Comment on the invalidity of the Brayshaw mechanism to generate peaks in three-hadron systems

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(Received 2 July 1979)

The recently proposed mechanism by Brayshaw is studied quantitatively. The already well-known subenergy singularities of the triangle diagram are indeed present. However, the proper unitarization of this diagram leads (in general) to no observable effects in cross sections, suggesting that no dynamical mechanism for producing three-body resonances can be associated with these singularities.

In recent papers[1-3] Brayshaw proposed a three-body mechanism to predict resonances at energies easily calculable via simple analytic formulas. The so-called Brayshaw mechanism is concerned with graphs like Fig. 1 and consists essentially of two statements.

(1) If we define the energy variable $E = \sqrt{s}$ $- 2m_a - m_b$ and the subenergy variable $\epsilon = \sqrt{\sigma}$ $- m_a - m_b$, where s and σ are the total invariant squares of the masses of the whole system and the resonating subsystem R, respectively, then the graph of Fig. 1 develops a singularity in the total energy E when the subenergy ϵ is near threshold $\epsilon \simeq 0$. The position of the singular point is given by

$$E_{1,2} = \frac{(\epsilon + \tilde{\epsilon}_R) \pm 2\alpha\sqrt{\epsilon\tilde{\epsilon}_R}}{1 - \alpha^2}, \tag{1}$$

where $\alpha = m_a/(m_a + m_b)$ and $\tilde{\epsilon}_R = \epsilon_R - i\Gamma/2 = M_R - m_a$ $- m_b - i\Gamma/2$. In (1) we have made the nonrelativis-

tic approximation $E, \epsilon \ll m_a, m_b$ only for reasons of simplicity (all the arguments also hold in the relativistic case).

(2) In general, diagrams like Fig. 1(a) are to be considered as input for some dynamical mechanism such as a multiple-scattering series, N/D equations, unitarity equations, etc. Then the statement of Brayshaw is that the resulting amplitude, using diagram 1(a) as dynamical input, produces a physical resonance, i.e., it contains a pole at the resonance position. In Ref. 1 a whole list of states including mesonic, $N\bar{N}\pi$, and $S\bar{S}M$ systems has been given.

Statement (1) is fully justified. However, many years ago the same mechanism had already been noted by Anisovich *et al.*[4] by Valuev,[4] and independently by Aitchison and Kacser.[5] To find on which Riemann sheets the singularities given in Eq. (1) actually lie, one may use the following expression for the amplitude of diagram 1(b):
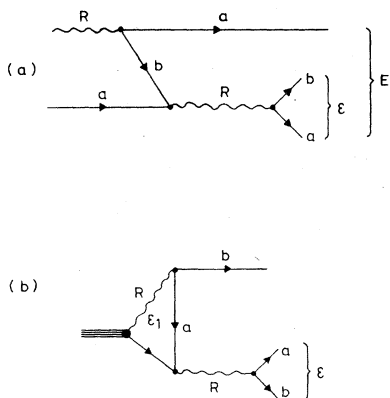

FIG. 1. Exchange graphs for three particles, m_a, m_a, and m_b. R is a resonance formed by a and b.
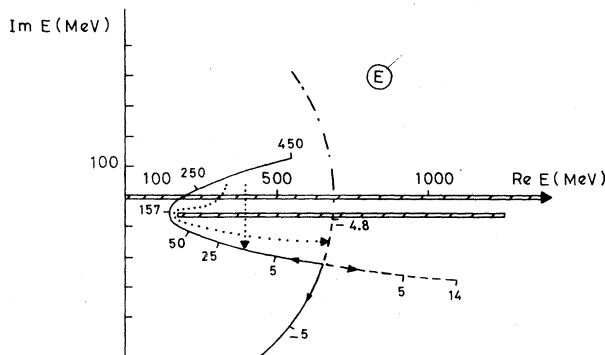

FIG. 2. Trajectories of Landau singularities. Full and dashed-dotted curves are on the second sheet, dashed on the first sheet with respect to the resonance-particle cut. Numbers indicate values of ϵ in MeV.

$$B_1(E,\epsilon) = \frac{B_0}{[2M^*(E-\epsilon)]^{1/2}} \int_0^\infty d\epsilon_1 \frac{\Gamma(\epsilon_1)g(\epsilon)g(\epsilon_1)}{|\epsilon_1 - \epsilon_R + i\Gamma(\epsilon_1)/2|^2} \ln\left(2\frac{(E-\epsilon_1)^{1/2} - \alpha(E-\epsilon)^{1/2} + (1-\alpha^2)^{1/2}\sqrt{\epsilon}}{(E-\epsilon_1)^{1/2} + \alpha(E-\epsilon)^{1/2} + (1-\alpha^2)^{1/2}\sqrt{\epsilon}}\right), \qquad (2)$$

where $M^* = m_a(m_a + m_b)/(2m_a + m_b)$ and Γ and g are defined below. The normalization constant B_0 is arbitrary. The positions of the initial points of the cuts (Landau singularities) in the total energy plane are shown in Fig. 2. The trajectories shown by the solid and dashed-dotted lines lie on the second sheet of the resonance-particle cut. The trajectory shown by the dashed line lies on the first sheet with respect to this cut. Not shown in Fig. 2 are the singularities which lie on the second sheet of the logarithmic cut in the upper half-plane (the three-particle unitarity cut). These singularities lie far from the physical region. (Their position can be obtained by a reflection with respect to the real axis of the trajectories shown in Fig. 2.)

For $\epsilon > \alpha^2(\epsilon_R - \Gamma/2)$ the initial point of the cut cannot produce any bump in the cross section since in this case the path from the physical sheet to that point should round the resonance-particle branch point $E = \epsilon_R - i\Gamma/2$. For $\epsilon < \alpha^2(\epsilon_R - \Gamma/2)$ the path from the physical region to the Landau branch point is straightforward and so we see that the first part of the Brayshaw mechanism is valid (as was also shown in a different context by Anisovich et al.,[4] Valuev,[4] and Aitchison and Kacser[5].

We finally note the Brayshaw statement[3] that for $\epsilon < 0$ ($m_a + m_b < m_{a'} + m_{b'}$) the singularity is shifted to a position very close to the physical region. However, from Fig. 2 we can see that for $\epsilon < 0$ the singularity moves far away from the physical sheet. The singularity which moves towards the real axis lies on a distant Riemann sheet (Fig. 2). Therefore the statement of Brayshaw[3] seems to be incorrect.

Concerning statement (2) we note that the essential step is the integration over the subenergy variable ϵ together with some phase-space factors. Brayshaw only considers the case $\epsilon \simeq 0$, but this region is usually suppressed by phase space. To see quantitatively what the consequences of such an integration are we made a detailed investigation of diagram 1(b). This diagram may be considered as a first step in a dynamical iteration process (by means of some integral equation) with diagram 1(a) as input. Moreover, the cross section calculated from diagram 1(b) is directly related to the norm of the kernel of the dynamical integral equation.

We evaluated expression (2), which is again for simplicity reasons nonrelativistic, for the $N\Delta$ system numerically with the following parametrization[6]:

$$\Gamma(\epsilon) = \Gamma_R\left(\frac{\epsilon}{\epsilon_R}\right)^{1/2} g^2(\epsilon),$$

$$g^2(\epsilon) = \frac{q^2(\epsilon_R) + \gamma^2}{q^2(\epsilon) + \gamma^2}, \qquad (3)$$

$$q^2(\epsilon) = \frac{M_N}{M_\Delta}\epsilon(\epsilon + 2m_\pi),$$

where we used as the radius for the form factor $\gamma = 3m_\pi$. The cross sections are defined by

$$\frac{d\sigma_1(E,\epsilon)}{d\epsilon} = |B_1(E,\epsilon)|^2 \frac{(E-\epsilon)^{1/2}}{|\epsilon - \epsilon_R + i\Gamma(\epsilon)/2|^2}, \qquad (4)$$

$$\sigma_1(E) = \int_0^E d\epsilon \frac{d\sigma_1(E,\epsilon)}{d\epsilon}. \qquad (5)$$

In Fig. 3 we show the shape of $d\sigma_1/d\epsilon$. It is essentially determined by the position of 2 branch points: the branch point of the resonance-particle cut at $E_R = \epsilon_R - i\Gamma/2 = (157 - 57.5i)$ MeV and the starting point $E_s(\epsilon)$ of the triangle singularity. For $\epsilon = \epsilon_R/100 = 1.57$ MeV there is a broad bump in the cross section. In this case $E_s(\epsilon_R/100) = (548 - 220i)$ MeV whereas $E_s(0) = (658 - 241i)$ MeV. So for small values of ϵ the singular point moves rather quickly. For $\epsilon = \epsilon_R/2 = 78.5$ MeV the triangle singularity lies close to the resonance-particle threshold and we see a pronounced maximum for $E \simeq E_R$. For $\epsilon > \epsilon_R - \Gamma/2$ the cross section is again a smooth function of the energy as has been noted above.

The Brayshaw mechanism[1-3] predicts a bump in the cross section $\sigma_1(E)$ near $E = \text{Re}E_s(0) \simeq 658$ MeV. However, from Fig. 4 we see that there is no bump at all near this energy. There is only a weak maximum of the cross section near $E \simeq E_R$
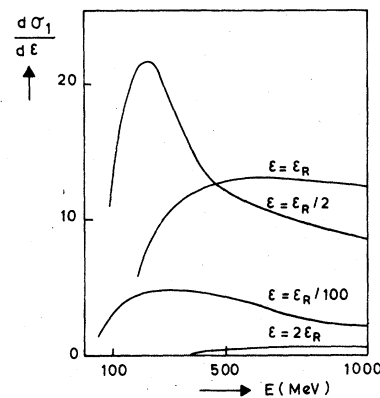

FIG. 3. Cross sections $d\sigma_1/d\epsilon$ for various values of ϵ as a function of E (units are arbitrary).
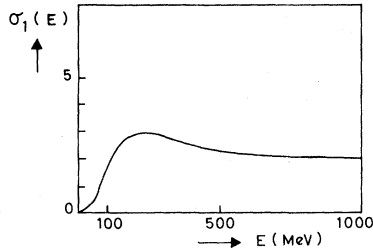
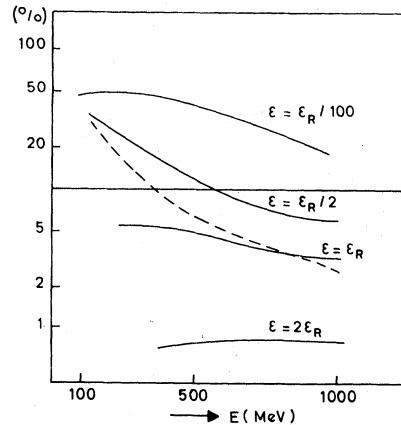FIG. 4. The total cross sections $\sigma_1(E)$ in arbitrary units.

FIG. 5. Ratios of cross sections in percent. Solid curve corresponds to $(d\sigma_1/d\epsilon)/(d\sigma_1/d\epsilon + d\sigma_0/d\epsilon)$ and dashed curve to $\sigma_1/(\sigma_0 + \sigma_1)$.

which corresponds to the resonance-particle threshold (resonance-particle unitarity cut). Therefore we see that in this case the Brayshaw mechanism does not work [at least concerning statement (2)].

Moreover, the contribution of the triangle graph to the cross section (σ_1) is small compared to the contribution of the graph without rescattering (σ_0). The cross sections $d\sigma_0/d\epsilon$ and σ_0 are easily obtained from Eqs. (4) and (5) by replacing the amplitude $B_1(E, \epsilon)$ by the constant B_0. In Fig. 5 we plot the ratios $(d\sigma_1/d\epsilon)/(d\sigma_1/d\epsilon + d\sigma_0/d\epsilon)$ and $\sigma_1/(\sigma_0 + \sigma_1)$ which are now of course independent of B_0.

We believe the above results will also hold for a completely relativistic treatment. Explicit calculations have shown that the relativistic trajectories of the Landau singularities lie very close to the nonrelativistic ones. Also the inclusion of spins, angular momenta, and isospins will not alter our conclusions since they cannot change the positions of the singularities.

Note that apart from the above-mentioned Brayshaw mechanism we also have the usually much stronger singularities arising from the normal resonance-particle cut. Detailed investigations by Badlyan and Simonov[7,8] have shown that the relevant parameter in this case is $\lambda = (\Gamma/\epsilon_R)f$ where Γ and ϵ_R are the width and excitation energy of the resonance R and f is a

smooth function of m_a, m_b and energy. For $\lambda \gg 1$ the strong resonance-particle interaction may produce bound states and resonances.[7] However, most systems in nature have $\lambda \ll 1$, for example the $\rho\pi$, whereas for the $N\Delta$ system $\lambda \lesssim 1$.[7] Therefore in those cases even the strong resonance-particle interaction is too weak to produce any bound states or resonances (see also the work of Ascoli and Wyld[9]).

Keeping the experience with the resonance-particle interaction in mind and noticing that the Brayshaw mechanism refers to an even weaker process, we feel that this mechanism will turn out to be unable to account for the proposed resonances in Ref. 1.

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