Partons at low P_T

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We propose a fragmentation mechanism to interpret in a parton framework the Pomeron contribution which dominates high-energy low-P_T hadronic reactions. We successfully compare our predictions to experimental data on one-particle inclusive distributions. We derive a pure parton expression of the Pomeron contribution which clarifies the theoretical understanding of high-energy interactions.

I. INTRODUCTION

In a preceding paper[1] (hereafter denoted as CTHKP) we have proposed a new approach to the confinement problem which consists of looking for correspondences between quantum chromodynamics (QCD) and the dual topological unitarization scheme (DTU). These correspondences allow, on one hand, the use of Regge phenomenological tools to get information on quark structure[2] and fragmentation functions and, on the other hand, the clarification of the parton interpretations of soft, or low-P_T hadronic reactions.

In the present paper, we focus on the Pomeron contribution which dominates high-energy low-P_T hadronic reactions. Whereas we had proposed a recombination interpretation of this contribution in CTHKP, we derive now an alternative interpretation, basically a fragmentation mechanism, in which quark-fragmentation functions are convoluted with a dual weight describing the jet configuration.

The dual weight is derived from a dual multiperipheral model.[3] We get in this way a model with no free parameter, which is successfully compared to experimental data on meson fragmentation.

Finally, we propose a model-independent parton expression for the dual weight itself, leading to a pure parton interpretation of the Pomeron contribution. We comment on the important theoretical implications of this scheme.

II. DERIVATION OF THE MODEL

A. Fragmentation and planar cross sections

Dual topological unitarization consists of decomposing the S-matrix elements into an infinite series of topological components characterized by the number of boundaries and handles of the associated dual diagrams. This topological expansion, which is expected to converge rapidly, is dominated by the first two components: the planar and the cylindrical ones (see Figs. 1 and 2) corresponding, through unitarity, to Reggeon and Pomeron exchange.

The parton interpretation of the one-particle inclusive planar cross section, sketched in Ref. 4 and proposed in the DTU scheme in the CTHKP paper, is exhibited in the factorized expression (see Fig. 3)

$$x_H \frac{d\sigma^{\text{planar}}}{dx_H}(A \underset{B}{\to} H) = g^2 \left(\frac{s}{s_0}\right)^{\alpha_{ii}(0)-1} x_H D_a^H(x_H)\,, \qquad (1)$$

where $g^2(s/s_0)^{\alpha_{ii}(0)-1}$ is the total planar cross section interpreted as the cross section of the quark-liberating process $AB \to ab$, and $D_a^H(x_H)$ is the quark-fragmentation function of quark a into hadron H with fraction of momentum x_H. However, this fragmentation interpretation of DTU is suited only for the planar contribution. In order to interpret in a parton framework the cylindrical (dominant at high energy) contribution, a recombination mechanism was proposed. What we want to do here is to show that it is actually possible to generalize Eq. (1) to the Pomeron contribution, and thus to provide this contribution with a fragmentation interpretation.

B. Generalization to the cylindrical contribution

In DTU, the cylindrical contribution is built out of two planar dual sheets. Since the multiproduction in a one-dual-sheet process is interpreted as the fragmentation of the quarks bordering the sheet, the fragmentation interpretation of the cylindrical contribution implies, for the final hadronic state, a four-jet structure where the jets are initiated by the four valence quarks of the incoming hadrons (supposed to be mesons for simplicity) and are elongated along the scattering axis.

Following the notations of Fig. 4, the fraction of momentum carried by each quark, x_A and $(1-x_A)$ for the quarks of A, x_B and $(1-x_B)$ for the quarks of B, can be intrinsically defined in duality by the sum of the energy-momenta of all the particles of each sheet, expressed in the c.m. system:

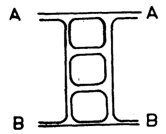

FIG. 1. The planar term.

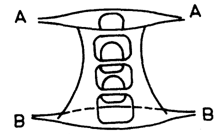

FIG. 2. The cylindrical term.

$$E_1 = x_A \tfrac{1}{2}\sqrt{s} + (1-x_B)\tfrac{1}{2}\sqrt{s}, \quad k_1 = x_A \tfrac{1}{2}\sqrt{s} - (1-x_B)\tfrac{1}{2}\sqrt{s},$$

$$E_2 = (1-x_A)\tfrac{1}{2}\sqrt{s} + x_B \tfrac{1}{2}\sqrt{s}, \quad k_2 = (1-x_A)\tfrac{1}{2}\sqrt{s} - x_B \tfrac{1}{2}\sqrt{s}.$$

Therefore, considering a dual weight $\mathcal{P}(x_A, x_B)$ for a given two-sheet configuration (x_A, x_B), the one-particle inclusive distribution is obtained by making the convolution of $\mathcal{P}$ with the appropriate quark-fragmentation functions. So, again with the notations of Fig. 3, we have

$$x_H \frac{d\sigma}{dx_H}(A \underset{B}{\to} H) = \iint_0^1 dx_A dx_B \,\mathcal{P}(x_A, x_B)\left[\frac{x_H}{x_A} D_a^H\left(\frac{x_H}{x_A}\right)\Theta(x_A - x_H) + \frac{x_H}{1-x_A} D_b^H\left(\frac{x_H}{1-x_A}\right)\Theta(1 - x_A - x_H)\right]. \tag{2}$$

C. Derivation of the dual weight $P(x_A, x_B)$

The dual weight is related to the total inelastic cross section through

$$\sigma_{\text{inel}} = \iint_0^1 dx_A dx_B \,\mathcal{P}(x_A, x_B). \tag{3}$$

In order to give an expression for $\mathcal{P}(x_A, x_B)$ we use DTU arguments. At the first step of the topological unitarization σ_{tot} is determined, through unitarity, by the dual multiperipheral production mechanism. Nontwisted diagrams give the planar, Reggeon, contribution whereas the sum of all twisted diagrams gives the cylindrical, Pomeron, contribution.

The rapidity ordering, which is characteristic of multiperipheral dynamics,[5] is maximal in the planar case, while the summation over all possible twists leads, for the Pomeron contribution, to a weaker ordering in the sense that there are now two uncorrelated sets of ordered particles (two dual planar sheets).

Following Chiu and Matsuda,[3] we write a typical contribution to the inelastic cross section $\sigma_{n_1 n_2}$ (n_1 particles in one sheet, n_2 in the other)

$$\sigma_{n_1 n_2} = \frac{\Lambda}{s}\int \left(\prod_{i=1}^{2} dy_1^{(i)} dy_{n_i}^{(i)}\right)\delta\left(\sqrt{s} - \sum_{j=1}^{2}(\mu \cosh y_1^{(j)} + \mu \cosh y_{n_j}^{(j)})\right)\delta\left(\sum_{k=1}^{2}(\mu \sinh y_1^{(k)} + \mu \sinh y_{n_k}^{(k)})\right)e^{2\alpha\Delta}$$
$$\times \int \prod_{l=1}^{2}\left[dy_2^{(l)} \cdots dy_{n_l-1}^{(l)} (g^2 N_f)^{n_l}\right], \tag{4}$$

where the rapidity variables $y_m^{(i)}$ are defined in Fig. 5, the index i labels the dual sheet and the lower index m labels the particle inside each sheet, N_f is the number of flavors [for the moment we assume exact $SU(N_f)$], g^2 is the universal coupling constant, α the usual Regge intercept, and Λ is a constant factor which will disappear in the expression of the density $(x/\sigma_{\text{inel}})(d\sigma/dx)$.

It is now a matter of algebraic calculations to perform the integrations and resummations. Using the planar-bootstrap equations

$$\alpha_R = 2\alpha - 1 + g^2 N_f,$$
$$\alpha_P = 2\alpha - 1 + 2g^2 N_f, \tag{5}$$

one finally gets

$$\sigma_{\text{inel}} = \sum_{n_1, n_2} \sigma_{n_1 n_2} = \Lambda \iint_0^1 dx_A dx_B \, e^{\alpha_R \Delta_A} e^{(\alpha_P - 1)\Delta_P} e^{\alpha_R \Delta_B}, \tag{6}$$

where Δ_A, Δ_B, Δ_P are the rapidity intervals defined in Fig. 5.

The actual values of α_R and α_P depend on the model one uses to solve planar-bootstrap constraints.[6,7] For instance, in the one-dimensional model of Huan Lee,[6] $\alpha_R = \alpha$ and thus $\alpha_P = 1$. For our purpose, we shall use the Reggeon and Pomeron intercepts according to the standard Regge phenomenology, taking into account the effects of $SU(N_f)$ breaking.

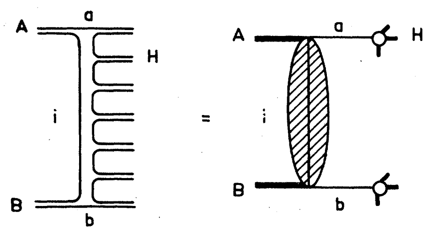

FIG. 3. The one-particle inclusive planar cross-section.

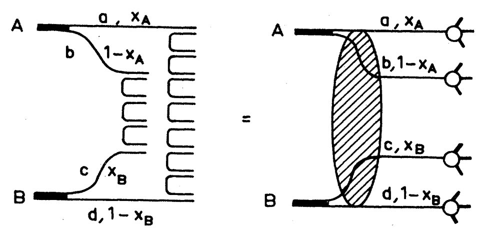

FIG. 4. The two-sheet configuration. The quark content of A is (ab). For a meson, a and b are quarks; for a baryon, they are quark or diquark.

Summing over the four possible configurations which are characterized by the relative magnitudes of $(1-x_A)$ vs x_A, $(1-x_B)$ vs x_B (see Fig. 6), we straightforwardly get

$$\sigma_{\text{inel}} = \Lambda s^{\alpha_P - 1} \iint_0^1 dx_A dx_B \left[\left(\frac{1-x_A}{x_A}\right)^{\alpha_{a\bar{a}}} x_A{}^{\alpha_P-1}\Theta(\tfrac{1}{2}-x_A) + \left(\frac{x_A}{1-x_A}\right)^{\alpha_{b\bar{b}}}(1-x_A)^{\alpha_P-1}\Theta(x_A-\tfrac{1}{2})\right]$$
$$\times\left[\left(\frac{1-x_B}{x_B}\right)^{\alpha_{c\bar{c}}} x_B{}^{\alpha_P-1}\Theta(\tfrac{1}{2}-x_B) + \left(\frac{x_B}{1-x_B}\right)^{\alpha_{d\bar{d}}}(1-x_B)^{\alpha_P-1}\Theta(x_B-\tfrac{1}{2})\right], \quad (7)$$

where $\alpha_{q\bar{q}}$ is the Regge intercept of the $q\bar{q}$ trajectory.

We write the dual weight $\mathcal{P}(x_A, x_B)$ in the factorized form

$$\mathcal{P}(x_A, x_B) = \Lambda s^{\alpha_P-1} U^{(A)}(x_A) U^{(B)}(x_B), \quad (8)$$

where

$$U^{(A)}(x_A) = \left[\left(\frac{1-x_A}{x_A}\right)^{\alpha_{a\bar{a}}} x_A{}^{\alpha_P-1}\Theta(\tfrac{1}{2}-x_A) + \left(\frac{x_A}{1-x_A}\right)^{\alpha_{b\bar{b}}}(1-x_A)^{\alpha_P-1}\Theta(x_A-\tfrac{1}{2})\right]. \quad (9)$$

Recollecting Eqs. (2), (3), and (7), we write the one-particle inclusive distribution:

$$\frac{x_H}{\sigma_{\text{inel}}}\frac{d\sigma}{dx_H}(A \underset{B}{\rightarrow} H) = \frac{1}{\int_0^1 dx_A U^{(A)}(x_A)} \int_0^1 dx_A U^{(A)}(x_A)\left[\frac{x_H}{x_A} D_a^H\left(\frac{x_H}{x_A}\right)\Theta(x_A - x_H) + \frac{x_H}{1-x_A} D_b^H\left(\frac{x_H}{1-x_A}\right)\Theta(1-x_A-x_H)\right]. \quad (10)$$

It is straightforward to verify that Eq. (10) satisfies the energy-momentum sum rule

$$\sum_H \int_0^1 dx_H \frac{x_H}{\sigma_{\text{inel}}}\frac{d\sigma}{dx_H} = 1,$$

which is a consequence of

$$\sum_H \int_0^1 x D_q^H(x)\, dx = 1.$$

III. COMPARISON WITH DATA

Looking for a clean test of the model, we are led to use data for the nondiffractive part of hadron-hadron processes which do not contain planar contributions. A good choice is the inclusive process $K^+ \underset{p}{\rightarrow} \pi^-$. In this reaction the s channel (K^+p) and the t channel $(K^+\pi^+)$ are exotic, which means no planar contribution and a negligible diffraction.

Owing to its factorization property, formula (10) does not depend on the target and can be applied to meson fragmentation on protons. The inputs are the usual Regge-trajectory intercepts $\alpha_f = 0.5$, $\alpha_\phi = 0$, and $\alpha_P = 1$, and the D-fragmentation functions taken from the Field and Feynman parametrizations.[8] As shown in Fig. 7, our prediction, with no free parameter, for the density distribution

$$\frac{x_H}{\sigma_{\text{inel}}}\frac{d\sigma}{dx_H}(K^+ \underset{p}{\rightarrow} \pi^-)$$

is in good agreement with the experimental data[9] in the whole fragmentation region $(0.1 < x_H < 1)$.

For other meson fragmentations, the analysis is less straightforward. Thus, for the reaction $\pi^\pm \underset{p}{\rightarrow} \pi^\mp$ one has to consider the highest energy data in order to avoid the planar contributions. In Fig. 8 we show the comparison of our predictions with the data[10] at 100 GeV. The difference between π^+ and π^- data gives an idea of the effect of planar contributions. In the case of reaction $\pi^+ \underset{p}{\rightarrow} \pi^+$ we use the data from Ref. 11 where the diffractive component has been subtracted by means of a triple-Regge parametrization [see Fig. 9]. In both cases, the agreement with our predictions

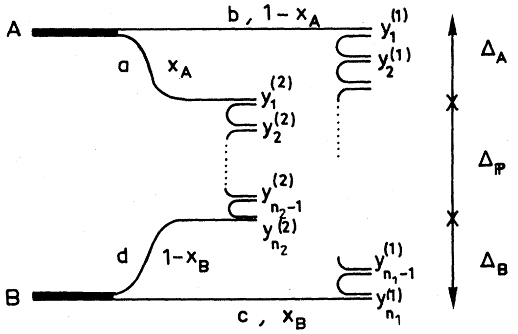

FIG. 5. Definition of rapidity variables. Δ_A, Δ_P, Δ_B are the differences of rapidity of the three sections. $\Delta = \Delta_A + \Delta_P + \Delta_B$ is the total rapidity interval.

is good.

However, it was already noticed by several authors[12] that convolution formulas involving fragmentation functions generally give wrong predictions and in order to fit data one has to use *only D* functions, assuming that the valence quark, which fragments into hadrons, carries all the momentum of the incident particle.[13] This empirical hypothesis implies for the final hadronic state, in the considered fragmentation region, a planar (or one-jet) configuration.

Although in our model the two-sheet structure is explicit, it hardly shows up in actuality. This is due to the fact that for light quarks, $\alpha_{a\bar{a}}(\alpha_{b\bar{b}})$ equal to $\alpha_f = 0.5$, and with $\alpha_P = 1$, the function

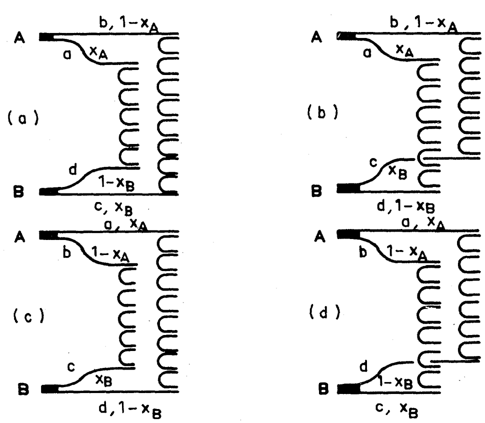

FIG. 6. The four configurations of the two dual sheets. (a) $x_A < 1 - x_A$, $x_B > 1 - x_B$, $\Delta_A = \ln[(1-x_A)/x_A]$, $\Delta_P = \ln[x_A(1-x_B)s/\mu^2]$, $\Delta_B = \ln[x_B/(1-x_B)]$. (b) $x_A < 1 - x_A$, $x_B < 1 - x_B$, $\Delta_A = \ln[(1-x_A)/x_A]$, $\Delta_P = \ln(x_A x_B s/\mu^2)$, $\Delta_B = \ln[(1-x_B)/x_B]$. (c) $x_A > 1 - x_A$, $x_B < 1 - x_B$, $\Delta_A = \ln[x_A/(1-x_A)]$, $\Delta_P = \ln[(1-x_A)x_B s/\mu^2]$, $\Delta_B = \ln[(1-x_B)/x_B]$. (d) $x_A > 1 - x_A$, $x_B > 1 - x_B$, $\Delta_A = \ln[x_A/(1-x_A)]$, $\Delta_P = \ln[(1-x_A)(1-x_B)s/\mu^2]$, $\Delta_B = \ln[x_B/(1-x_B)]$. μ is the mass of the outgoing hadrons.

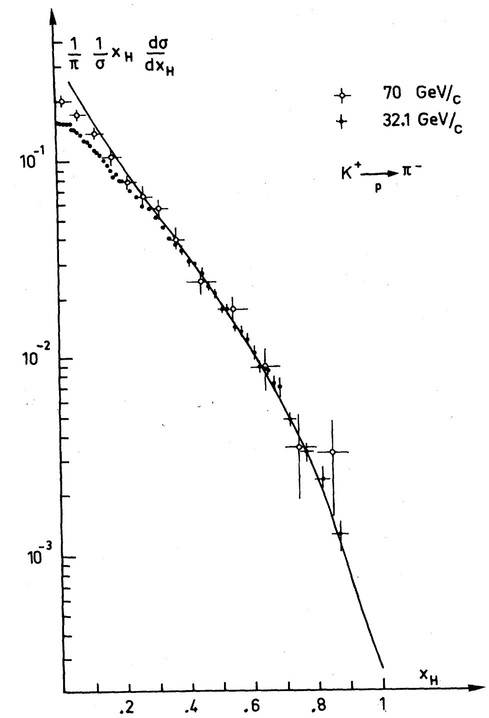

FIG. 7. The density distribution $(1/\pi)(x/\sigma_{\rm inel})(d\sigma/dx)(K^+ \underset{p}{\to} \pi^-)$, calculated from formula (10), compared to data taken from Ref. 9, with $\sigma_{\rm inel}(K^+p) = 17$ mb.

$$U^{(A)}(x_A) = \left(\frac{1-x_A}{x_A}\right)^{\alpha_{a\bar{a}}} \Theta(\tfrac{1}{2} - x_A) + \left(\frac{x_A}{1-x_A}\right)^{\alpha_{b\bar{b}}} \Theta(x_A - \tfrac{1}{2}) \qquad (11)$$

has a singularity at $x_A = 0$ ($x_A = 1$). This implies that in formula (10) the contributions to inclusive cross sections will be dominated by the configurations where a (b) takes a small fraction of momentum, while b (a) carries all the rest. In order to give a quantitative estimate of this effect, we show in Table I the mean values of the fraction of the momentum $\langle x\rangle_i^j$ carried by the fast quark q_i (the leading quark), calculated as

$$\langle x\rangle_i^j = \frac{\int_{0.5}^1 x U^{(A)}(x)dx}{\int_{0.5}^1 U^{(A)}(x)dx} = \frac{\int_{0.5}^1 x\left(\frac{x}{1-x}\right)^{\alpha_{q_j}} dx}{\int_{0.5}^1 \left(\frac{x}{1-x}\right)^{\alpha_{q_j}} dx},$$

where α_{q_j} denotes the Regge trajectory built up by the slow quark (the nonleading one). Furthermore, it is interesting to point out that this mean value $\langle x\rangle_i^j$ does not depend on the quantum numbers of the fast quark q_i but only on those of the slowest one q_j.

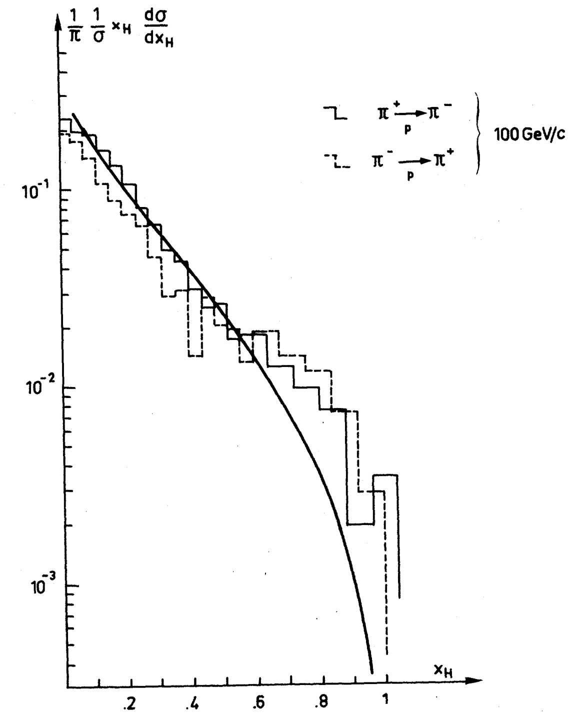

FIG. 8. The density distribution $(1/\pi)(x/\sigma_{\text{inel}})(d\sigma/dx)(\pi^\pm \xrightarrow{p} \pi^\mp)$, calculated from formula (10), compared to data for both reactions $\pi^+ \xrightarrow{p} \pi^-$ and $\pi^- \xrightarrow{p} \pi^+$, taken from Ref. 10, with $\sigma_{\text{inel}}(\pi^+p)=\sigma_{\text{inel}}(\pi^-p)=21$ mb.

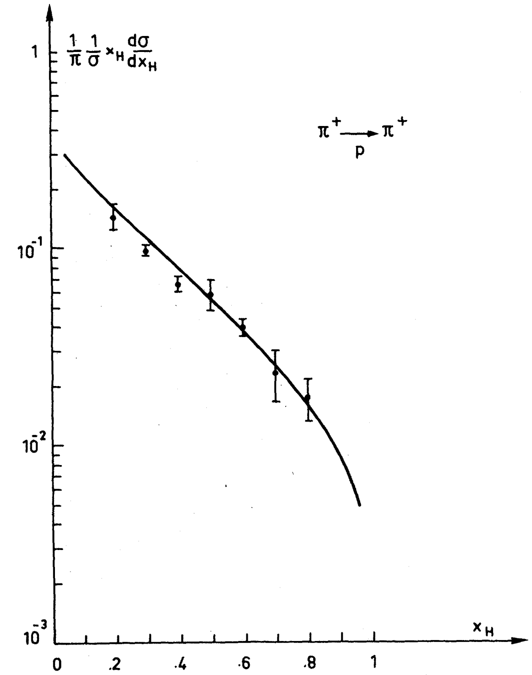

FIG. 9. The nondiffractive part of $(1/\pi)(x/\sigma_{\text{inel}})(d\sigma/dx)(\pi^+ \xrightarrow{p} \pi^+)$, calculated from formula (10), compared to data taken from Ref. 11.

IV. PARTON INTERPRETATION OF THE DUAL WEIGHT AND A NEW THEORETICAL UNDERSTANDING OF THE POMERON

For a really complete parton interpretation of inclusive hadronic reactions, it is necessary to provide the dual weight $\mathcal{P}(x_A, x_B)$ of Eqs. (8) and (9) with a parton interpretation.

We first remark that the behavior $x_A^{-\alpha_{a\bar a}}$ is the behavior of the valence structure function $G^v_{A\to a}(x_A)$ for $x_A \to 0$, whereas the behavior $(1-x_A)^{-\alpha_{b\bar b}}$ is the one of $G^v_{A\to b}(1-x_A)$ for $x_A \to 1$. This observation suggests the replacement of

$$\left(\frac{1-x_A}{x_A}\right)^{\alpha_{a\bar a}}\Theta(\tfrac{1}{2}-x_A) \text{ by } G^v_{A\to a}(x_A),$$

and

$$\left(\frac{x_A}{1-x_A}\right)^{\alpha_{b\bar b}}\Theta(x_A-\tfrac{1}{2}) \text{ by } G^v_{A\to b}(1-x_A).$$

This replacement leads to a straightforward interpretation of all the convolutions involved in Eq. (10). For instance,

$$G^v_{A\to a}(x_A)x_A{}^{\alpha_p-1}\frac{x_H}{x_A}D_a^H\left(\frac{x_H}{x_A}\right)$$

describes H as a fragmentation product of valence quark a which has interacted with a quark of B [see Fig. 10(a)] and

$$G^v_{A\to b}(1-x_A)(1-x_A)^{\alpha_p-1}\frac{x_H}{x_A}D_a^H\left(\frac{x_H}{x_A}\right)$$

describes H as a fragmentation product of valence quark a which was spectator, while quark b was interacting with a quark of B [see Fig. 10(b)]. The same applies for fragmentation of quark b. One has to add the four contributions since, in low-P_T collisions, spectator jets and interacting quark jets are all mixed along the incident longitudinal direction. We point out that our parton interpretation of low-P_T inclusive reaction is very similar to the standard interpretation of large-P_T hadronic

TABLE I. Leading-quark effect. q_j is the slow quark.

q_j	u,d	s	qq
α_{q_j}	0.5	0	-1.5
$\langle x\rangle_i^j$	0.85	0.75	0.62

collisions which also involves jets of interacting quarks and spectator jets.

For $\alpha_P = 1$, the expression $U^{(A)}(x_A)$ in Eq. (9) takes a very simple and physically meaningful form

$$Q_a(x_A) = \frac{U^{(A)}(x_A)}{\int U^{(A)}(x_A)dx_A} = \tfrac{1}{2}[G^v_{A\to a}(x_A) + G^v_{A\to b}(1-x_A)] \quad (12)$$

and the density distribution becomes

$$\frac{x_H}{\sigma_{\text{inel}}}\frac{d\sigma(A \underset{B}{\to} H)}{dx_H} = \int_0^1 dx_A \frac{G^v_{A\to a}(x_A) + G^v_{A\to b}(1-x_A)}{2}\left[\frac{x_H}{x_A}D^H_a\left(\frac{x_H}{x_A}\right)\Theta(x_A - x_H) + \frac{x_H}{1-x_A}D^H_b\left(\frac{x_H}{1-x_A}\right)\Theta(1-x_A-x_H)\right]. \quad (13)$$

We have verified that Eq. (13) leads to a distribution which is very similar to the one obtained from Eq. (10). We have studied $\pi^+ \to \pi^\pm$ fragmentation, using for the valence structure functions of the pion,

$$xG^v_{\pi^+\to u}(x) = xG^v_{\pi^+\to \bar{d}}(x) = 0.75\sqrt{x}(1-x),$$

which is in good agreement with lepton-pair production data[14] and which is compatible with the expectations of our dual approach.[2] For the quark-fragmentation functions we use the Field-Feynman parametrizations.[8] The discrepancy with the distribution obtained with the weight given by Eq. (9) hardly exceeds 1%, i.e., it never exceeds the thickness of the curves in the drawing of Figs. 8 and 9.

In Eq. (13), $Q_a(x)$ can be interpreted as the probability of finding a constituent quark Q_a in A with a fraction of momentum x_A. Indeed, we remark that since

$$Q_b(1-x_A) = Q_a(x_A),$$

$$\int_0^1 Q_a(x)dx = 1,$$

$$\int_0^1 x[Q_a(x) + Q_b(x)]dx = 1,$$

the meson A is just a Q_aQ_b system.

To our knowledge, it is the first time one has such a simple expression for the density distribution associated with the bare Pomeron. Usually, the derivation of the Pomeron properties requires the solution of planar-bootstrap equations. Since one does not know yet how to solve exactly these equations, one is obliged to build simplified models. This is the reason why the Pomeron one gets is never completely model-independent. However, we propose expression (13) as a model-independent expression of the density distribution associated with the bare Pomeron.

The fact that the dominant contribution to high-energy hadronic reactions depends only on *free-parton probabilities* suggests a new theoretical understanding of hadronic reactions. One usually expects that low-P_T hadronic interactions are due to complicated residual forces in molecular physics. On the contrary, the picture which emerges from our approach is the simplest one could imagine: The interaction between hadrons results from the reshuffling of quarks which can be considered as *free partons*. That is why the hadronic scattering amplitudes, at the lowest order in DTU, can be considered as providing the "zeroth-order" approximation to QCD.[15]

Note added. After the completion of this work we were informed of a work by H. Minakata [Phys. Rev. D 20, 1656 (1979)] which reaches similar conclusions.

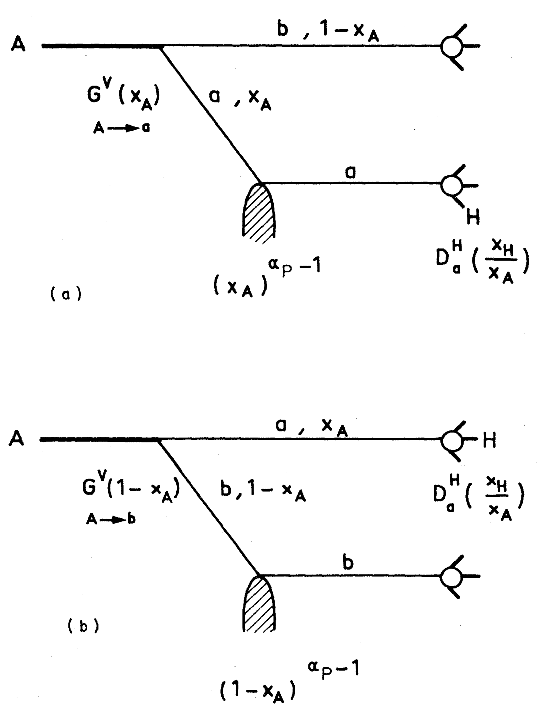

FIG. 10. Two of the diagrams occurring in the parton interpretation of the fragmentation $A \to H$, in a low-P_T process. (a) Quark a interacts with a quark of B and gives H. (b) Quark b interacts with a quark of B while quark a gives H.

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