octet quark-antiquark state, and is given by

$$a_{\gamma 8} = (\tfrac{1}{3})^{1/2} a_{\gamma u} - (\tfrac{2}{3})^{1/2} a_{\gamma s} . \qquad (22)$$

It is pointed out in Ref. 2 that if the η_γ is identified with the 1416 MeV E meson, the $(K^*\overline{K} + \overline{K}^*K)$ partial width of the E is a measure of $a_{\gamma 8}$.[2] The experimental partial width of ~12 MeV leads to the result $a_{\gamma 8}^2 \sim 0.22$.[19] This number is consistent with the amplitudes of columns 1 and 2 of Table I.[20]

One of the most striking results of Table I is that for all amplitudes in the range of Eq. (21) [column 1 of the table], the calculated amplitude $a_{\beta s}$ is very large. No experimental numbers exist that can be used to determine $a_{\beta s}$ accurately. However, it has been shown by Marzano *et al.* that at a laboratory momentum of 4.2 GeV/c, the differential $K^- p \to n_\beta \Lambda$ cross section is significantly larger than the corresponding $K^- p \to n_\alpha \Lambda$ cross section in the range of momentum-transfer squared $t' < 0.6$ (GeV/c)^2, where t' is $|t - t(0°)|$.[17] It is pointed out in Ref. 2 that it is difficult to understand this phenomenon unless $|a_{\beta s}|$ is large.

For the parameters of column 2 and for almost all the range of column 1, the gluon-state mixing consistency conditions of Eq. (18) are satisfied. In all these cases $|M_{u0}|$ is large. The only part of the column 1 range for which $|M_{us}| > |M_{s0}|$ corresponds to $a_{\alpha u}$ near its upper limit and $a_{\beta u}/a_{\alpha u}$ near its lower limit. The very small value $M_{us} = 58$ MeV corresponds to $a_{\alpha u} = 0.57$ and $a_{\beta u}/a_{\alpha u} = 0.70$.

The calculated range of the "gluon-state mass" M_{00} (discussed in Sec. II A) is reasonable. In column 1 the smaller and larger values for M_{00} correspond, respectively, to smaller and larger values of $a_{\alpha u}$. The ψ_u mass M_{uu} seems large. However, in this mixing model all the expectation values M_{uu}, M_{ss}, and M_{00} must lie between the experimental masses of the $\eta_\alpha(549)$ and $\eta_\gamma(1416)$. Thus, the large M_{uu} results in part from the large experimental mass of the η_α; the η_α mass is not understood theoretically.

It is interesting that the results of this section correspond to a large inert compound in the η_α, but not in the η_β. The numbers of Eq. (21) lead to the result

$$0.25 < a_{\alpha 0}^2 < 0.48 ,$$

while the amplitudes of column 1 of Table I lead to the corresponding η_β result

$$0 < a_{\beta 0}^2 < 0.12 .$$

The conclusion that $a_{\alpha 0}^2$ is appreciable is different from the conclusions of some other analyses of η particles; a more usual conclusion is that $a_{\alpha 0}^2$ is small while η_β may have an appreciable inert part.[21,22] Therefore I will discuss here the origin of the differences between my result and the more conventional result of Refs. 21 and 22.

Most of the available experimental data have to do with hadrons that contain only up and down quarks and antiquarks. Therefore $a_{\alpha u}$ and $a_{\beta u}$ are known much better than $a_{\alpha s}$ and $a_{\beta s}$. The average value of $a_{\alpha u}$ used here and in Ref. 2 is close to the values given in Refs. 21 and 22. The range of $a_{\alpha u}$ used here and in Ref. 2 is taken from the analysis of Okubo and Jagannathan[23]; the $a_{\beta u}$ of Ref. 22 is in this range.

On the other hand, the $a_{\alpha s}$ of Eq. (21) is very different from the values of Refs. 21 and 22. In these references this parameter is determined primarily from $a_{\alpha u}$ and the Gell-Mann–Okubo mass formula.[7] When written in terms of the squares of the masses, this formula is

$$M_8^2 = \langle \psi_8 | M^2 | \psi_8 \rangle = \tfrac{1}{3}(4M_K^2 - M_\pi^2) = (567\ \text{MeV})^2 , \qquad (23)$$

where ψ_8 is the octet $q\overline{q}$ state

$$\psi_8 = (\tfrac{1}{3})^{1/2}\psi_u - (\tfrac{2}{3})^{1/2}\psi_s .$$

Presumably M_8 is much smaller than either the corresponding singlet mass or the masses of any other states that may be mixed in the physical η's. Since M_8 is close to the physical mass of the η_α, it is not surprising that when the mass formula is imposed the resulting predictet $q\overline{q}$ octet probability of the η_α is close to unity. This forces the probabilities for all states other than ψ_u and ψ_s to be small.

In contrast to this, our analysis (Ref. 2) does not use the mass formula. The mass formula is ignored because in the P-meson multiplet the SU(3) mass splitting is comparable in size to the average mass. Therefore, there is no good reason to assume that any mass-splitting effect is present only to low orders. This difficulty is illustrated by the fact that quite different results are obtained if the formula is applied to the masses rather than to the squares of masses.

Unfortunately, the mass formula does not predict the mass of any member of a nonet from those of all the others, so the validity of the formula is not tested easily.

If the mass formula is not used, then $a_{\alpha s}$ must be determined directly from experiment. In Ref. 2, $a_{\alpha s}$ is determined from the $\phi \to \eta_\alpha \gamma$ partial width, and the result is shown to be consistent with the presence of a dip in the $K^-p \to \eta_\alpha \Lambda$ forward differential cross section.[17] However, this determination is not conclusive, since the simple model used for the $P\gamma$ decays of vector mesons in Ref. 2 does not fit the presently accepted value of the $K^{0*} \to K^0\gamma$ partial width; the $\rho \to \pi\gamma$ width also does not fit if the photon has no SU(3)-singlet part.[24]

Another phenomenon that depends on $a_{\alpha s}$ is the $\gamma\gamma$ partial width of the η_α; the appropriate expression is Eq. (20) of Ref. 2. If $\Gamma(\pi_0 \to \gamma\gamma)$ is 7.86 eV, this expression may be written for any η in the form

$$\Gamma(\eta_i \to \gamma\gamma) = (8.88\ \text{keV}) M_i c^2/1\ \text{GeV})^3 (a_{iu} + \tfrac{1}{5}\sqrt{2}\, a_{is})^2 . \quad (24)$$

Unfortunately, the coefficient of a_{is} is not large, so this equation is useful for determining a_{is} only if a_{iu} is known fairly accurately. Thus, if $\Gamma(\eta_\alpha \to \gamma\gamma) = 324$ eV and $a_{\alpha u}$ is taken to be in the range of Eq. (21), Eq. (24) leads only to the equation $a_{\alpha s} < -0.36$. However, Eq. (24) would be very useful if $a_{\alpha u}$ were known more accurately.

We next consider the $\eta_\beta \to \gamma\gamma$ width. The amplitudes of column 1 of Table I, when substituted into Eq. (24), lead to $\Gamma(\eta_\beta \to \gamma\gamma)$ in the range 2.2–4.6 keV. This is in agreement with the recent measurement of 5.4 ± 2.1 keV by Binnie *et al.*[25] and the measurement of $5.9 \pm 1.6 \pm 1.2$ keV, by Abrams *et al.*[26] Although the measurements are not accurate enough to be used in Eq. (24) to determine $a_{\beta s}$, it is clear that these results favor a large value of $a_{\beta s}$.

B. The scalar-meson multiplet

We next turn to the scalar-meson multiplet, assuming again that there are three physical isoscalar states that are mixtures of the quark states ψ_u and ψ_s and one gluon bound state ψ_0. We use the Euler angles of Eq. (19), with the particles $S^*(980)$ and $\epsilon(1300)$ identified with the states α and β, respectively. The third physical particle (labeled γ) is assumed to exist, but is not identified with any particular experimental effect. The phase convention is that of Eq. (20).

I take $A_{\alpha u}{}^2$ and the scalar-PP coupling constant G^2 to be given by Eqs. (13) and (11), respectively. The amplitude $A_{\alpha s}$ is taken within the range of Eq. (14). If $A_{\alpha s}$ is at the lower end of this range, i.e., $A_{\alpha s} = 0.10$, then the orthogonality of A implies that $A_{\alpha 0}{}^2 \sim 0.87$. In this case the S^* is nearly a gluon bound state. If the condition that $|M_{us}|$ be small [Eq. (18)] is assumed, this implies either that the two physical, isoscalar, scalar particles are nearly pure ψ_u and ψ_s states, or that they have comparable masses. The first of these possibilities has been suggested by Robson.[27] Because of the evidence of a strong coupling of the $\epsilon(1300)$ with $K\bar{K}$ states, it is unlikely that the ϵ is a nearly pure ψ_u state.[14] On the other hand, if the ϵ is a nearly pure ψ_s state, then our value of G^2 and Eqs. (2), (8), and (9) may be used to predict the following partial widths:

$$\Gamma(\epsilon \to K\bar{K}) \sim 200\ \text{MeV},$$

$$\Gamma(\epsilon \to \pi\pi) = \text{small}.$$

Clearly, better measurements of the ϵ partial widths are necessary.

Next, I consider the possibility that the analysis of Ref. 11 is fairly accurate, in which case $A_{\alpha s}$ may be taken as 0.60, the upper limit of Eq. (14). The wave function for the $S^*(890)$ is then,

$$\psi_\alpha = 0.34\psi_u + 0.60\psi_s - 0.73\psi_0 . \quad (25)$$

We see that in this case the S^* contains appreciable components both of quark states and of a gluon bound state.

If Eq. (25) is used, the Euler angles may be determined if the $\epsilon \to \pi\pi$ partial width is known. Unfortunately, this width is not known at all accurately at present, so we make the weak assumption

$$50 < \Gamma_{\epsilon\pi\pi} < 300\ \text{MeV} . \quad (26)$$

It turns out that this weak assumption leads to significant conclusions. For any assumed $\Gamma_{\epsilon\pi\pi}$ there are two solutions corresponding to a positive or negative value of S_3 in Eq. (19). However, the solution with negative S_3 has some disadvantages. First, it leads to a $\epsilon \to K\bar{K}$ partial width less than 70 MeV for any $\Gamma_{\epsilon\pi\pi}$ in the range of Eq. (26); it appears experimentally that $\Gamma_{\epsilon K\bar{K}}$ is larger than this.[14] (If the input $\epsilon\pi\pi$ width is larger than 150 MeV, the resulting calculated $\epsilon K\bar{K}$ width is very small, less than 25 MeV.) A second disadvantage of a negative S_3 is that $|M_{us}|$ cannot be small unless the third isoscalar meson γ is much heavier than the ϵ. Therefore, I assume that $S_3 > 0$.

We consider the two cases corresponding to the extremes of Eq. (26). If $\Gamma_{\epsilon\pi\pi}$ is 50 MeV, then the ϵ wave function is

$$\psi_\beta = -0.33\psi_u - 0.65\psi_s - 0.69\psi_0 .$$

The $\epsilon \to K\bar{K}$ partial width, computed from Eqs. (2) and (9), is 159 MeV. On the other hand, if $\Gamma_{\epsilon\pi\pi} = 300$ MeV, the calculated ϵ wave function and $K\bar{K}$ partial width are

$$\psi_\beta = -0.81\psi_u - 0.21\psi_s - 0.55\psi_0 ,$$

$$\Gamma_{\epsilon K\bar{K}} = 124\ \text{MeV} .$$

With this solution, and $\Gamma_{\epsilon\pi\pi}$ anywhere in the range of Eq. (26), the condition that $|M_{us}|$ be small would require that the third physical, isoscalar, scalar meson have a mass between those of the S^* and ϵ. Throughout this $\Gamma_{\epsilon\pi\pi}$ range the calculated "gluon-state mass" M_{00} satisfies the equation

$$M_{00} = 1131 \text{ MeV} - A_{\gamma 0}{}^2(m_\epsilon - m_0),$$

where $A_{\gamma 0}{}^2$ satisfies the inequality $A_{\gamma 0}{}^2 < 0.17$. Therefore, if the third isoscalar state is no heavier than the ϵ, M_{00} is less than 1131 MeV. This is in agreement with the general considerations of Sec. II A, although the data on P mesons and scalar mesons are not sufficiently complete to make clear whether or not the scalar gluon-bound-state mass is less than that of the pseudoscalar gluon bound state.

V. CONCLUDING REMARKS

We have postulated that in the pseudoscalar- and scalar-meson multiplets one of more gluon bound states is mixed strongly with the light $q\bar{q}$ states, and that the strongest interaction leading to this mixing is between a gluon bound state and the $q\bar{q}$ state made of nonstrange quarks. A previous analysis of the pseudoscalar mesons is extended and shown to be consistent with this postulate. [This follows because the parameter $|M_{u0}|$ of Eqs. (17) and (18) is large.]

Although the data concerning scalar mesons are not complete, there is evidence that the $S^*(980)$ may have a strong admixture of a gluon bound state. In fact, if the S^*, ϵ, and κ are part or all of an SU(3) multiplet, the multiplet is anomalous if there is not any strong mixing with a gluon bound state or another state with similar properties. Furthermore, in this mixing model the evidence is that the mass parameter of the scalar gluon bound state is not large, and may well be smaller than that of the pseudoscalar gluon bound state. This is in agreement with naive expectations.

In the numerical analysis of this paper and of Ref. 2, it was assumed that the scalar and pseudoscalar gluon bound states are "inert" in the sense that they do not interact with two-boson states. (In the scalar-meson case, the relevant two-boson states are two P states, while in the P-meson case, the relevant two-boson states are a state of a P and a 2^+ meson, a two-photon state, a photon–vector-meson state, and a P-meson–Reggeon state.) Actually, there must be some interaction of the gluon bound state with a two-boson state. If the third isoscalar member of the scalar and P nonets is identified, some of the equations given here provide measures of the deviations from inertness. For example, if the third isoscalar, scalar meson is found and included in the sums of Eq. (10), the interaction of the gluon bound state with a $\pi\pi$ or $K\bar{K}$ state should lead to an excess of the sums over the value $(\frac{9}{5})G^2$. Of course, these deviations may not be measured in this way if they turn out to be smaller than deviations typical of SU(3) breaking.

In the case of the pseudoscalar mesons, it has been suggested that the inert component of the $\eta'(958)$ is a radially excited $q\bar{q}$ state.[28] In the results of Sec. IV A of the present paper the same suggestion could apply to the $\eta(549)$, since the η' does not have an appreciable inert component. At present, one cannot distinguish with certainty between this possibility and the gluon-bound-state possibility. However, it seems unlikely that radially excited states are responsible for the unusual properties of the light scalar mesons, because this would require that the lightest isoscalar, scalar meson mixes with excited states much more than does the lightest isovector, scalar meson.

The model predicts the existence of a third isoscalar, scalar meson in the mass region 800–1400 MeV/c^2. Experimental information on $\pi\pi$ and $K\bar{K}$, S-wave amplitudes in this region will be very important.

ACKNOWLEDGMENT

This work was supported in part by the U.S. Department of Energy.

---

[1]D. Robson, Nucl. Phys. B130, 328 (1977). This paper contains references to previous papers on gluon bound states.
[2]R. H. Capps, Phys. Rev. D 17, 1862 (1978).
[3]S. Okubo, Phys. Lett. 5, 165 (1963); G. Zweig, CERN Report No. 8419/Th 412, 1964 (unpublished); J. Iizuka, K. Okada, and O. Shito, Prog. Theor. Phys. 35, 1061 (1966).
[4]R. L. Jaffe and K. Johnson, Phys. Lett. 60B, 201 (1976).
[5]See, for example, R. H. Capps, Phys. Rev. D 15, 171 (1977).
[6]A compilation of data on scalar mesons is given by the Particle Data Group, Phys. Lett. 75B, 1 (1978). See the discussions on pages 115 and 143 of this reference.
[7]S. Okubo, Prog. Theor. Phys. 27, 949 (1962).
[8]This can be seen from the following argument. The mass formula requires that the K (strange meson) mass is intermediate between those of the isotriplet and the "pure-octet" isosinglet. If an SU(3) singlet meson is introduced with arbitrary mass and arbitrary

mixing with the octet state, this cannot result in both physical isosinglets and the isotriplet being on the same side of the *K*.

[9]Coupling-constant ratios predicted from the OZI rule and SU(3) symmetry are given in Ref. 5. Convenient tables of SU(3) Clebsch-Gordan coefficients are given by P. McNamee, S. J. Chilton, and Frank Chilton, Rev. Mod. Phys. 36, 1005 (1964).

[10]A. C. Irving, Phys. Lett. 70B, 217 (1977).

[11]A. D. Martin, E. N. Ozmutlu, and E. J. Squires, Nucl. Phys. 121, 514 (1977).

[12]N. M. Cason *et al* , Phys. Rev. Lett. 41, 271 (1978).

[13]D. Morgan, Phys. Lett. 51B, 71 (1974).

[14]A. J. Pawlicki *et al* , Phys. Rev. D 15, 3196 (1977).

[15]R. L. Jaffe, Phys. Rev. D 15, 267 (1977); A. W. Hendry and I. Hinchliffe, *ibid.* 18, 3453 (1978).

[16]R. H. Capps, Phys. Rev. D 15, 171 (1977).

[17]F. Marzano *et al*., Nucl. Phys. B123, 203 (1977).

[18]Two types of arguments lead to the relation $a_{\alpha s} \approx -a_{\alpha u}$ First, it should be noted that the best measured of the a parameters is $a_{\alpha u}$. If one takes $a_{\alpha u}{}^2$ to be about $\frac{1}{2}$ [consistent with Eq. (20)] and assumes there is no inert component, then the orthogonality of a implies $a_{\alpha s}{}^2 \approx a_{\alpha u}{}^2$. Second, if there is no inert state, $a_{\alpha s} = -a_{\alpha u}$ corresponds to a singlet-octet mixing angle of about $-10°$, close to the value suggested by the quadratic mass formula.

[19]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[20]Recently, evidence has been reported for another isoscalar, pseudoscalar stata at about 1275 MeV by N. L. Stanton *et al*., Phys. Rev. Lett. 42, 346 (1979). The data are consistent with still another such state at about 1400 MeV. At present it is not clear that the structures of these states are in a quark-gluon model.

[21]N. Fuchs, Phys. Rev. D 14, 1912 (1976).

[22]I. Cohen and H. J. Lipkin, Nucl. Phys. B151, 16 (1979). I refer here to the model discussed in Sec. I–V of this reference.

[23]S. Okubo and K. Jagannathan, Phys. Rev. D 15, 177 (1977).

[24]A recent discussion of the problems involved in obtaining these partial widths from the data is given by A. N. Kamal and G. L. Kane, Phys. Rev. Lett. 43, 551 (1979).

[25]D. M. Binnie *et al*., Phys. Lett. 83B, 141 (1979).

[26]G. S. Abrams *et al*., Phys. Rev. Lett. 43, 477 (1979).

[27]The possibility that the S^* is a gluon bound state is discussed by Robson in Ref. 1.

[28]Harry J. Lipkin, Phys. Lett. 67B, 65 (1977).