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Remarks on causality and electromagnetic mass shifts

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We remark on the work of Bardek and Zovko related to our previous work on the effect of causality on hadron electromagnetic mass differences.

With reference to the work of Bardek and Zovko[1] related to our paper[2] on causality and the electromagnetic proton-neutron mass difference, we believe that the issue would be further clarified by noting the following points:

(1) The causality sum rules that we use are not of the fixed-mass form of Eq. (9) in Ref. 1, but are of the form[2,3]

$$\int_{-\infty}^{\infty} t_i(q^2, \nu) dq_0 = 0\,, \quad i = 1, 2\,. \tag{1}$$

Reduction of these integrals to the fixed-mass form is possible only under special conditions.[4] Consequently the causality sum rules (1) are not "just the zero-moment superconvergence sum rules for $t_{1,2}$." In particular the issue of the convergence of the integrals in these sum rules cannot be decided by consideration of Regge asymptotic behavior. One further notes that the causality-modified electromagnetic mass difference formula[2] is obtained by imposing the causality conditions (1) as they stand into the original Cottingham formula, before transformation into a fixed-mass form.

(2) We have remarked[2] on the possible validity of these sum rules for the nucleon electromagnetic amplitudes, referring to the work of Leutwyler and Stern[5] on the nucleon electromagnetic structure functions and noting that the asymptotic conditions on the Jost-Lehmann-Dyson spectral functions are the same for these two cases. This furnishes a plausible basis for our application of the causality-modified formula to the proton-neutron electromagnetic mass difference. It is not clear to us whether there are grounds for the assumption that the integrals in these sum rules also converge for the electromagnetic pion amplitudes.

(3) In Ref. 2 we remarked that causal amplitudes satisfy relations "in which the Born contributions are exactly balanced by the non-Born contributions. This feature indicates that the expectation of a satisfactory result from the Born term by itself is rather unfounded." This is substantiated by both our calculation[2] and that of Bardek and Zovko.[1] Let us, however, consider a case in which the original Cottingham formula gives a reasonably good—or, for the sake of argument, an exact—result from the Born term alone. Assume, further, that conditions for the validity of the causality sum rules are satisfied. Denote by Δm_0, Δm_c, and Δm_e the original, causality-modified, and experimental mass difference respectively. Then

$$\Delta m_0^B = \Delta m_e\,, \tag{2}$$

where B refers to the Born contribution. But

$$\Delta m_0^B = \Delta m_c^B + \frac{1}{4\pi i} \int (3t_1 + t_2)^B d^4q\,. \tag{3}$$

Thus the expectation $\Delta m_c^B = \Delta m_e$ will be realized only if

$$\int (3t_1 + t_2)^B d^4q = 0\,. \tag{4}$$

This then requires that the causality sum rule

$$\int (3t_1 + t_2) d^4q = 0 \tag{5}$$

be saturated by the Born-term contribution. When this is not realized $\Delta m_0^B = \Delta m_e$ does not imply $\Delta m_c^B = \Delta m_e$, and one should regard the success of the Born-term calculation to be purely accidental. It is obvious that, in the case of the pion electromagnetic mass difference, saturation of the causality sum rules by the Born term is not realized, so that $\Delta m_0^B \neq \Delta m_c^B$.

(4) Finally, we would like to suggest an extension of this work by the construction of parameter-free causal continuations of the Born contributions to t_1 and t_2. When these are substituted into the original mass-shift expression one must have $\Delta m_0^B = \Delta m_c^B$, since Eq. (4) would automatically hold for the causally-continued Born terms. It would then be interesting to see if approximate agreement with the experimental value for Δm_π is still maintained.

[1]V. Bardek and N. Zovko, preceding paper, Phys. Rev. D 21, 2706 (1980).
[2]A. M. M. Abdel-Rahman and M. O. Taha, Phys. Rev. D 15, 2472 (1977).
[3]J. W. Meyer and H. Suura, Phys. Rev. 160, 1366 (1967).
[4]M. O. Taha, Nuovo Cimento 30A, 144 (1975). In particular see Eq. (2.18) of this reference.
[5]H. Leutwyler and J. Stern, Phys. Lett. 31B, 458 (1970).