Comments and Addenda

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Impact of causality on electromagnetic mass shifts of hadrons

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The impact of some asymptotic bounds and causality conditions for pion and nucleon Compton amplitudes on pion and nucleon mass shifts is considered.

I. INTRODUCTION

By imposing certain causality conditions, Abdel-Rahman and Taha[1] have recently come to a rather revolutionary result that in the well-known Cottingham formula for electromagnetic mass shifts only the invariant amplitude $t_2(q^2,\nu)$ contributes to the proton-neutron mass difference:

$$m_p - m_n = -\frac{i}{2\pi}\int \frac{d^4q}{q^2}\nu^2 t_2(q^2,\nu)\,, \quad \nu \equiv p\cdot q/m\,. \tag{1}$$

This causality-modified Cottingham formula relies strongly on a fundamental dynamical assumption: The Jost-Lehmann-Dyson (JLD) spectral function $\psi(u,s)$ is assumed to be so bounded at high energies s that $s\psi(u,s)\to 0$ for $s\to\infty$. However, the evaluation of the t_2 contribution in the nucleon case leads to regularizational difficulties,[1] so that it is not possible to draw a clear-cut conclusion.

In order to test these ideas, we considered the mass shift of the pion multiplet. If it is not a misleading illusion, this is the only case theoretically well understood. Calculations within different frameworks have shown that the generalized Born term explains the observed mass shift in a satisfactory way[2]; this means that the contribution of inelastic intermediate states must be small. Either all these terms are small (in accord with Harari's arguments[3] along the line of absence of relevant Regge recurrences and validity of unsubtracted dispersion relations), or some among them are of significant size and cancellations occur. In either case, the pion mass shift, properly evaluated, is determined only by the elastic contribution.

II. CAUSALITY-MODIFIED PION MASS SHIFT

The elastic contribution to the pion mass shift, known for two decades, reads[4]

$$\delta(m_\pi{}^2)_{\rm el} = -\frac{i\alpha}{4\pi^3}\frac{1}{2}\int \frac{d^4q\,F_\pi(q^2)}{q^2}g_{\mu\nu}\left[\frac{(2p+q)^\mu(2p+q)^\nu}{q^2+2p\cdot q}+\frac{(2p-q)^\mu(2p-q)^\nu}{q^2-2p\cdot q}-2g^{\mu\nu}\right], \tag{2}$$

where the factor $\frac{1}{2}$ in front of the integral comes from the $q\rightleftarrows -q$ symmetrization of the integrand, which ensures the manifest gauge covariance. Comparing this expression with the relation

$$\delta(m_\pi{}^2) = \frac{i}{2(2\pi)^4}\int \frac{d^4q}{q^2}T_{\mu\nu}(p,q)g^{\mu\nu}\,, \tag{3}$$

where $T_{\mu\nu}$ is conventionally decomposed as

$$T^{\mu\nu}(p,q) = D_1^{\mu\nu}t_1(q^2,\nu) + D_2^{\mu\nu}t_2(q^2,\nu)\,, \tag{4}$$

$$D_1^{\mu\nu} \equiv q^2 g^{\mu\nu} - q^\mu q^\nu\,, \tag{5}$$

$$D_2^{\mu\nu} \equiv \nu^2 g^{\mu\nu} + q^2\frac{p^\mu p^\nu}{m^2} - \frac{\nu}{m}(p^\mu q^\nu + p^\nu q^\mu)\,, \tag{6}$$

we easily find the elastic contributions to the invariant amplitudes t_1 and t_2:

$$t_1^{\rm el}(q^2,\nu)=\frac{8\pi\alpha q^2 F_\pi{}^2(q^2)}{q^4-4m_\pi{}^2\nu^2}\,, \tag{7}$$

$$t_2^{\rm el}(q^2,\nu)=-\frac{32\pi\alpha m_\pi{}^2F_\pi{}^2(q^2)}{q^4-4m_\pi{}^2\nu^2}\,. \tag{8}$$

By inserting the decomposition (4)–(6) into Eq. (3) and applying the causality sum rules[1]

$$\int_{-\infty}^{\infty} d\nu\, t_{1,2}(q^2,\nu)=0\,, \tag{9}$$

we finally obtain the pion mass shift in the elastic approximation:

$$\delta(m_\pi{}^2)_{\rm el}=-\frac{i\alpha}{\pi^3}\int\frac{d^4q}{q^2}\,\frac{p\cdot q}{q^2-2p\cdot q}F_\pi{}^2(q^2)\,. \tag{10}$$

By evaluating the integral (10) using Feynman's standard method of symmetric integration, we see that the photon mass q^2 will always stay in the region of electron-pion scattering due to the $q_0\to iq_0$ rotation. In this region the pion form factor is real and, being far from the cut, may be parametrized only by a simple ρ pole. In this way we obtain

$$m_{\pi^+}-m_{\pi^0}=0.23\ \text{MeV}\,, \tag{11}$$

which, in comparison with the usual evaluation giving typically 4.2 MeV (Ref. 4) and the experimental value of 4.6 MeV, is too small by more than an order of magnitude.

III. DISCUSSION

The above disagreement most probably indicates that the high-energy bound[1] assumed for the JLD spectral function in the causal representation for the amplitudes $t_{1,2}(q^2,\nu)$ was too strong. It is, in fact, easier to incorporate certain dynamical constraints in the form of high-energy bounds directly for the amplitudes $t_{1,2}$. The causality sum rules (9) are just the zero-moment superconvergence sum rules for $t_{1,2}$.

Let us consider when these sum rules might hold. As discussed for the first time by Harari,[3] the Regge-pole theory suggests the asymptotic behavior[4]

$$t_1\propto\nu^{\alpha(0)}\,,\quad t_2\propto\nu^{\alpha(0)-2}\,,\quad \nu\to\infty \tag{12}$$

where $\alpha(0)$ is the zero-energy intercept of relevant trajectories. It is negative for the pion, $\alpha(0)<0$, and positive for the nucleon, $\alpha(0)\simeq0.5$. Therefore, the usual analyticity properties require the validity of the zero-moment sum rule only for the t_2 amplitude in the pion case,

$$\int_{-\infty}^{\infty} t_2(q^2,\nu)\,d\nu=0\,. \tag{13}$$

Neither t_1 nor t_2 is superconvergent in the nucleon case; t_1 requires even a subtracted dispersion relation.

In the pion rest frame, $\vec{p}=0$, we have $dq_0=d\nu$, and in the full contribution

$$\delta m_\pi{}^2\sim\int d^3q\int_{-\infty}^{\infty}d\nu\left(3t_1+t_2+\frac{2\nu^2}{q^2}t_2\right), \tag{14}$$

only the middle term may be left out as a consequence of the sum rule (13). In this case we obtain

$$m_{\pi^+}-m_{\pi^0}=3.64\ \text{MeV}\,, \tag{15}$$

which is not too far from 4.2 MeV.

Finally, we also have to allow for the possibility that the Regge asymptotics here adopted are not fully justified, and that the causality sum rules (9) have indeed to be fulfilled. In this case the application of these ideas to the pion multiplet clearly leads to violent disagreement with experiments. We believe that this would also happen to the nucleon if we were able to evaluate properly the t_2 contribution (1). This suggests that the asymptotic bound imposed on the JLD spectral function in Ref. 1 is too strong.

[1] A. M. M. Abdel-Rahman and M. O. Taha, Phys. Rev. D 15, 2472 (1977).
[2] See, for example, A. Zee, Phys. Rep. 3C, 127 (1972).
[3] H. Harari, Phys. Rev. Lett. 17, 1303 (1966).
[4] See, for example, F. M. Renard, Nucl. Phys. B7, 183 (1968).