Nonpolynomial scalar interactions in four dimensions. I. Quartic interaction perturbed by bounded interaction

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We study the N-component scalar field theory with a general O(N)-symmetric interaction Lagrangian. Performing a suitable reparametrization of the model we demonstrate that it is renormalizable in the limit $N\rightarrow\infty$ if the potential has the form of a quartic interaction perturbed by a bounded interaction. The nth derivative of the perturbing term should vanish at infinity faster than the argument to the power $1-n$. We calculate the effective potential, effective action and derive several Green's functions. Although the technique of performing calculations requires modifications, the main qualitative features of our model are the same as of the ordinary $(1/N)(\Phi^4)_4$ theory. In particular, there is no symmetry breaking. Spectral properties of Green's functions are correct.

I. INTRODUCTION

This is the first of two papers which are a continuation of an earlier study[1,2] of the application of the $1/N$ expansion as a tool for investigating field theories with nonpolynomial interactions which are expandable in a Taylor series in powers of field variables. The scope of Refs. 1 and 2 has been restricted to models in one-, two-, and three-dimensional space-time. It turned out that the main properties of nonpolynomial interactions obtained in the leading order[1] in $1/N$ were the same as those in the ordinary (Ref. 3) $(1/N)\Phi^4$ model. Next-to-leading-order results[2] resemble those found in the standard (Ref. 4) $(1/N)\Phi^6$ model. Up to three dimensions renormalization was particularly simple because in the leading and next-to-leading orders there is no need to perform more than one subtraction in Feynman integrands. In four dimensions, self-closing loops occurring in the leading order require two subtractions. This causes additional difficulties in arranging counterterms into existing constants of the model.

By reparametrizing our model with use of the intermediate collective field (which we introduce in a way which is different from that in Refs. 1–4), we considerably simplify the renormalization procedure and show that arranging counterterms causes no problems, if for large Φ^2/N the interaction Lagrangian (and all its derivatives) does not increase faster than

$$N[\tfrac{1}{2}m_0(\Phi^2/N)+\tfrac{1}{8}g_0(\Phi^2/N)^2+W(\Phi^2/N)]$$

(corresponding derivatives of), where $W^{(n)}(\Phi^2/N)=d^nW/d(\Phi^2/N)^n$ satisfies $W^{(n)}(x)=o(x^{1-n})$ as $x\rightarrow\infty$ for $n\geqslant 2$. Then we derive renormalized expressions for the effective potential and action for the model with the interaction part exactly as above, with coupling of the bounded term relatively weak compared to g_0. As in lower-dimensional space-time the results do not differ much from what was found in the $(1/N)(\Phi^4)_4$ model.[5] The effective potential has two minima, one leading to the symmetric, the other to the asymmetric ground state. For large values of classical fields the potential becomes complex. Symmetry breaking is impossible because the symmetric minimum is always deeper than the asymmetric one. Expansion around the correct ground state does not yield poles for spacelike momenta.

This would be a rather uninteresting result, if the similarity between $(1/N)\Phi^4$ and a general interaction were model independent as was the case in one, two, and three dimensions. In four dimensions this is no longer the case. In the accompanying paper[6] it is shown that drastic changes arise when the interaction is bounded. Then the potential is everywhere real, there is only one minimum, and for the positive sign of coupling constant symmetry breakdown takes place.

II. RENORMALIZATION IN THE LEADING-ORDER APPROXIMATION

Formal similarity of the leading-order results for the model under consideration and of the $(1/N)(\Phi^4)_4$ theory[5] whose careful analysis is already available, allows us to neglect some details of calculations which can be anticipated from the existing findings.

The effective action $\Gamma(\varphi, G)$ is[7]

$$\Gamma(\varphi,G)=I(\varphi)+\tfrac{1}{2}i\hbar\,\mathrm{Tr}\ln G^{-1}+\tfrac{1}{2}i\hbar\,\mathrm{Tr}\mathfrak{D}^{-1}(\varphi)G+\Gamma_2(\varphi,G)\,, \tag{1}$$

where I is the classical action,

$$i\mathfrak{D}^{-1}{}_{ab}(\varphi)=\delta^2I(\varphi)/\delta\varphi_a(x)\delta\varphi_b(y)\,,$$

and the implicit definition of G_{ab} is

$$\delta\Gamma(\varphi, G)/\delta G_{ab}(\varphi; x,y) = 0\,.$$

$\Gamma_2(\varphi, G)$ is the sum of all two-particle irreducible vacuum graphs with propagators G and vertices generated by the Lagrangian whose field arguments Φ^i are shifted by their classical values φ^i. In the leading order the relevant Feynman rules for Γ_2 are generated by the effective interaction Lagrangian

$$L_{\rm int}(\varphi, \Phi) = -N\sum_{k=2}^{\infty}\frac{1}{k!}(\Phi^2/N)^k V_0^{(k)}(\varphi^2/N)$$

so that the relevant $2n$-point functions are[1]

$$-iN(2/N)^k V_0^{(k)}(\varphi^2/N)$$
$$\times(\delta_{a_1a_2}\cdots\delta_{a_{2n-1}a_{2n}} + \text{distinct permutations}).$$

The diagrams which contribute to Γ_2 are presented in Fig. 1 and yield

$$\Gamma_2(\varphi, G) = -N\sum_{k=2}^{\infty}\frac{1}{k!}(\hbar/N)^k\int d^4x[G_{aa}(x,x)]^k \times V_0^{(k)}(\varphi^2/N)\,.$$

Defining

$$G_{ab}(x,y) = N\delta_{ab}\,g(x,y) + O(1)\,,$$

we have to leading order

$$g^{-1}(\varphi^2; x,y) = i[\Box - 2V_0^{(1)}(\varphi^2/N - \hbar g(x,x))]\delta(x-y)$$

and

$$\Gamma(\varphi) = I(\varphi) + \tfrac{1}{2}Ni\hbar\,\mathrm{Tr}\ln g^{-1} - N\int d^4x[V_0(\varphi^2/N + \hbar g) - V_0(\varphi^2/N) - \hbar g V_0^{(1)}(\varphi^2/N + \hbar g)]\,.$$

Introducing

$$\chi = \varphi^2/N + \hbar g\,, \tag{2}$$

we observe that to leading order our model can be equivalently replaced by the "comparison theory"[1,4] with the Lagrangian

$$L(\Phi, \chi_0) = \tfrac{1}{2}(\partial_\mu\Phi)^2 - N[V_0(\chi_0) - (\chi_0 - \Phi^2/N)V_0^{(1)}(\chi_0)]\,, \tag{3}$$

the unrenormalized effective action

$$\Gamma(\varphi, \chi_0) = \int d^4x\{\tfrac{1}{2}(\partial_\mu\varphi)^2 - N[V_0(\chi_0) - (\chi_0 - \varphi^2/N)V_0^{(1)}(\chi_0)]\} + \tfrac{1}{2}Ni\hbar\,\mathrm{Tr}\ln[\Box + 2V_0^{(1)}(\chi_0)]\,, \tag{4}$$

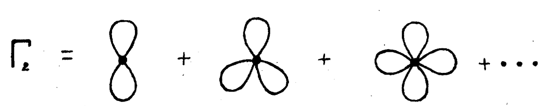

FIG. 1. Leading contributions to Γ_2.

and the unrenormalized effective potential

$$V_{\rm eff}(\varphi, \chi_0) = N[V_0(\chi_0) - (\chi_0 - \varphi^2/N)V_0^{(1)}(\chi_0)] - \tfrac{1}{2}iN\hbar\int\frac{d^4p}{(2\pi)^4}\ln[-p^2 - 2V_0^{(1)}(\chi_0)]\,. \tag{5}$$

Two subtractions are required to renormalize the divergent integral in (5). The first derivative of (5) is

$$\frac{\partial V_{\rm eff}}{\partial\chi_0} = NV_0^{(2)}(\chi_0)\left[\frac{\varphi^2}{N} - \chi_0 + i\hbar\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2 - 2V_0^{(1)}(\chi_0)}\right]. \tag{6}$$

On the physical orbit defined by the gap equation

$$\frac{\varphi^2}{N} = \chi_0 - i\hbar\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2 - 2V_0^{(1)}(\chi_0)}\,, \tag{7}$$

we obtain for the second derivative of (5)

$$\frac{\partial^2 V_{\rm eff}}{\partial\chi_0^{\,2}} = -NV_0^{(2)}(\chi_0) + 2iN\hbar\,[V_0^{(2)}(\chi_0)]^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{[p^2 - 2V_0^{(1)}(\chi_0)]^2}\,. \tag{8}$$

Now the problem is how to arrange counterterms into the definitions of χ_0 and $V_0^{(k)}(0)$ in such a way that expressions (6)–(8) will be finite. We shall see that the solution is not as immediate as in the ordinary $(1/N)(\Phi^4)_4$ model. Dimensionally regularizing the integrals occurring in (6)–(8) and introducing an arbitrary mass scale μ^2 we obtain

$$i\int\frac{d^{4-\epsilon}p}{(2\pi)^4}\frac{1}{p^2 - 2V_0^{(1)}(\chi_0)} = -\frac{V_0^{(1)}(\chi_0)}{4\pi^2}\frac{1}{\epsilon} + \frac{V_0^{(1)}(\chi_0)}{8\pi^2}\left(\ln\frac{V_0^{(1)}(\chi_0)}{\mu^2}\right) \tag{9}$$

and

$$i\int\frac{d^{4-\epsilon}p}{(2\pi)^4}\frac{1}{[p^2 - 2V_0^{(1)}(\chi_0)]^2} = -\frac{1}{8\pi^2}\frac{1}{\epsilon} + \frac{1}{16\pi^2}\ln\frac{V_0^{(1)}}{\mu^2}\,, \tag{10}$$

so that the conditions of renormalizability are

$$\chi = \chi_0 - \frac{2c}{\epsilon}V_0^{(1)}(\chi_0) = \text{finite}$$

and

$$V_0^{(2)}(\chi_0) + \frac{2c}{\epsilon}[V_0^{(2)}(\chi_0)]^2 = \text{finite}\,, \tag{11}$$

where we have introduced $c = \hbar/8\pi^2$.

In two and three dimensions only the mass renormalization is required in the leading order, so that the second condition in (11) is not necessary. In that case the renormalization condition can be equivalently represented in the form[1]

$$V^{(j)}(0)=V_0^{(j)}(C)=\sum_{k=0}^{\infty}\frac{1}{k!}C^k V_0^{(j+k)}(0)=\text{finite for all } j$$

where C stands for the pole part of the integral (9) calculated in two- and three-dimensional space-time. Let us adopt the formula

$$V^{(j)}(0)=V_0^{(j)}(\chi_0-\chi) \tag{12}$$

for our purposes, remembering that now (12) is a bare definition of the coefficients $V^{(j)}(0)$ as a power series in $(2c/\epsilon)V_0^{(1)}(\chi_0)$, and not a renormalization condition. The coefficients $V^{(j)}(0)$ must not be finite. Substituting (12) into (11) we obtain

$$\chi=\chi_0-\frac{2c}{\epsilon}V^{(1)}(\chi)=\text{finite},$$
$$V^{(2)}(\chi)+\frac{2c}{\epsilon}[V^{(2)}(\chi)]^2=\sum_{k=0}^{\infty}\frac{1}{k!}\left[V^{(2+k)}(0)+\frac{2c}{\epsilon}\sum_{l=0}^{k}\binom{k}{l}V^{(l+2)}(0)V^{(k-l+2)}(0)\right]\chi^k=\text{finite}. \tag{11$'$}$$

The solution of (11′) offers no problem if the theory is renormalizable, i.e., if $V^{(k)}(0)=0$ for $k>2$.[3,5] In our case the infinite number of nonvanishing coefficients $V^{(k)}(0)$ makes it much harder to represent $V^{(k)}(0)$ in the form of the power series in $1/\epsilon$,

$$V^{(k)}(0)=V_R^{(k)}(0)+\sum_{i=1}^{\infty}(1/\epsilon)^i A_i^{(k)}(V_R^{(j)}(0)),\quad \text{all } j \tag{13}$$

such as to ensure the finiteness of (11′). With all nonrenormalizable theories our model seems to share the distinction of having an infinite number of counterterms so that any renormalized action or effective potential depends on an infinite number of arbitrary parameters.

Things look much better if the interaction Lagrangian has the form of the renormalizable interaction perturbed by a bounded interaction. Then as indicated in Ref. 2 one can hope that only a finite number of parameters of the model require subtractions, so that the number of arbitrary renormalization conditions is also finite, as in renormalizable theories.

The argument hinges on the particular property of the effective Feynman rules of the $1/N$ expansion, in which conventional Feynman diagrams are partially resummed and produce effective vertices which are derivatives of the bare interaction Lagrangian. As we have just seen, the arguments of these derivatives involve contributions from the self-closing loops which are divergent in the limit of the removed regularization ($\epsilon\to 0$). In contradistinction to the general polynomial case, now only a few low-order derivatives tend to infinity in this limit, so that the structure of counterterms may be the same as in the case of renormalizable theories.

The derivatives of the bounded term should vanish at infinity fast enough as to ensure the finiteness of all derivatives of the effective potential, so additional constraints must be imposed on the shape of the interaction Lagrangian. In order to find these constraints we shall again reparametrize our model. The merit of the new reparametrization is that it considerably simplifies the procedure of arranging counterterms. In the case of the $(1/N)\Phi^4$ model our reparametrization reduces to that used in Refs. 3 and 5 and other papers on a similar subject. Let us return to the unrenormalized expression (5) for $V(\varphi,\chi_0)$ and introduce the new field variable ψ,

$$\psi=V_0^{(1)}(\chi_0),\quad \chi_0=f_0(\psi). \tag{14}$$

If the classical potential $V(\varphi^2/N)$ has only one extremum the function $\psi(\chi_0)$ is monotonic, so that $f_0(\psi)$ is single valued. If not, we have to consider all branches of $f_0(\psi)$.

In terms of the new variables we have

$$V_{\rm eff}(\varphi,\psi)=N\left[V_0(f_0(\psi))-(f_0(\psi)-\varphi^2/N)-\tfrac{1}{2}iN\hbar\int\frac{d^4p}{(2\pi)^4}\ln(-p^2+2\psi)\right],$$
$$\frac{\partial V_{\rm eff}(\varphi,\psi)}{\partial\psi}=N\left[\varphi^2/N-f_0(\psi)-i\hbar\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-2\psi}\right], \tag{15}$$
$$\frac{\partial^2 V_{\rm eff}(\varphi,\psi)}{\partial\psi^2}=-N\left[f_0^{(1)}(\psi)+2i\hbar\int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-2\psi)^2}\right].$$

Dimensionally regularizing we obtain

$$\frac{\partial V_{\rm eff}(\varphi,\psi)}{\partial\psi}=N[\varphi^2/N-f_0(\psi)+(2c/\epsilon)\psi+c\psi(\ln(\psi/\mu^2)-1)] \tag{16}$$

and

$$\frac{\partial^2 V_{\rm eff}(\varphi,\psi)}{\partial\psi^2} = -N[f_0^{(1)}(\psi) - 2c/\epsilon + c\ln(\psi/\mu^2)]\,. \quad (17)$$

Expanding $f_0(\psi)$ in powers of ψ,

$$f_0(\psi) = \sum_{n=0}^{\infty} (1/n!) f_0^{(n)}(0)\psi^n\,, \quad (18)$$

we find that the renormalization prescription should be

$$f_0^{(k)}(0) = f_0^{(k)}(0) + (2c/\epsilon)\delta_{1k}\,, \quad (19)$$

where all $f^{(k)}(0)$ are finite. The fact that we need not renormalize $f_0^{(0)}(0)$ is due to the dimensional regularization. Dimensionally regularized theories are void of power divergences. Comparing (19) with (14) we find that

$$\chi_0 = \chi + (2c/\epsilon)\psi = \chi + (2c/\epsilon)V^{(1)}(\chi)\,, \quad (20)$$

where χ is finite and we have introduced

$$V^{(k)}(\chi) = V_0^{(k)}(\chi_0)\,. \quad (21)$$

Let us note that in changing the field variables according to the definition (14) we have dodged the problem of how to ensure the finiteness of $V^{(1)}(\chi)$, given finite coefficients $V_R^{(k)}(0)$. Now $V^{(1)}(\chi)$ is known to be finite, but the definition (21) is only formal and it should be separately verified under the additional conditions that the requirement $V^{(k)}(0)$ = finite is satisfied for all k. To this purpose we expand the expression

$$V_0^{(1)}(\chi + (2c/\epsilon)V^{(1)}(\chi)) = V^{(1)}(\chi)$$

in powers of χ. Equating the first coefficients of the expansion we obtain

$$V_0^{(1)}((2c/\epsilon)V^{(1)}(0)) = V^{(1)}(0)\,.$$

Denoting

$$V^{(1)}(0) = M \quad \text{and} \quad (2c/\epsilon)M = \Lambda\,,$$

we have

$$V_0^{(1)}(\Lambda) = M\,. \quad (22)$$

Comparing the second coefficients we get

$$V_0^{(2)}(\Lambda)[1 + (\Lambda/M)V^{(2)}(0)] = V^{(2)}(0)\,,$$

so that in the limit $\Lambda\to\infty$ (i.e., $\epsilon\to 0$)

$$V_0^{(2)}(\Lambda) = \frac{V^{(2)}(0)}{1 + (\Lambda/M)V^{(2)}(0)} \sim M/\Lambda = O(\Lambda^{-1})\,. \quad (23)$$

For the third coefficient we have

$$V_0^{(3)}(\Lambda)[1 + (\Lambda/M)V^{(2)}(0)]^2 + V_0^{(2)}(\Lambda)(\Lambda/M)V^{(3)}(0) = V^{(3)}(0)\,.$$

Then, making use of (23) we obtain

$$\lim_{\Lambda\to\infty} V_0^{(3)}(\Lambda)[1 + (\Lambda/M)V^{(2)}(0)]^2 = 0\,,$$

and in general

$$\lim_{\Lambda\to\infty} V_0^{(k)}(\Lambda)[1 + (\Lambda/M)V^{(2)}(0)]^{k-1} = 0\,, \quad k \geq 3$$

so that

$$V_0^{(k)}(\Lambda) = o(\Lambda^{1-k}) \text{ as } \Lambda\to\infty \text{ for } k\geq 3\,. \quad (24)$$

There is an essential difference between the conditions (23) and (24). The former results from the renormalization procedure [also in the ordinary $(1/N)g\Phi^4$ theory we have $g_0(1 + (\Lambda/M)g_R) = g_R$] and allows us to make an arbitrary choice of the value of the finite coefficient $V^{(2)}(0)$. The latter condition should be understood as a restriction on the asymptotic behavior of the interaction Lagrangian. The examination of the equation for the third and higher coefficients that we have done above, has led us to the equations which involve only $V_0^{(k)}(\Lambda)$, $k\geq 3$, and not the coefficients $V^{(k)}(0)$.

The constraints on $V_0^{(k)}(\Lambda)$ are conditions on the asymptotic behavior in the limit of the infinite value of Λ. The condition (24) is satisfied by a large class of functions whose coefficients $V_0^{(k)}(0)$ of the expansion in power series are finite. Therefore we are allowed to write

$$V_0^{(k)}(0) = V^{(k)}(0) \text{ finite for } k > 2\,,$$

and restrict ourselves to interaction Lagrangians of the form

$$V_0(\Phi^2/N) = \tfrac{1}{2}m_0^2(\Phi^2/N) + (g_0/8N)(\Phi^2/N)^2 + W(\Phi^2/N)\,, \quad (25)$$

where

$$W^{(n)}(x) = o(x^{1-n}) \text{ as } x\to\infty \text{ for } n > 0\,.$$

Such Lagrangians can be considered as renormalizable in the usual sense because in (25) only m_0 and g_0 require renormalization conditions, the term $W(x)$ being irrelevant in the large-x region.

III. THE EFFECTIVE POTENTIAL

Let us integrate (16) using (19) to get the renormalized effective potential

$$V_{\rm eff}(\varphi,\psi) = N\left[(\varphi^2/N) - \int_0^\psi f(x)dx + (c/2)\psi^2(\ln(\psi/\mu^2) - \tfrac{3}{2})\right],$$

while after renormalization the gap equation (7) is

$$(\varphi^2/N) = f(\psi) - c\psi(\ln(\psi/\mu^2) - 1)\,.$$

Both formulas are manifestly μ^2 dependent and should be reexpressed in a renormalization-independent manner. First we remark that

$$f(\psi) - \psi f^{(1)}(0)$$

is a renormalized-independent quantity, so is κ^2 defined by

$$\kappa^2 = \mu^2 \exp[f^{(1)}(0)/c].$$

Then we rewrite the effective potential and the gap equation in the μ-independent manner:

$$V_{\text{eff}}(\varphi, \psi) = N\left[\frac{\varphi^2}{N}\psi - \int_0^\psi (f(x) - xf^{(1)}(0))dx + \tfrac{1}{2}c\psi^2(\ln(\psi/\kappa^2) - \tfrac{3}{2})\right], \tag{26}$$

$$\varphi^2/N = f(\psi) - \psi f^{(1)}(0) - c\psi(\ln(\psi/\kappa^2) - 1). \tag{27}$$

Eliminating φ^2/N from (26) with use of the gap equation we obtain the constrained effective potential $V_{\text{con}}(\psi)$,

$$\begin{aligned} V_{\text{con}}(\psi) &= N\left[\psi f(\psi) - \int_0^\psi f(x)dx - \tfrac{1}{2}\psi^2 f^{(1)}(0) - \tfrac{1}{2}c\psi^2(\ln(\psi/\kappa^2) - \tfrac{1}{2})\right] \\ &= N\left[\int_0^\psi x(f^{(1)}(x) - f^{(1)}(0)) - \tfrac{1}{2}c\psi^2(\ln(\psi/\kappa^2))\right]. \end{aligned} \tag{28}$$

From now on we shall focus our attention on interactions of the form which is strictly the same as (25), i.e., to the massive Φ^4 theory "perturbed" by the bounded interaction. The line of argument will not suffer qualitative alterations with change of the actual form of $W(\Phi^2/N)$, so we shall, to illustrate general considerations, make recourse to the specific example, namely

$$\frac{1}{N} V(\Phi^2/N) = \tfrac{1}{2}m(\Phi^2/N) + (g/8)(\Phi^2/N)^2 + \lambda \exp(-l\Phi^2/N), \quad l > 0. \tag{29}$$

The first derivative of (29) equals

$$\frac{1}{N} V^{(1)}(\Phi^2/N) = \tfrac{1}{2}m^2 - (g/4)(\Phi^2/N) - l\lambda \exp(-l\Phi^2/N) \tag{30}$$

and can not be explicitly inverted. However, we shall not need the actual expression for $f(\psi)$. The behavior of f is visualized in Fig. 2. We have to distinguish four cases: (a) $\lambda > 0$, $g > 0$, f increases; (b) $\lambda < 0$, $g < 0$, f decreases; (c) $\lambda < 0$, $g > 0$; (d) $\lambda > 0$, $g < 0$. In cases (c) and (d) f has two branches, one of them increasing, the other decreasing. Above the ψ axis the asymptotic behavior of f is linear while under this axis it is logarithmic. Since our procedure requires both f and $f^{(1)}$ to be defined at $\psi = 0$ we ask parameters m, g, λ to satisfy additional constraints[8]:

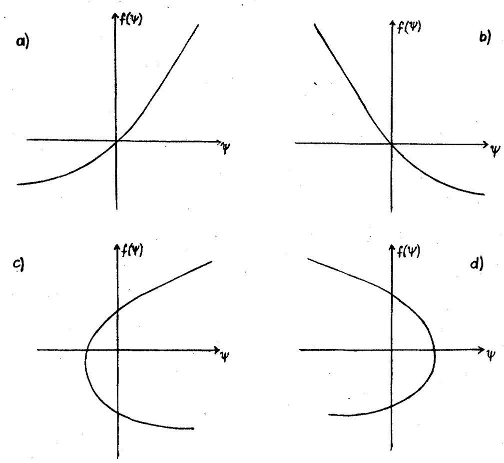

FIG. 2. $f(\psi)$ vs ψ. (a) for $\lambda > 0$, $g > 0$; (b) for $\lambda < 0$, $g < 0$; (c) $\lambda < 0$, $g > 0$; (d) $\lambda > 0$, $g < 0$. In cases (c) and (d) parameters obey constraints (31).

$$\begin{aligned} &m^2 < (g/2l)\ln(-g/4el^2\lambda) \quad [\text{case (c)}], \\ &m^2 > (g/2l)\ln(-g/4el^2\lambda) \quad [\text{case (d)}]. \end{aligned} \tag{31}$$

For $|l^2\lambda| \ll g$ (this is what one should mean when stating that $g\Phi^4$ is perturbed by bounded interaction) lower branches in case (c) and (d) are unphysical i.e., $\varphi^2/N < 0$ for all ψ. This allows us to consider in what follows only upper branches of f.

Differentiating (28) with respect to ψ we have

$$V'_{\text{con}}(\psi) = N\psi[f^{(1)}(\psi) - f^{(1)}(0) - c\ln(\psi/\kappa^2)]. \tag{32}$$

Hence V_{con} has extrema at $\psi = 0$ (minimum) and possibly at $\psi = \tilde{\psi}_i$, where $\tilde{\psi}_i$'s are solutions of the equation

$$f^{(1)}(\tilde{\psi}) - f^{(1)}(0) = c\ln(\tilde{\psi}/\kappa^2). \tag{33}$$

The expression in brackets in (32) is the derivative of the gap equation (27) so the gap function $\psi(\varphi^2)$ has branch points at each $\tilde{\psi}$ where V_{con} has an extremum. If $\varphi^2(\psi = 0) \geq 0$, the ground state can possibly exist at $\psi = 0$ [symmetry breaking if $\varphi^2(\psi = 0) > 0$] or at zeros of $\varphi^2(\psi)$, i.e., at ψ_0 satisfying

$$f(\psi_0) - \psi_0 f^{(1)}(0) = c\psi_0(\ln(\psi_0/\kappa^2) - 1) \tag{34}$$

(no symmetry breaking). If there are two such zeros $\psi_{01} < \psi_{02}$, say, separated by a maximum of $\varphi^2(\psi)$ and $V_{\text{con}}(\psi)$ at $\psi = \psi$, then $V_{\text{con}}(\psi_{02}) < V_{\text{con}}(\psi_{01})$. Indeed,

$$\begin{aligned} V_{\text{con}}(\psi_{02}) - V_{\text{con}}(\psi_{01}) = N\Big\{ &-\int_{\psi_{01}}^{\psi_{02}} [f(x) - xf^{(1)}(0)]dx + \psi_{02}f(\psi_{02}) - \psi_{01}f(\psi_{01}) - (\psi_{02}{}^2 - \psi_{01}{}^2)f^{(1)}(0) \\ &- (c/2)\psi_{02}{}^2(\ln(\psi_{02}/\kappa^2) - \tfrac{1}{2}) + (c/2)\psi_{01}{}^2(\ln(\psi_{01}/\kappa^2) - \tfrac{1}{2})\Big\}. \end{aligned}$$

Eliminating $\psi_{0i}f(\psi_{0i})$ with the help of (34), we get

$$V_{\rm con}(\psi_{02})-V_{\rm con}(\psi_{01})=N\left\{-\int_{\psi_{01}}^{\psi_{02}}[f(x)-xf^{(1)}(0)]dx+(c/2)\psi_{02}{}^2(\ln(\psi_{02}/\kappa^2)-\tfrac{3}{2})-(c/2)\psi_{01}{}^2(\ln(\psi_{01}/\kappa^2)-\tfrac{3}{2})\right\}.$$

In the interval $\psi_{01}<\psi<\psi_{02}$, $\varphi^2(\psi)>0$; therefore

$$\int_{\psi_{01}}^{\psi_{02}}[f(x)-xf^{(1)}(0)]dx>c\int_{\psi_{01}}^{\psi_{02}}x[\ln(x/\kappa^2)-1],$$

and performing the integration we get

$$V_{\rm con}(\psi_{02})-V_{\rm con}(\psi_{01})<0. \tag{35}$$

To the right of $\psi=0$, $\varphi^2(\psi)$ increases while $\varphi^2(\psi)\to-\infty$ for $\psi\to\infty$; therefore $\varphi^2(\psi)$ must have a maximum at some $\tilde{\psi}>0$, and if $f(0)\geqslant 0$ a zero at $\psi_0>\tilde{\psi}$. $V_{\rm con}(\psi=0)=0$, while $V_{\rm con}(\psi_0)<0$ by the same argument as above.

Concluding, the vacuum is always symmetric because its site is at the deepest of the minima of $V_{\rm con}$. At the ground state

$$\frac{d}{d\psi}\frac{\varphi^2}{N}(\psi_0)=f^{(1)}(\psi_0)-f^{(1)}(0)-c\ln(\psi_0/\kappa^2)<0, \tag{36}$$

because ψ_0 is situated to the right of a maximum of $\varphi^2/N(\psi)$. Let us show that in our specific example one encounters only one zero satisfying the condition (36). First let us prove this for cases (a) and (b). From (27) it is apparent that maximum occurs for $\psi>\kappa^2$ because $f(\psi)-f^{(1)}(0)$ is an increasing function. To the right of κ^2 both sides of (34) are increasing but the second derivative of the left-hand side (lhs) of (34) is negative while that of the right-hand side (rhs) is positive; therefore for $\psi_0>\kappa$ the lhs intersects the rhs in no more than one point. One can not have two intersection points to the left of κ^2 because then the lhs increases while the rhs decreases.

In case (c) the upper branch of f increases while its first derivative decreases and so does the lhs of (33). The rhs of (33) increases from minus to plus infinity; hence (33) has only one root and the gap equation has only one minimum at $\tilde{\psi}$. There is no more than one zero to the right (left) of $\tilde{\psi}$. Finally, in case (d) $f(\psi)$ decreases on the upper branch, and so does $f^{(1)}(\psi)$. To the right of the branch point f goes out of existence but if $|\lambda l^2|\ll g$ the solution(s) of (34) exists. The decrease of $f^{(1)}$ implies the position of zeros is the same as in case (c).

IV. THE SPECTRUM

The effective action is

$$\Gamma(\varphi^2,\psi)=\int d^4x[\tfrac{1}{2}\varphi\Box\varphi-\varphi^2\psi-NV_0(f_0(\psi))+Nf_0(\psi)\psi] + \tfrac{1}{2}Ni\hbar\,{\rm Tr}\ln(\Box+2\psi). \tag{37}$$

Shifting the field variable ψ by ψ_0,

$$\varphi^2\to\varphi^2,\quad \psi\to\psi+\psi_0,$$

we obtain

$$\Gamma(\varphi^2,\psi)=\int d^4x[\tfrac{1}{2}\varphi\Box\varphi-\varphi^2\psi-\varphi^2\psi_0 - NV_0(f_0(\psi+\psi_0)) + Nf_0(\psi+\psi_0)\psi+Nf_0(\psi+\psi_0)\psi_0] + \tfrac{1}{2}Ni\hbar\,{\rm Tr}\ln(\Box+2\psi+2\psi_0). \tag{38}$$

This yields for the ψ tadpole diagram

$$\left.\frac{\delta\Gamma}{\delta\psi}\right|_{\substack{\psi=0\\ \varphi^2=0}}=N\left[f_0(\psi_0)+i\hbar\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-2\psi_0}\right] =N[f(\psi_0)-\psi_0f^{(1)}(0)-c\psi_0(\ln(\psi_0/\kappa^2)-1)]$$

which vanishes in view of the gap equation (27). The propagator of the φ field equals

$$(p^2-2\psi_0)^{-1}. \tag{39}$$

There are no mixed φ-ψ lines in the Feynman rules and the ψ-ψ propagator $D_{\psi\psi}(k^2)$ is given by

$$-D_{\psi\psi}(k^2)^{-1}=\left.\frac{\delta^2\Gamma}{\delta\psi^2}\right|_{\psi=\varphi^2=0}=N\left[f_0^{(1)}(\psi_0)-2i\hbar\int\frac{d^4p}{(2\pi)^4}\frac{1}{(p+k)^2-2\psi_0}\frac{1}{p^2-2\psi_0}\right] =N\{[f^{(1)}(\psi_0)-f^{(1)}(0)-c\ln(\psi_0/\kappa^2)]-[(c/2)B(k^2,2\psi_0)-2]\}, \tag{40}$$

where

$$B(k^2,m^2)=2(1-1/x)^{1/2}\ln((1-x)^{1/2}+(-x)^{1/2})\ \text{ for } x=k^2/4m^2\leqslant 0,$$
$$B(k^2,m^2)=2(1/x-1)^{1/2}\arctan(x^{1/2}(1-x)^{1/2})\ \text{ for } 0\leqslant x\leqslant 1, \tag{41}$$
$$B(k^2,m^2)=(1-1/x)^{1/2}\,[-i\pi+2\ln((x-1)^{1/2}+x^{1/2})]\ \text{ for } x>1.$$

The first expression in square brackets in the second line of (40) is negative in view of (36); furthermore, $B\,(k^2=0, 2\psi_0)=2$ and increases if k^2 increases so that $D_{\psi\psi}(k^2)$ has no (tachyon) poles for spacelike momenta. This conclusion ends our discussion which has shown that bounded perturbation of the $(1/N)\Phi^4$ theory results in the leading order only in the need to modify some details of the computational technique. Essential features of the model remain unchanged: Symmetry breakdown is impossible and expanding around the symmetric minimum does not lead to ghost poles in the intermediate field propagator.

In the accompanying paper[6] we shall show that the above conclusion is not model independent. Changing the interaction potential we may obtain essentially different results.

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[8]In case (c) one could, of course, take the parameters from outside of the region defined by (31), but then one should renormalize at certain $\psi > 0$ where both f and $f^{(1)}$ exist.