Particle production by white holes

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(Received 18 December 1979)

A white hole is the time reverse of a spacetime in which gravitational collapse has occurred to form a black hole. We find that in quantum field theory in a white-hole background, for any initial state of the field which is a product of a state on the horizon with a state at past null infinity, an infinite particle and energy flux occurs at future null infinity when the white-hole horizon is seen to terminate. This may be interpreted as a quantum version of the classical white-hole instability discussed by Eardley. Consequently, there appear to be considerable difficulties in incorporating white holes into a consistent picture of a thermodynamic self-gravitating quantum system. This provides evidence that the laws of quantum gravity may not be time-reversal invariant.

I. INTRODUCTION

It is by now well known that calculations in the framework of quantum field theory in curved spacetime yield the result[1,2] that a Schwarzschild black hole radiates particles with a thermal spectrum. The process can be thought of as arising from the production of pairs of particles near the event horizon of the black hole. One might ask what effects are predicted by a similar calculation when the background spacetime is that of a white hole. By a white hole we mean simply the time reverse of a spacetime in which gravitational collapse to a black hole occurs. A conformal diagram for the classical white-hole geometry we shall consider is given in Fig. 1 below.

Our reason for investigating particle creation by white holes is the following: One of the most intriguing aspects of the theory of black holes, both classical and quantum (semiclassical), is the very close analogy between the laws of black-hole physics and the laws of thermodynamics. It has been argued[3] that "black-hole thermodynamics" is plausibly just ordinary thermodynamics applied to a quantum self-gravitating system. However, "ordinary" thermodynamic systems satisfy time-reversible laws of physics; the time reverse of any dynamical motion is also a possible dynamical motion. If the standard views on quantum black-hole dynamics are correct, then in a closed self-gravitating system, black holes can form and evaporate. Hence, if such a system also displays time-reversible behavior, it should be possible for white holes to form and disappear. It is therefore of interest to examine the particle-creation effects of white holes to see if they can be consistently incorporated into the picture of a quantum self-gravitating thermodynamic system. Remarkably, although neither general relativity nor the theory of quantum fields in curved spacetime display any manifest time-reversal noninvariance, we shall see that there are substantial difficulties in doing so.

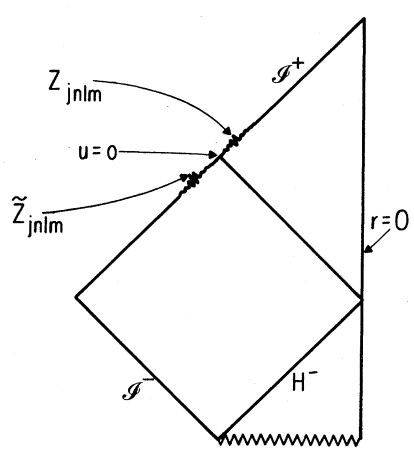

FIG. 1. A conformal diagram for a white-hole spacetime, showing the locations on $\mathscr{I}^+$ of the wave packets Z_{jnlm}, $\tilde{Z}_{jnlm}$ used in the calculation.

In the theory of particle creation, both by black holes and by white holes, an ambiguity arises concerning the definition of "horizon states"—i.e., particles which go into the black hole or come out of the white hole—as this requires a notion of "positive frequency," which does not arise naturally on the horizon. In the black-hole case, it turns out that this ambiguity does not affect the calculation of the density matrix describing observations at $\mathscr{I}^+$, i.e., what a distant observer would see at late times. However, in the case of a white hole, the definition of "positive frequency" as well as the choice of initial white-hole state once this definition has been made affect significantly the state of the field at $\mathscr{I}^+$. Hence, in the white-hole case, a definite prediction of what a distant observer would see cannot be made without some rather arbitrary choices. Therefore, it might be thought that no meaningful predictions can be made at all. However, we shall show that under only the assumption that the initial "in" state is a product of a state at $\mathscr{I}^-$ with a horizon state, a lower bound can be derived for the particle and energy flux which reaches $\mathscr{I}^+$ at a time near the "disappearance time" u_0 of the white hole (see Fig. 1). This bound depends neither on the definition of "positive frequency" on the past horizon, nor on the choice of the particular horizon and $\mathscr{I}^-$ states constituting the "in" product state. Since observation of the initial state at $\mathscr{I}^-$ should prepare the system as a product state, consideration by such product states should be of physical relevance if white holes exist.

Our calculations show a larger and larger flux of particles at higher and higher energies reaching $\mathscr{I}^+$ as $u \to u_0$. Estimating the energy flux associated with this particle flux, we shall obtain in Sec. II the lower bound

$$\frac{dE}{du} \gtrsim \frac{10^{-3}\hbar}{(u-u_0)^2}. \tag{1}$$

Thus, neglecting back-reaction effects, an infinite burst of energy would be seen at the time of disappearance of the white hole. We also have reason to believe that the expectation value of the stress-energy tensor is singular on the white-hole horizon.

Some of our conclusions have been obtained independently in unpublished work by Schutz and Sorkin. Similar effects have been considered previously by Eardley[4] and by Zel'dovich, Novikov, and Starobinski,[5] who investigated the possible existence of astrophysical white holes arising as "delayed pieces of the big bang." Although Eardley's analysis was purely classical, and the quantum analysis of Zel'dovich *et al.* was carried out before the tools used in Hawking's black-hole calculation were available, their conclusion that white holes are unstable is fully consistent with our results. Indeed, the infinite energy flux we find at $\mathscr{I}^+$ at time u_0 doubtless arises from the "blue-sheet" instability of the white hole discussed by Eardley. Our conclusions strengthen Eardley's classical result by showing that the instability occurs for any incoming state whatsoever at $\mathscr{I}^-$ (including no incoming particles at all), provided only that the total state vector be a product of this state with a horizon state. If there are no incoming particles, spontaneous particle creation by the white hole is sufficient to induce the instability.

II. CALCULATION OF PARTICLE CREATION

In this section we shall derive a lower bound on the particle and energy flux emerging from a white hole as seen from $\mathscr{I}^+$. For simplicity, we shall treat only a massless Klein-Gordon scalar field, but similar results should apply to all other fields. In addition, we shall take as our model of a white-hole spacetime simply the time reverse of a spherically symmetric spacetime describing gravitational collapse to a black hole (see Fig. 1). Thus, the white hole is assumed to have mass M (and surface gravity $\kappa = 1/4M$) in the asymptotic past and the horizon of the white hole is assumed to terminate at retarded time $u=u_0$. For convenience we take $u_0 = 0$. Natural units $G=c=\hbar=1$ are used throughout.

As already mentioned in the Introduction, the particle creation one calculates as emerging from the white hole depends on the definition of positive frequency on the past horizon H^- of the white hole, and the choice of initial white-hole state, as well, of course, as the incoming state from $\mathscr{I}^-$. Without some restriction on the incoming state at H^- and $\mathscr{I}^-$, any result could be obtained. Indeed by choosing the time reverse of the state arising from spontaneous particle creation by a black hole, we could produce an incoming state for the white-hole spacetime which would result in no particles reaching $\mathscr{I}^+$. However, such an incoming state has an enormously high degree of initial correlation between H^- and $\mathscr{I}^-$. It appears physically unnatural to consider such incoming states. Indeed, if one stations an observer at $\mathscr{I}^-$ to measure the incoming particles, then by the usual interpretive rules of quantum mechanics, he would "knock" the incoming state Ψ_{in} into a product of a state $\Psi_{\mathscr{I}^-}$ at $\mathscr{I}^-$ with a horizon state Ψ_{H^-}:

$$\Psi_{\text{in}} = \Psi_{\mathscr{I}^-} \otimes \Psi_{H^-}. \tag{2}$$

Thus, it appears most physically relevant to consider incoming states of the form Eq. (2). It is for incoming states of this form that we shall derive our lower bound on the outgoing particle and energy flux. Note that the product form of Ψ_{in} is preserved under changes of the definition of positive frequency at H^-.

The number operator for particles in the outgoing state τ is

$$N(\tau) = b^\dagger(\tau) b(\tau), \tag{3}$$

where $b(\tau)$ is the "out" annihilation operator for the state τ. As discussed more fully elsewhere,[6] the "out" annihilation and creation operators b, $b^\dagger$ are related to the corresponding "in" operators a, $a^\dagger$ by

$$b(\tau) = a(A\tau) - a^\dagger(B\tau). \tag{4}$$

Here $A\tau$ denotes the "in" state obtained by propagating the "out" wave packet associated with τ into the past and taking its positive-frequency part there, while $B\tau$ is the "in" state associated with its negative-frequency part. Further properties of A and B as well as the role of Eq. (4) in deriving the S matrix are described in Ref. 6.

For any given "in" state the expected number $\langle N(\tau)\rangle$ of "out" particles in the state τ can be calculated directly from Eqs. (3) and (4). Our aim is to obtain a lower bound on $\langle N(\tau)\rangle$ for appropriate choices of τ. We begin by deriving the following general result.

Lemma. Suppose the one-particle "in" Hilbert space $H_{\rm in}$ is the direct sum of two subspaces H_1 and H_2, and hence that the "in" Fock space $\mathfrak{F}(H_{\rm in})$ is the tensor product of $\mathfrak{F}(H_1)$ with $\mathfrak{F}(H_2)$. Let τ be a one-particle "out" state. Let α_1 denote the projection of $A\tau$ onto H_1, and let α_2 be its projection onto H_2. Similarly, let β_1 and β_2 be the projections of $B\tau$ onto H_1 and H_2, respectively. Then for all "in" states of a product form $\Psi=\Psi_1\otimes\Psi_2$, a lower bound on $\langle N(\tau)\rangle$ is

$$\langle N(\tau)\rangle \geq \|\beta_1\|^2 - \|\alpha_1\|^2 . \tag{5}$$

To prove Eq. (5), we calculate $\langle N(\tau)\rangle$ using Eq. (4). We find

$$\begin{aligned}\langle\Psi|N(\tau)|\Psi\rangle &= \langle\Psi_1\otimes\Psi_2|[a^\dagger(\alpha_1)+a^\dagger(\alpha_2)-a(\beta_1)-a(\beta_2)][a(\alpha_1)+a(\alpha_2)-a^\dagger(\beta_1)-a^\dagger(\beta_2)]|\Psi_1\otimes\Psi_2\rangle\\ &= \langle\Psi_1|[a^\dagger(\alpha_1)-a(\beta_1)][a(\alpha_1)-a^\dagger(\beta_1)]|\Psi_1\rangle + \langle\Psi_2|[a^\dagger(\alpha_2)-a(\beta_2)][a(\alpha_2)-a^\dagger(\beta_2)]|\Psi_2\rangle\\ &\quad + 2\,\mathrm{Re}\langle\Psi_1|a^\dagger(\alpha_1)-a(\beta_1)|\Psi_1\rangle\langle\Psi_2|a(\alpha_2)-a^\dagger(\beta_2)|\Psi_2\rangle .\end{aligned} \tag{6}$$

We rewrite the first term as

$$\begin{aligned}\langle\Psi_1|[a^\dagger(\alpha_1)-a(\beta_1)][a(\alpha_1)-a^\dagger(\beta_1)]|\Psi_1\rangle &= \langle\Psi_1|[a(\alpha_1)-a^\dagger(\beta_1)][a^\dagger(\alpha_1)-a(\beta_1)]|\Psi_1\rangle\\ &\quad + \langle\Psi_1|[a^\dagger(\alpha_1),a(\alpha_1)]|\Psi_1\rangle + \langle\Psi_1|[a(\beta_1),a^\dagger(\beta_1)]|\Psi_1\rangle\\ &= \|[a^\dagger(\alpha_1)-a(\beta_1)]\Psi_1\|^2 - \|\alpha_1\|^2 + \|\beta_1\|^2 .\end{aligned} \tag{7}$$

The third term in Eq. (6) cannot be less than minus its absolute value. By the Schwarz inequality we can bound the absolute value of the first factor by

$$|\langle\Psi_1|[a^\dagger(\alpha_1)-a(\beta_1)]|\Psi_1\rangle| \leq \|[a^\dagger(\alpha_1)-a(\beta_1)]\Psi_1\| \tag{8}$$

and we can similarly bound the factor involving Ψ_2. Thus, we obtain

$$\begin{aligned}\langle N(\tau)\rangle &\geq \|\beta_1\|^2 - \|\alpha_1\|^2 + \|[a^\dagger(\alpha_1)-a(\beta_1)]\Psi_1\|^2 + \|[a(\alpha_2)-a^\dagger(\beta_2)]\Psi_2\|^2\\ &\quad - 2\|[a^\dagger(\alpha_1)-a(\beta_1)]\Psi_1\|\,\|[a(\alpha_2)-a^\dagger(\beta_2)]\Psi_2\|\\ &\geq \|\beta_1\|^2 - \|\alpha_1\|^2 ,\end{aligned} \tag{9}$$

since the last three terms are a perfect square. Equation (9) is the desired result.

We shall apply this result to the white-hole case by taking H_1 to be the one-particle states at $\mathscr{I}^-$, while H_2 is the space of "horizon states." The next step is to find appropriate out states τ for which the lower bound [Eq. (9)] is nonzero. Fortunately, most of the work required for that purpose has already been carried out in the analysis of particle production by black holes.[1,2] In the black-hole case, wave packets P_{jnlm} were constructed at $\mathscr{I}^+$ centered about retarded time $u=2\pi n/\epsilon$ with a small frequency spread ϵ about $\omega_j=(j+\frac{1}{2})\epsilon$ and with angular dependence Y_{lm}. Propagation of this wave packet back into the past produced a highly blue-shifted wave packet Z_{jnlm} at $\mathscr{I}^-$ which as $n\to\infty$ becomes more and more concentrated near the advanced time v_0 at $\mathscr{I}^-$ corresponding to the formation of the black hole. Decomposition of Z_{jnlm} into its positive- and negative-frequency parts at $\mathscr{I}^-$ yielded the results on particle creation by black holes.

The black-hole analysis suggests that in the white-hole case we look for particle production in modes related to the analogs of the Z_{jnlm} wave packets at $\mathscr{I}^+$. As shown in more detail in Ref. 2, positive-frequency modes can be constructed from Z_{jnlm} as follows: The "blue-shifted spherical waves" $Z_{\omega lm}$ are given at $\mathscr{I}^+$ by

$$Z_{\omega lm} = \begin{cases}\exp\left(-i\dfrac{\omega}{\kappa}\ln(u/C)\right), & u>0\\ 0, & u<0\end{cases} \tag{10}$$

where the constant C depends on the choice of origin of time at $\mathscr{I}^-$. The wave packet Z_{jnlm} constructed from the $Z_{\omega lm}$ is

$$Z_{jnlm}=\begin{cases}\epsilon^{1/2}(2\pi\omega_j)^{-1/2}\exp[-i(j+\tfrac{1}{2})L(u)]\dfrac{\sin L/2}{L/2}\,Y_{lm}(\theta,\varphi), & u>0\\ 0, & u<0\end{cases} \tag{11}$$

where

$$L(u)=(\epsilon/\kappa)\ln|u/C|+2\pi n\,. \tag{12}$$

We define $\tilde{Z}_{jnlm}$ to be the time reflection of Z_{jnlm} at $\mathscr{I}^+$ about retarded time $u=0$:

$$\tilde{Z}_{jnlm}=\begin{cases}0, & u>0\\ \epsilon^{1/2}(2\pi\omega_j)^{-1/2}\exp[-i(j+\tfrac{1}{2})L(u)]\dfrac{\sin L/2}{L/2}, & u<0\,.\end{cases} \tag{13}$$

Then, by the analysis of Appendix A of Ref. 2, it follows that $\tilde{Z}_{jnlm}+\exp(-\pi\omega_j/\kappa)Z_{jnlm}$ contains no negative-frequency components at $\mathscr{I}^+$. We define

$$W_{jnlm}=\left[\tilde{Z}_{jnlm}+\exp\left(-\pi\frac{\omega_j}{\kappa}\right)Z_{jnlm}\right]\Big/\left[1-\exp\left(-2\pi\frac{\omega_j}{\kappa}\right)\right]^{1/2}. \tag{14}$$

Then the $\{W_{jnlm}\}$ are positive-frequency wave packets at $\mathscr{I}^+$ which are orthonormal in the Klein-Gordon norm. (They are not complete, but this is not of importance since we are interested only in deriving a lower bound on particle production.) The energy flux at $\mathscr{I}^+$ associated with the wave packet W_{jnlm} is given by

$$\begin{aligned}\frac{dE_{jnlm}}{du}&=\int T_{ab}u^a u^b d\Omega=\int\left|\frac{dW_{jnlm}}{du}\right|^2 d\Omega\\
&=\begin{cases}\left[1-\exp\left(-2\pi\dfrac{\omega_j}{\kappa}\right)\right]^{-1}\displaystyle\int\left|\frac{dZ_{jnlm}}{du}\right|^2 d\Omega, & u<0\\ \exp\left(-2\pi\dfrac{\omega_j}{\kappa}\right)\left[1-\exp\left(-2\pi\dfrac{\omega_j}{\kappa}\right)\right]^{-1}\displaystyle\int\left|\frac{d\tilde{Z}_{jnlm}}{du}\right|^2 d\Omega, & u>0\end{cases}\\
&\approx\begin{cases}\dfrac{\epsilon\omega}{2\pi\kappa^2u^2}\left[1-\exp\left(-2\pi\dfrac{\omega_j}{\kappa}\right)\right]^{-1}\dfrac{\sin^2[\frac{1}{2}L(u)]}{[\frac{1}{2}L(u)]^2}, & u<0\\ \dfrac{\epsilon\omega}{2\pi\kappa^2u^2}\exp\left(-2\pi\dfrac{\omega_j}{\kappa}\right)\left[1-\exp\left(-2\pi\dfrac{\omega_j}{\kappa}\right)\right]^{-1}\dfrac{\sin^2[\frac{1}{2}L(u)]}{[\frac{1}{2}L(u)]^2}, & u>0\,.\end{cases}\end{aligned} \tag{15}$$

We shall now compute a lower bound for particle production in the mode $\tau=W_{jnlm}$. By the time reverse of the analysis of the black-hole case,[2] the propagation of W_{jnlm} into the past yields the following: The portion, $\tilde{Z}_{jnlm}$, of W_{jnlm} with $u<0$ propagates directly into the white hole. A fraction Γ_{jnlm} of the portion, Z_{jnlm}, with $u>0$ propagates back to $\mathscr{I}^-$ while the remainder gets scattered into the white hole. The portion reaching $\mathscr{I}^-$ is purely negative frequency there. Thus, we find that the norms of the positive- and negative-frequency parts, α_1 and β_1, of the wave packet at $\mathscr{I}^-$ are

$$\|\alpha_1\|^2=0\,, \tag{16a}$$

$$\|\beta_1\|^2=\frac{\Gamma_{jnlm}e^{-2\pi\omega_j/\kappa}}{1-e^{-2\pi\omega_j/\kappa}}\,. \tag{16b}$$

(Γ_{jnlm} equals the absorption coefficient of the wave packet P_{jnlm} in the static Schwarzschild geometry.)

Thus, from Eq. (5) for any product "in" state we obtain

$$\langle N(W_{jnlm})\rangle\geq\frac{\Gamma_{jnlm}}{e^{2\pi\omega_j/\kappa}-1}\,, \tag{17}$$

which is a lower bound on the expected number of W_{jnlm} particles reaching $\mathscr{I}^+$. We can use Eqs. (15) and (17) to estimate a lower bound on the total energy flux reaching $\mathscr{I}^+$ near $u=0$. Neglecting interference effects between different modes, we find, using the $u<0$ estimate for dE_{jnlm}/du,

$$\begin{aligned}\frac{dE}{du}&\gtrsim\sum_{jnlm}N_{jnlm}\frac{dE_{jnlm}}{du}\\
&\geq\sum_{jnlm}\frac{\Gamma_{jnlm}}{e^{2\pi\omega_j/\kappa}-1}\frac{dE_{jnlm}}{du}\\
&=\sum_{jnlm}\frac{\Gamma_{jnlm}}{e^{2\pi\omega_j/\kappa}-1}\frac{\epsilon\omega_j}{2\pi\kappa^2u^2}\frac{e^{2\pi\omega_j/\kappa}}{e^{2\pi\omega_j/\kappa}-1}\frac{\sin^2 L/2}{(L/2)^2}\,.\end{aligned} \tag{18}$$

To evaluate this expression, we approximate $\Gamma_{jnlm}=1$ for $l\leq 3\sqrt{3}\,\omega_j M$ and $\Gamma_{jnlm}=0$ for $l > 3\sqrt{3}\,\omega_j M$ (the geometrical-optics approximation). We obtain

$$\frac{dE}{du}=\sum_{j,n}\sum_{l=0}^{3\sqrt{3}\omega_j M}\sum_{m=-l}^{l}\frac{\epsilon\omega_j}{2\pi\kappa^2u^2}\frac{e^{2\pi\omega_j/\kappa}}{(e^{2\pi\omega_j/\kappa}-1)^2}\frac{\sin^2 L/2}{(L/2)^2}$$
$$=\frac{27M^2}{2\pi\kappa^2u^2}\sum_{j=0}^{\infty}\frac{\epsilon\omega_j{}^3e^{2\pi\omega_j/\kappa}}{(e^{2\pi\omega_j/\kappa}-1)^2}\times\sum_{n=-\infty}^{\infty}\frac{\sin^2[n\pi+(\epsilon/2\kappa)\ln|u/C|]}{[n\pi+(\epsilon/2\kappa)\ln|u/C|]^2}. \quad (19)$$

For small ϵ, the sum over n approaches 1, while the sum over j can be written as

$$\int_0^\infty\frac{d\omega\,\omega^3e^{2\pi\omega/\kappa}}{(e^{2\pi\omega/\kappa}-1)^2}=\left(\frac{\kappa}{2\pi}\right)^4\int_0^\infty\frac{dx\,x^3e^x}{(e^x-1)^2}=\left(\frac{\kappa}{2\pi}\right)^4\Gamma(4)\zeta(3). \quad (20)$$

Hence

$$\frac{dE}{du}\gtrsim\frac{27M^2\kappa^2}{(2\pi)^5u^2}\Gamma(4)\zeta(3)=\frac{81\zeta(3)}{256\pi^5}\frac{1}{u^2}. \quad (21)$$

Thus, we find that the energy flux diverges as $1/u^2$ as $u\to 0$. Consequently, back-reaction effects must be of importance. These and other issues are discussed in Sec. III.

III. DISCUSSION

The calculation of Sec. II produced the rather pathological result of an infinite energy flux at $\mathscr{I}^+$ at $u=0$. The first issue we wish to address is the physical origin of the burst of particles. Since nothing of local geometric importance occurs near the event marking the termination of the white-hole horizon, it is not plausible that these particles were created there. We believe that the correct interpretation is that pairs of particles were produced near the white-hole horizon throughout the history of the white hole. These particles propagated along the horizon, becoming more and more blue-shifted, until the horizon terminated and they were free to propagate out to $\mathscr{I}^+$. Additional particles sent in from $\mathscr{I}^-$ add to this burst directly as well as stimulate the production of more pairs.[7]

The infinite flux at $u=0$ clearly means that back-reaction effects must be important. Indeed, when dE/du becomes greater than 1 (in geometrized units $G=c=1$), a classical propagation of the particles back into the past shows that they would have been within their own Schwarzschild radius near the termination of the white-hole horizon. (Since the background spacetime is nearly flat over this propagation route, such a classical propagation should be justified.) Thus, we believe that for retarded times u when $dE/du<1$, the results of Sec. II are valid. However, the burst of particles which is predicted by our calculation to arrive after this time probably collapses instead to form a black hole, and thus no further radiation should reach $\mathscr{I}^+$. Thus, a black hole should form around the termination point of the white holes, as previously described in a classical context by Eardley.[4]

The back-reaction effect of black-hole formation should thereby avoid the prediction of an infinite flux of particles reaching $\mathscr{I}^+$. However, it seems likely—particularly in view of the above physical interpretation—that the expected stress-tensor $\langle T_{ab}\rangle$ at the white-hole horizon will still be infinite, and thus that back reaction will also convert the white-hole horizon into a naked singularity. This belief is supported by a calculation of Unruh,[8] who found that for the "in" vacuum state, $\langle T_{ab}\rangle$ is singular on the past horizon of a two-dimensional analog of the Schwarzschild spacetime.

In view of this pathological behavior of white holes for "reasonable" choice of "in" state, it appears that—unlike black holes—it is very difficult to incorporate white holes into a consistent picture of the dynamics of a self-gravitating quantum system. Although there remain some possibilities for doing so—for example, one could postulate that white holes are always "born" in states with an enormously high degree of correlation with $\mathscr{I}^-$ or employ nonstandard interpretations of quantum theory such as the notion of observer-dependent geometry[9]—it appears to us most likely that there is simply a fundamental lack of time-reversal invariance in quantum gravity; that in quantum gravity, black holes can exist but white holes cannot. Independent arguments for time asymmetry in quantum gravity have been given recently by Penrose.[10] Further evidence of perhaps an even stronger nature for time asymmetry comes from the idea that when a black hole is formed and evaporated, an initial pure state will evolve to a density matrix.[2,11] To maintain time-reversal invariance, one also must require an initial density matrix to evolve to a final pure state[12] which (under some minimal assumptions about the scattering) is seen to be impossible.[13] Interestingly, a weaker, asymptotic form of time-reversal symmetry could still hold, so that one would have to make measurements in strong gravitational fields in order to detect the time asymmetry. These ideas will be discussed more fully by one of us in the following paper.[13]

ACKNOWLEDGMENTS

We wish to thank R. Sorkin for helpful discussions. This research was supported in part by NSF Grant No. PHY 78-24275 to the University of Chicago and by the Alfred P. Sloan Foundation.

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