Parton-model relation without quantum-chromodynamic modifications in lepton pair production

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It is pointed out that, in contrast to the recently discovered large quantum-chromodynamics (QCD) correction to the Drell-Yan formula, a certain quark-parton-model relation between structure functions for lepton pair production is not subject to any first-order QCD modification. Both parallelism and contrast to the Callan-Gross relation in deep-inelastic scattering are spelled out. Implications on the lepton angular distribution for both low and high $q_\perp$ are discussed. The case is made that this relation provides a unique opportunity to test the "QCD-improved quark-parton model."

Recent quantum-chromodynamics (QCD) calculations[1] reveal a large (~100%) order-α_s correction to the Drell-Yan cross-section formula[2] for lepton pair production (LPP) in hadron collisions. This makes the integrated cross section a dubious testing ground for the QCD-corrected parton model, and raises fundamental questions about the viability of the perturbative QCD approach. It is, therefore, important to study other aspects of this approach.

There is much more to the quark-parton model and its QCD modifications in LPP than just the integrated Drell-Yan cross-section formula. The lepton angular distributions are controlled by structure functions which obey parton-model relations[3,4] similar to those between F_1 and F_2 in deep-inelastic scattering (DIS). How are these relations affected by perturbative QCD corrections? The answer to this question is quite surprising: At least one of these relations—the exact counterpart of the Callan-Gross[5] relations—is not modified at all by first-order QCD corrections, although individual terms in this relation may be subject to large corrections. In the rest of this note, we spell out explicitly the parallelism as well as the contrast between the DIS and LPP cases in the perturbative QCD approach and discuss the experimental implications of these results.

The LPP process is described by a tensor amplitude[6] $W_{\mu\nu}$ corresponding to the current correlation function

$$W_{\mu\nu}=\mathcal{S}\int d^4z\, e^{iq\cdot z}\,\langle P_1P_2|J_\mu(z)J_\nu(0)|P_1P_2\rangle \qquad (1)$$

similar to that of DIS. Here P_1 and P_2 are the four momenta of the colliding hadrons, $\mathcal{S}=-(P_1+P_2)^2$, and q is the momentum of the virtual photon (hence the lepton pair). The trace of this tensor is related to the cross section (integrated over the lepton center-of-mass angles) via the formula[2,4]

$$\frac{d\sigma}{d^4q}=\left(\frac{\alpha}{M\mathcal{S}}\right)^2\frac{1}{(2\pi)^3}W^\mu_\mu\,, \qquad (2)$$

wherein M represents the effective mass of the lepton pair and α the fine structure constant.

The tensor $W_{\mu\nu}$ can be decomposed into four structure functions[4] in much the same way as in DIS:

$$W_{\mu\nu}=W_1\tilde{g}_{\mu\nu}+W_2\tilde{P}_\mu\tilde{P}_\nu-W_3(\tilde{P}_\mu\tilde{p}_\nu+\tilde{P}_\nu\tilde{p}_\mu)+W_4\tilde{p}_\mu\tilde{p}_\nu\,, \qquad (3)$$

where $\tilde{g}_{\mu\nu}=g_{\mu\nu}-q_\mu q_\nu/q^2$, $P=P_1+P_2$, $p=P_1-P_2$, $\tilde{P}_\mu=\tilde{g}_{\mu\nu}P^\nu/\sqrt{\mathcal{S}}$, and $\tilde{p}_\mu=\tilde{g}_{\mu\nu}p^\nu/\sqrt{\mathcal{S}}$. These structure functions can be determined experimentally from Eq. (2) together with lepton angular distribution measurements. Equation (3) closely resembles the corresponding formula defining W_1 and W_2 in DIS (where the last two terms are absent).

A basic result of the quark-parton model in DIS is the Callan-Gross relation.[5] It is usually written as

$$W_L=-W_1+\left(\frac{\nu^2}{q^2}-1\right)W_2=0\,, \qquad (4)$$

where W_L is the helicity structure function for the longitudinal virtual photon. We can recast this result in the not-so-familiar form:

$$W^\mu_\mu=2W_1\,. \qquad (5)$$

In Ref. 4 we showed that *the same equation follows as a general consequence of the quark-parton model in LPP.* In terms of the invariant structure functions W_1 to W_4 of LPP, Eq. (5)

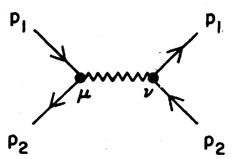

FIG. 1. Lowest-order diagram for LPP cross section.

takes the form

$$W_1 + \left(\frac{(q\cdot P)^2}{sM^2} - 1\right) W_2 - \frac{(q\cdot P)(q\cdot p)}{sM^2}\, W_3 + \left(1 + \frac{(q\cdot p)^2}{sM^2}\right) W_4 = 0, \qquad (6)$$

whereas in terms of the helicity structure functions

$$W_L = 2W_{\Delta\Delta}\,. \qquad (7)$$

Here $W_{\Delta\Delta}$ is the "double-flip" structure function.[4] These results are to be compared with Eq. (4) for DIS. Of the three equivalent versions, Eqs. (5), (6), and (7), only the first one is form-invariant when going over from LPP to DIS.

It is known that in perturbative QCD the structure functions become functions of q^2. In addition:

(i) for DIS, the Callan-Gross relation is modified by small first-order QCD terms[7]; and

(ii) for LPP, the Drell-Yan cross-section formula, which is essentially W^μ_μ [(cf. Eq. (2)] is subject to very substantial first-order corrections.[1]

It is natural, therefore, to ask how Eq. (5) is affected by QCD effects in LPP.

If we represent the basic parton-parton amplitude by Fig. 1, then first-order QCD corrections come in three forms: the "annihilation" diagrams, Fig. 2; the "Compton" diagrams, Fig. 3; and the "vertex correction" diagrams, Fig. 4. The annihilation and Compton diagrams have

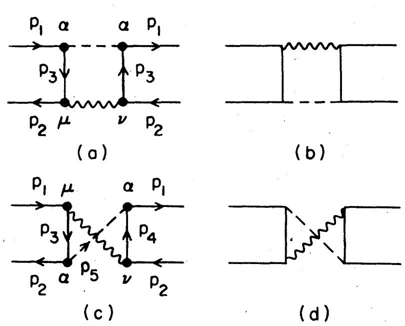

FIG. 2. Order-α_s "annihilation diagrams" for LPP cross section.

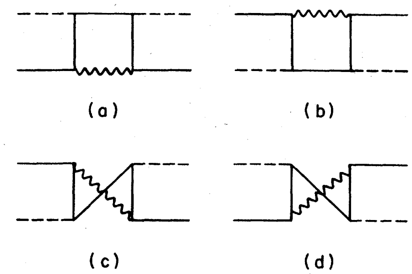

FIG. 3. Order-α_s "Compton diagrams" for LPP cross section.

been studied by us[8] and others[9,10] in connection with high-q events for which they should represent the dominant mechanism. It has been noticed[8] that the parton-model structure-function relation, Eq. (5), is also satisfied by these QCD diagrams. Since this result holds for all values of $q_\perp$, it means that these diagrams do not introduce any first-order QCD correction to the parton relation Eq. (5) even at low $q_\perp$. We now point out that the remaining first order-diagrams Fig. 4, give rise to an amplitude with the same tensor structure as the zeroth-order amplitude of Fig. 1—as any anomalous magnetic moment term from the modified vertex drops out upon taking the trace of the Dirac matrices. It follows then, even though the overall normalization may be corrected by some factor, the decomposition into invariant structure functions is not affected and Eq. (5) remains intact.

We conclude, therefore, in LPP the parton-model relation $W^\mu_\mu = 2W_1$ is not modified by first-order QCD corrections at all—in contrast to both the Callan-Gross relation (for DIS) and the Drell-Yan cross-section formula [essentially W^μ_μ, cf. Eq. (2)]. This appears to be a rather remarkable result; we are not aware of any other parton-model result which is not affected by QCD corrections. For this reason, we sketch in the Appendix a derivation of Eq. (5) from the diagrams Figs. 1–4 which is more direct than those given before.[8-10] It will be interesting to find out to what extent this result can be extended to higher-order diagrams.

We now make a few additional remarks about Eq. (5) and its experimental implications:

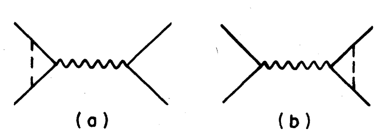

FIG. 4. Order-α_s "vertex-correction diagrams" for LPP cross section.

(i) The relation $W^{\mu}_{\mu}=2W_1$ is closely related to the spin-$\frac{1}{2}$ nature of the charged parton (i.e., quark). The fact that this relation is not modified by first-order QCD effects when both sides themselves may be subject to corrections of order 100% certainly represents a unique signature of the QCD-quark-parton picture.

(ii) In terms of helicity structure functions, this relation takes the form $W_L=2W_{\Delta\Delta}$, Eq. (7). Although for LPP, the helicity structure functions depend on the choice of coordinate axes[4] (e.g., Gottfried-Jackson, Collins-Soper, etc.), *this relation remains frame independent*—i.e., if the QCD-quark-parton model is correct, the two structure functions W_L and $W_{\Delta\Delta}$ must be related by Eq. (7), for *any* choice of axes in the lepton-pair center-of-mass frame. This strong result again demonstrates the significance of this relation.

We know the angular distribution of the leptons in the rest frame of the pair is given by[4]

$$\frac{d\sigma}{d^4q\,d\Omega^*}=\frac{1}{2}\,\frac{\alpha}{(2\pi)^4}\left(\frac{\alpha}{M\mathcal{S}}\right)^2 \times\left[W_T(1+\cos^2\theta^*)+W_L(1-\cos^2\theta^*)+W_\Delta\sin2\theta^*\cos\phi^*+W_{\Delta\Delta}\sin^2\theta^*\cos2\phi^*\right]. \tag{8}$$

Written in the form

$$\frac{d\sigma}{d^4q\,d\Omega^*}\propto 1+\alpha\cos^2\theta^*+\beta\sin2\theta\cos\phi^*+\gamma\sin^2\theta^*\cos2\phi^*, \tag{9}$$

Eq. (7) implies

$$1-\alpha=4\gamma \tag{10}$$

independent of the choice of axes.

(iii) For $q_\perp^2\ll M^2$, kinematic constraints require $W_{\Delta\Delta}$ to be small[4] and of order $q_\perp^2/M^2$. The dynamical relation Eq. (7) restricts W_L to stay the same as $2W_{\Delta\Delta}$, hence it must also be small. The angular distribution of the leptons should be close to $1+\cos^2\theta^*$. [W_Δ is required by kinematics to be $O(q_\perp/M)$ and small also.] Now, make the transition to the large-$q_\perp$ region. There is no reason for $W_{\Delta\Delta}$ to stay small: the angular distribution will be very different from $1+\cos^2\theta^*$, but Eq. (7) is still in force and W_L must keep pace with the change in $W_{\Delta\Delta}$ as $q_\perp$ increases. Note, although the rate of high-$q_\perp$ events is of order $\alpha_s(M^2)$, the predicted change in angular distribution from low to high $q_\perp$ is of order 1 and the relation between W_L and $W_{\Delta\Delta}$ remains precise throughout. In other words, the test of QCD furnished by this relation does not concern just small correction effects; it involves quantities of order unity.

For comparison, in DIS, the Callan-Gross relation reads $W_L=0$ in the parton limit and acquires a small α_s correction when QCD is taken into account. Since there is no $q_\perp$ variable in DIS, the transition in angular distribution discussed above simply does not exist here.

(iv) So far, we have neglected all masses (m_i) and intrinsic transverse momenta ($P_{i\perp}$) of the partons inside the hadron. It is relatively easy to see that incorporating these effects in the zeroth-order parton model (Fig. 1) incurs order m_i^2/M^2 and $p_{i\perp}^2/M^2$ corrections to the basic relation Eq. (5).[4] In current QCD jargon, these represent "higher-twist" effects. In DIS as well as elsewhere, the higher-twist effects occur alongside with calculable QCD (twist two) correction terms, thereby complicating the phenomenology considerably. Because of the unique situation here in LPP where first-order QCD correction is absent, deviations from Eqs. (5)–(7) can only come from the higher-twist effects. Therefore, we have a cleaner source of information on the size of $\langle p_{i\perp}^2\rangle$, etc.

Incorporating parton intrinsic transverse momenta in first-order QCD diagrams, Figs. 2–4, is much more complicated. However, here the effects are of order $\alpha_s p_{i\perp}^2/M^2$ and represent only a very small perturbation if M^2 is large.

(v) One may wonder why Eq. (5) is not subject to first-order QCD corrections in LPP while the same relation (i.e., Callan-Gross relation) in DIS is—after all, the Feynman diagrams involved in the two cases are identical except for line reversal. The technical explanation for this discrepancy lies in the fact that in DIS, corrections to the Callan-Gross relation come from Figs. 2 and 3 as a result of integration over the momenta of the two final-state partons; by contrast, in LPP, there is only one parton in the final state—the momentum of which is fixed by conservation, hence, there is no integration involved.

(vi) We mention, for completeness, that in addition to Eq. (5), there are other parton-model relations[4] which follow from the Drell-Yan mechanism for low $q_\perp$ and from the QCD annihiliation diagrams, Fig. 2, for high $q_\perp$.[8,10] Those relations are interesting in their own right and, in conjunction with Eq. (5), yield definite predictions on the angular distribution of the leptons. However, these predictions *do* depend on the choice of helicity frames and are subject to QCD corrections, hence are not as striking as Eq. (5).

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APPENDIX

In the literature, all structure-function relations were derived by first calculating the full tensor amplitude. A more direct derivation of $W^{\mu}_{\mu}=2W_1$ is given in this appendix. The Feynman gauge is used and partons are assumed to be massless.

In Figs. 1 and 2(a), the cross-section tensor $W_{\mu\nu}$ are, respectively, proportional to

$$T^{(1)}_{\mu\nu}=\mathrm{Tr}(\gamma_\mu\gamma\cdot p_1\gamma_\nu\gamma\cdot p_2)\equiv T^{(1)}_1 g_{\mu\nu}+\cdots \tag{A1}$$

and

$$T^{(2(a))}_{\mu\nu}=\mathrm{Tr}(\gamma_\mu\gamma\cdot p_3\gamma_\alpha\gamma\cdot p_1\gamma^\alpha\gamma\cdot p_3\gamma_\nu\gamma\cdot p_2)$$
$$=T^{(2(a))}_1 g_{\mu\nu}+\cdots \; . \tag{A2}$$

Using the identities

$$\gamma_\mu\gamma_{\alpha_1}\gamma_{\alpha_2}\cdots\gamma_{\alpha_{2n+1}}\gamma^\mu=-2\gamma_{\alpha_{2n+1}}\cdots\gamma_{\alpha_2}\gamma_{\alpha_1}, \tag{A3}$$

$$\mathrm{Tr}(\gamma_\mu\gamma_{\alpha_1}\gamma_{\alpha_2}\cdots\gamma_{\alpha_{2n+1}}\gamma_\nu\gamma_{\beta_1}\gamma_{\beta_2}\cdots\gamma_{\beta_{2m+1}})$$
$$=-g_{\mu\nu}\mathrm{Tr}(\gamma_{\alpha_1}\gamma_{\alpha_2}\cdots\gamma_{\alpha_{2n+1}}\gamma_{\beta_1}\cdots\gamma_{\beta_{2m+1}})+\cdots \, , \tag{A4}$$

we immediately obtain the relation $T^{\mu}_{\mu}=2T_1$ for both Fig. 1 and Fig. 2(a). Similarly we can drive the relation for Figs. 2(b), 3(a), and 3(b).

The derivation for Figs. 2(c), 2(d), 3(c), and 3(d) are more complicated in that the massless nature of the partons have to be used. We merely exhibit how this is done for Fig. 2(c). There

$$T^{(2(c))}_{\mu\nu}=\mathrm{Tr}(\gamma_\mu\gamma\cdot p_1\gamma_\alpha\gamma\cdot p_4\gamma_\nu\gamma\cdot p_2\gamma^\alpha\gamma\cdot p_3)$$
$$=-2\;\mathrm{Tr}(\gamma_\mu\gamma\cdot p_1\gamma\cdot p_2\gamma_\nu\gamma\cdot p_4\gamma\cdot p_3)$$
$$=-2g_{\mu\nu}\mathrm{Tr}(\gamma\cdot p_1\gamma\cdot p_2\gamma\cdot p_4\gamma\cdot p_3)+\cdots \; .$$

Using $p_3=p_2+p_5$, $p_4=p_1-p_5$, $p_1^2=p_2^2=p_5^2=0$, we get

$$T^{(2(c))}_1=-8(p_1\cdot p_2 p_3\cdot p_4-p_1\cdot p_4 p_2\cdot p_3$$
$$+p_1\cdot p_3 p_2\cdot p_4)$$
$$=-16p_1\cdot p_2(p_1\cdot p_2-p_2\cdot p_5+p_1\cdot p_5)$$
$$=-16p_1\cdot p_2 p_3\cdot p_4=\tfrac{1}{2}T^{(2(c))\mu}_{\mu}\; .$$

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