

## Flavor-changing neutral currents involving $b$ quarks and $\tau$ leptons

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We examine the experimental implications of flavor-changing neutral currents (FCNC's) involving  $b$  quarks and/or  $\tau$  leptons. Several final-state configurations signaling the presence of FCNC's are discussed; we find that cascade decays by ordinary charged currents provide a substantial background when looking for FCNC decays of  $b$  quarks. Present limits on  $\tau$  rare decay modes do not as yet rule out FCNC decays. We compare our results with those previously obtained by other authors.

### I. INTRODUCTION

As is well known, the standard model of the weak interactions<sup>1-3</sup> provides a natural explanation<sup>4</sup> for the absence of strangeness-changing neutral currents at the level of  $G_F$  and  $G_F\alpha$  and gives good agreement with the experimental results for the  $K_L-K_S$  mass difference and the rate for  $K_L \rightarrow \mu^+\mu^-$ .<sup>5,6</sup> The model in its three-doublet form, known as the Kobayashi-Maskawa (KM) model,<sup>3</sup> also predicts the absence of flavor-changing neutral currents (FCNC's) of any kind, including systems involving heavy quarks, at the level of  $G_F$  and  $G_F\alpha$ . [Within the  $SU(2) \times U(1)$  model, the conditions for the natural absence of FCNC's were studied by Glashow and Weinberg.<sup>4</sup>]

So far, the standard model has been very successful; all of the present data on neutral currents<sup>7</sup> as well as charged currents can be accommodated within it. [This includes the decay modes of the  $D$  (Refs. 8 and 9) and the  $\tau$  as well.] There is, however, a substantial portion of the model which has not yet been subject to detailed experimental study since the particles have either not been produced (such as  $b$ -carrying mesons) or the data are still in a preliminary form (as in the charm-meson and  $\tau$ -lepton cases).

Recently, Buccella and Oliver<sup>10</sup> have made a thorough study of possible charm-changing neutral-current (CCNC) effects within the  $D$ -meson system and have obtained limits on the allowed range of possible couplings. (Their bound is strongest when the CCNC's are purely left-handed.)

In this paper we will make a parallel analysis for the system of  $b$ -carrying mesons; we will assume that the charged currents of the theory are those given by the standard KM model; we will also examine the possibility of  $e$ - $\tau$  or  $\mu$ - $\tau$  FCNC's.

Section II contains an analysis of  $B^0-\bar{B}^0$  transitions and compares the predictions of the standard model to one containing FCNC's for the  $B^0-\bar{B}^0$  mass difference; we will also discuss the decay

$B^0 \rightarrow \tau^-\tau^+$  (analogous to  $K_L^0 \rightarrow \mu^+\mu^-$ ). In Sec. III we discuss the possible effects of  $b$  FCNC in  $\nu$  interactions and the possible restrictions that can be obtained by looking for neutral-current  $b$ -quark decays.

In Sec. IV we discuss a possible  $e$ - $\tau$  or  $\mu$ - $\tau$  FCNC and its effects on  $\tau$  decay and  $\tau$  production in deep-inelastic  $eN$  reactions. Section V contains a comparison of our results and procedures with those of Ali and Aydin.<sup>11</sup> Our conclusions can be found in Sec. VI.

### II. THE $B^0-\bar{B}^0$ SYSTEM

In this section we will analyze  $B^0-\bar{B}^0$  transitions<sup>11</sup> and compare the predictions of the standard model to one containing a  $b$  FCNC. Here  $B^0$  will represent either the  $b\bar{d}$  or  $b\bar{s}$  meson ( $B^0 = b\bar{q}$ ;  $q = d$  or  $s$ ). In our analysis we will follow the work of Ref. 10.

We will take our effective Lagrangians to be

$$\mathcal{L}^{|\Delta B|=2} = \sqrt{2}G_F \left[ \sum_{q=d,s} g_L^q \bar{b} \gamma_{\mu} \frac{1}{2}(1-\gamma_5)q + g_R^q \bar{b} \gamma_{\mu} \frac{1}{2}(1+\gamma_5)q \right]^2 + \text{H.c.} \quad (2.1)$$

for  $|\Delta B| = 2$  and

$$\mathcal{L}^{|\Delta B|=1} = \sqrt{2}G_F \left[ \sum_{q=d,s} g_L^q \bar{b} \gamma_{\mu} \frac{1}{2}(1-\gamma_5)q + g_R^q \bar{b} \gamma_{\mu} \frac{1}{2}(1+\gamma_5)q \right] J_{\text{WS}}^{\mu} + \text{H.c.} \quad (2.2)$$

for  $|\Delta B| = 1$ . Here, we have taken (for any fermion  $f$ )

$$J_{\text{WS}}^{\mu} = \sum_{\text{fermions}} \bar{f} [(T_3^f - 2x_W Q^f) \gamma_{\mu} - T_3^f \gamma_{\mu} \gamma_5] f, \quad (2.3)$$

with  $T_3^f$  being the usual weak isospin,  $Q$  the charge of the fermion  $f$ , and  $x_W \equiv \sin^2 \theta_W \approx 0.23$ . (WS denotes Weinberg-Salam). Obviously the  $|\Delta B| = 2$  piece contributes directly to the  $B^0-\bar{B}^0$  mass difference while the  $|\Delta B| = 1$  piece contributes to  $b$ -particle production and decay.

Among the possible  $b$ -quark decay modes are the

following (we treat the antiquark in the initial state as a spectator):

$$b \rightarrow c\bar{u}d \sim V_{11}V_{23}^*, \quad (2.4a)$$

$$b \rightarrow c\bar{u}s \sim V_{12}V_{23}^*, \quad (2.4b)$$

$$b \rightarrow c\bar{c}d \sim V_{21}V_{23}^*, \quad (2.4c)$$

$$b \rightarrow c\bar{c}s \sim V_{22}V_{23}^*, \quad (2.4d)$$

$$b \rightarrow u\bar{u}d \sim V_{11}V_{13}^*, \quad (2.4e)$$

$$b \rightarrow u\bar{u}s \sim V_{12}V_{13}^*, \quad (2.4f)$$

$$b \rightarrow u\bar{c}d \sim V_{21}V_{13}^*, \quad (2.4g)$$

$$b \rightarrow u\bar{c}s \sim V_{22}V_{13}^*. \quad (2.4h)$$

Here the  $V_{ij}$  are the complex elements of the KM charged-current mixing matrix:

$$J_{\mu}^c = \mathcal{P}\gamma_{\mu}(1 - \gamma_5)V\mathcal{N} \quad (2.5)$$

with

$$\mathcal{P} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad \mathcal{N} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (2.6)$$

In our analysis we will always assume that

$$|V_{23}|^2 \gg |V_{13}|^2 \quad (2.7)$$

(consistent with the bounds of Ref. 6) such that  $b$  decays mainly into charm. This implies that from a single  $B_d^0$  decay we expect a single (anti) charmed particle with amplitude  $V_{11}V_{23}^*$  ( $V_{21}V_{13}^*$ ), with no accompanying  $K$ 's, as well as a negatively charged lepton if the decay is semileptonic. The  $B_s^0$  decay will yield a single (anti) charmed particle with amplitude  $V_{12}V_{23}^*$  ( $V_{22}V_{13}^*$ ), with an accompanying  $s\bar{s}$  pair.

Mixing must be taken into account, however, since the  $B_d^0$  and  $\bar{B}_d^0$  share common final states, e.g.,

$$\begin{aligned} B_d^0 &\rightarrow c\bar{u}d\bar{d} \sim V_{11}V_{23}^*, \\ \bar{B}_d^0 &\rightarrow \bar{u}c d\bar{d} \sim V_{21}^*V_{13}. \end{aligned} \quad (2.8)$$

Similarly, for the  $B_s^0$  and  $\bar{B}_s^0$  states,

$$\begin{aligned} B_s^0 &\rightarrow c\bar{u}s\bar{s} \sim V_{12}V_{23}^*, \\ \bar{B}_s^0 &\rightarrow \bar{u}c s\bar{s} \sim V_{22}^*V_{13}. \end{aligned} \quad (2.9)$$

Here, we have shown the initial spectator quark explicitly.

Neglecting  $CP$  violation, we form the  $CP$  eigenstates  $(B^0 \pm \bar{B}^0)/\sqrt{2}$ , with definite masses ( $m_{\pm}$ ) and inverse lifetimes ( $\lambda_{\pm}$ ). Following Ref. 10, we find the probabilities of getting a negative or a positive lepton from an initially pure  $B^0$  state:

$$\frac{N(B^0 - l^+ + x)}{N(B^0 - l^- + x)} = \frac{\delta m^2 + \delta\lambda^2/4}{2\lambda^2 + \delta m^2 - \delta\lambda^2/4}, \quad (2.10)$$

with

$$\lambda = \frac{1}{2}(\lambda_+ + \lambda_-), \quad \delta\lambda = \lambda_+ - \lambda_-, \quad \delta m = m_+ - m_-. \quad (2.11)$$

Similarly, one can consider such ratios as

$$\frac{N(B_d^0 - c^+ + \text{pions})}{N(B_d^0 - c^* + \text{pions})}, \quad (2.12)$$

with a similar expression for the  $B_s^0$  decay. (In what follows, we will not discuss the hadronic final states for  $B_s^0$  decay since they are quite complicated in signature, e.g.,  $c + s\bar{s}$ .) The expression (2.12) can be evaluated using Eq. (7) of Ref. 10; we denote by  $\alpha_i$  the amplitude for  $B_d^0 \rightarrow c^+ + \text{pions}$  ( $\sim V_{11}V_{23}^*$ ) and by  $\beta_i$  the amplitude for  $B_d^0 \rightarrow c^* + \text{pions}$  ( $\sim V_{21}V_{13}^*$ ).

To go further we must say something about the relative sizes of  $\lambda$ ,  $\delta\lambda$ , and  $\delta m$ . Here we employ the quark model; the calculation of  $\lambda$  is straightforward in terms of the  $V_{ij}$  and any possible non-leptonic enhancement factors.<sup>12</sup> In this calculation we take the quark masses as given by their current algebra values and we assume that the  $V_{ij}$  are given by the central values<sup>13</sup> obtained by Shrock, Trieman, and Wang<sup>6</sup> (STW). To calculate  $\delta\lambda$ , we consider the matrix

$$\Gamma_{ij} = \sum_n \langle B_i | H_W | n \rangle \langle n | H_W | B_j \rangle \rho_n, \quad i, j = 1, 2, \quad (2.13)$$

where  $B_i$  is  $B^0$  or  $\bar{B}^0$ .  $\rho_n$  is the appropriate phase-space factor for the final state  $|n\rangle$ . Note that for  $B_d^0$  the only possible intermediate states are (2.4c) and (2.4e); for  $B_s^0$  the only states are (2.4d) and (2.4f). For  $\Gamma_{11}$  and  $\Gamma_{22}$  we find, trivially,

$$\Gamma_{11} = \Gamma_{22} = \lambda \quad (2.14)$$

and, in the  $CP$ -conserving limit,

$$\Gamma_{12} = \Gamma_{21} = \frac{1}{2}\delta\lambda. \quad (2.15)$$

Tables I and II show the values for  $\lambda$  and  $\delta\lambda/\lambda$  for the  $B_d^0 - \bar{B}_d^0$  and  $B_s^0 - \bar{B}_s^0$  systems, respectively, under various assumptions. The values of  $\delta\lambda/\lambda$  obtained

TABLE I. Average inverse lifetime, lifetime difference, and  $\delta m$  for the  $B_d^0$  ( $= b\bar{d}$ ) meson system for four different models. A: STW solution (b) without nonleptonic enhancement factors. B: STW solution (a) without nonleptonic enhancement factors. C: Same as A with  $f_1^2 + 2f_2^2 = 4$ . D: Same as B with  $f_1^2 + 2f_2^2 = 4$ . For  $\delta m$  we have taken  $B(f_B/f_{\pi})^2$  equal to unity.

	$\lambda$ ( $10^{-2}$ eV)	$\delta\lambda/\lambda$	$\delta m$ (eV)
A	1.61	$1.30 \times 10^{-1}$	$1.9 \times 10^{-3}$
B	6.54	$5.04 \times 10^{-2}$	$5.4 \times 10^{-4}$
C	1.97	$1.42 \times 10^{-1}$	$1.9 \times 10^{-3}$
D	7.95	$5.54 \times 10^{-2}$	$5.4 \times 10^{-4}$

TABLE II. Same as Table I for the  $B_s^0$  ( $\equiv b\bar{s}$ ) meson system.

	$\lambda$ ( $10^{-2}$ eV)	$\delta\lambda/\lambda$	$\delta m$ (eV)
A	1.61	$3.58 \times 10^{-1}$	$8.0 \times 10^{-3}$
B	6.54	$3.32 \times 10^{-1}$	$4.1 \times 10^{-2}$
C	1.97	$3.90 \times 10^{-1}$	$8.0 \times 10^{-3}$
D	7.95	$3.64 \times 10^{-1}$	$4.1 \times 10^{-2}$

here are comparable to those obtained for the  $D^0$ - $\bar{D}^0$  system<sup>10</sup> ( $\sim 5 \times 10^{-2}$ ).

We now turn our attention to  $\delta m$ ; in the KM model with no flavor-changing neutral currents we have<sup>5,6</sup>

$$\delta m = \text{Re} \frac{-2Bf_B^2 m_B (G_F/\sqrt{2})(\alpha/4\pi)}{3x_W} \sum_{i,j=u,c,t} \lambda_i \lambda_j A_{ij}, \quad (2.16)$$

where  $\lambda_i = V_{ib}V_{id,s}^*$ ,  $x_W$  is  $\sin^2\theta_W$  ( $\approx 0.23$ ),  $f_B$  is the  $B$ -meson decay constant, and

$$A_{ij} = (1-x_i)^{-1}(1-x_j)^{-1} + (x_i-x_j)^{-1} \left[ \frac{x_i^2}{(1-x_i)^2} \ln x_i - \frac{x_j^2}{(1-x_j)^2} \ln x_j \right], \quad (2.17)$$

with  $x_i \equiv m_i^2/m_W^2$ . ( $m_W$  is the mass of the  $W$  in the WS model.) This results from calculating the diagrams of Fig. 1. The factor  $B$  is used to take into account corrections to the vacuum-insertion approximation and is less than unity; for the  $K^0$ - $\bar{K}^0$  system,  $B \approx 0.4$ .<sup>6</sup> We find ( $m_t \approx 20$  GeV)

$$\begin{aligned} \delta m_{B_d} &\sim 10^{-3} B(f_B/f_\pi)^2 \text{ eV}, \\ \delta m_{B_s} &\sim 10 \delta m_{B_d}, \end{aligned} \quad (2.18)$$

which, together with Tables I and II, imply [for

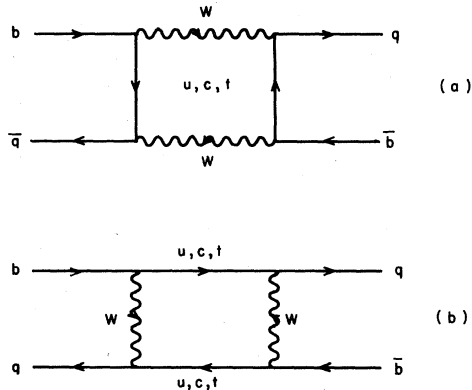


FIG. 1. Graphs contributing to the  $B^0$ - $\bar{B}^0$  mass difference in the KM model.

$$B(f_B/f_\pi)^2 = 1]$$

$$\left( \frac{\delta m}{\lambda} \right)_B \sim \begin{cases} 10^{-7} & \text{for } B_d \\ 10^{-5} & \text{for } B_s. \end{cases} \quad (2.19)$$

As a comparison, for the  $K^0$ - $\bar{K}^0$  system we find

$$(\delta m/\lambda)_K \approx 0.9, \quad (2.20)$$

whereas for the  $D^0$ - $\bar{D}^0$  system we expect (assuming no FCNC)

$$(\delta m/\lambda)_D \approx 10^{-6}. \quad (2.21)$$

Exact values can be obtained from Tables I and II; we remind the reader that the factor  $B(f_B/f_\pi)^2$  may be quite large ( $\gg 1$ ).

If only a  $|\Delta B| = 2$  neutral-current piece were present, there would be a contribution to  $\delta m$  of order  $G_F$ ; since this does not affect  $\delta\lambda$  we may write (2.10) as

$$\frac{N(B^0 \rightarrow l^+ + x)}{N(B^0 \rightarrow l^- + x)} \approx \frac{\delta m^2}{2\lambda^2 + \delta m^2}, \quad (2.10')$$

since we then expect  $\delta m^2 \gg \delta\lambda^2$  and, as before,  $\lambda^2 \gg \delta\lambda^2/8$ . If, however, both  $|\Delta B| = 2$  and  $|\Delta B| = 1$  pieces were present,  $\delta\lambda$  could be modified significantly, since  $B^0$  and  $\bar{B}^0$  would share more common final states:

$$\begin{aligned} b\bar{d} &\rightarrow d\bar{d} + \bar{q}q(l^+l^-, \bar{\nu}\nu), \\ b\bar{s} &\rightarrow s\bar{s} + \bar{q}q(l^+l^-, \bar{\nu}\nu). \end{aligned} \quad (2.22)$$

A clear signal then, for  $|\Delta B| = 1$  couplings of substantial size, is values of  $\lambda$  and  $\delta\lambda/\lambda$  much larger than expected.

In the former case, we can calculate  $\delta m$  and use (2.10') to put bounds on the coupling constants; following Ref. 10 we find, from Fig. 2,

$$\begin{aligned} \delta m_{\text{FC}} &= \frac{G_F}{\sqrt{2}} f_B^2 m_B \\ &\times \left\{ \frac{4}{3} (g_L^2 + g_R^2) f_1 \right. \\ &\quad \left. + 2g_L g_R \left[ -\frac{1}{9} (8f_2 + f_3) + \frac{2}{3} \frac{m_B}{m_b + m_q} \right] \right\}, \end{aligned} \quad (2.23)$$

where

$$f_1 = f, \quad f_2 = f^{1/2}, \quad f_3 = f^{-4}, \quad (2.24)$$

and

$$f = \left[ 1 + \frac{23}{3} \frac{\alpha_s(m_b^2)}{4\pi} \ln \left( \frac{m_W^2}{m_b^2} \right) \right]^{-6/23}, \quad (2.25)$$

where strong-interaction corrections have been taken into account. Assuming  $m_B \approx m_b + m_q$ , we

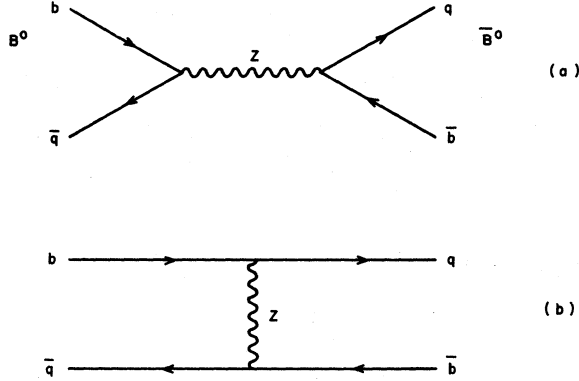


FIG. 2. Additional graphs contributing to the  $B^0 - \bar{B}^0$  mass difference if FCNC's are present.

can write  $\delta m_{FC}$  as

$$\delta m_{FC} = \frac{G_F}{\sqrt{2}} f_B^2 m_B [A(g_L^2 + g_R^2) + B g_L g_R], \quad (2.26)$$

with

$$A = \frac{4}{3} f_1, \quad B = \frac{2}{9} (5f_3 - 8f_2).$$

Given some knowledge of  $f$  and  $f_B$  and  $B^0$  decay properties, this can provide the same kind of

$$B(B^0 \rightarrow \tau^+ \tau^-) = \frac{G_F^2}{2\pi^4} \frac{(1 - 4m_\tau^2/m_B^2)^{1/2}}{(1 - m_\tau^2/m_B^2)^2} \frac{\left( \sum_{i=u,c,t} \text{Re}(V_{ib}^* V_{i\tau}) m_i^2 \right)^2}{|V_{qb}|^2} B(B^* \rightarrow \tau^+ \nu_\tau) \frac{\tau(B^0)}{\tau(B^*)}, \quad (2.27)$$

where  $q = d$  or  $s$ , depending on the meson. The dominant contribution to the sum is from the  $t$  quark and we find

$$B(B^0 \rightarrow \tau^+ \tau^-) \simeq 6.0 \times 10^{-14} (\text{Re} V_{tb})^2 (f_B/f_\tau)^2 m_t^4 \sim 2.10^{-15} (f_B/f_\tau)^2 m_t^4, \quad (2.28)$$

which for  $f_B/f_\tau \sim 3$ ,  $m_t = 25$  GeV is  $\sim 10^{-8}$ .

If  $|\Delta B| = 1$  FCNC's are present, the above decay can go directly via the diagram in Fig. 4. The rate for this process would be simply

$$\Gamma_{FCNC} = \frac{G_F^2 f_B^2 M_B^3}{8\pi} \left( \frac{m_\tau}{M_B} \right)^2 (1 - 4m_\tau^2/m_B^2)^{1/2} (g_L - g_R)^2 \simeq 8.88 \times 10^{11} (g_L - g_R)^2 f_B^2 / f_\tau^2 \text{ sec}^{-1} \quad (2.29)$$

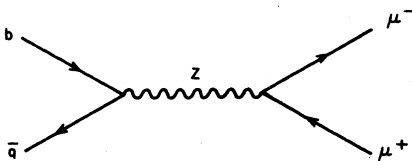


FIG. 4. Additional graph for  $B^0 \rightarrow \tau^+ \tau^-$  if FCNC's are present.

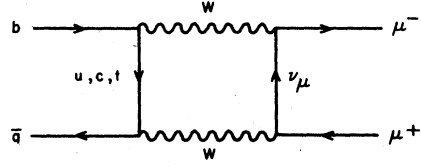


FIG. 3. Graph for  $B^0 \rightarrow \tau^+ \tau^-$  in the KM model.

bounds as found in Ref. 10 for the  $D^0$  case.

The values of  $\delta m^2$  provides the best bounds on  $|\Delta B| = 2$  couplings; if  $|\Delta B| = 1$  couplings are present, various bounds can be arrived at by measuring  $\delta\lambda$ , looking for unexpected final states in  $b$  decay, or looking for unexpected production rates of hadrons containing  $b$  quarks in  $\nu$  and  $\bar{\nu}$  interactions. Let us examine some of the decay modes for  $b$  quarks if FCNC's are present.

One of the best final states to look for if  $|\Delta B| = 1$  transitions are present is the decay  $B^0 \rightarrow \tau^+ \tau^-$ , since  $\tau$ 's are readily identified. As in the case of  $K_L \rightarrow \mu^+ \mu^-$ , this decay mode is expected due to one-loop graphs (shown in Fig. 3) as well as other contributions.<sup>14</sup>

Following Shrock and Voloshin<sup>14</sup> we find the "short-distance" contribution to the branching ratio for this mode to be

or, with a branching ratio,

$$B \sim 3 \times 10^{-2} (g_L - g_R)^2 (f_B^2 / f_\tau^2). \quad (2.30)$$

If we knew that  $B$  was less than ten times the value given by (2.28), we would find the bound

$$|g_L - g_R| \leq 3 \times 10^{-4} \quad (2.31)$$

which is comparable to the charm bound on  $g_L$ .<sup>10</sup>

### III. FURTHER CONSTRAINTS

In addition to pure leptonic decays, we would also have semileptonic decays as well if  $|\Delta B| = 1$  FCNC's existed; these data would not give bounds as good as (2.31). Such modes as  $b \rightarrow d l^+ l^-$  and  $b \rightarrow s l^+ l^-$  would be the simplest to detect; apart from phase-space factors these widths would be

$$\Gamma_{SL} = \frac{G_F^2 M_B^5}{768\pi^3} (g_L^2 + g_R^2) (1 - 4x_w + 8x_w^2) \simeq 1.6 \times 10^{-2} (g_L^2 + g_R^2) \text{ eV} \quad (x_w = 0.23). \quad (3.1)$$

Note that this is comparable to the lifetimes found above, apart from the factor  $(g_L^2 + g_R^2)$ . If  $\Gamma_{SL} / \Gamma_{\text{tot}}$  was  $\leq 10^{-2}$  we could conclude only that

$$(g_L^2 + g_R^2) \leq 10^{-2}, \quad (3.2)$$

which is not as good as (2.31). Similar bounds can be obtained by looking for  $b$ -quark production by  $\nu$  or  $\bar{\nu}$  reactions. In the valence-quark limit we find (as in Ref. 10) for an isoscalar target

$$R^\nu \equiv \frac{\sigma(\nu N \rightarrow \nu B)}{\sigma(\nu N \rightarrow \nu X)} \simeq \frac{\frac{1}{4}(g_L^2 + g_R^2/3)}{\frac{1}{2} - x_W + \frac{20}{27}x_W^2}, \quad (3.3)$$

$$R^{\bar{\nu}} \equiv \frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} B)}{\sigma(\bar{\nu} N \rightarrow \bar{\nu} X)} \simeq \frac{\frac{1}{4}(g_L^2 + 3g_R^2)}{\frac{1}{2} - x_W + \frac{20}{9}x_W^2} \quad (3.4)$$

for a large  $b$ - $d$  coupling. If the  $b$ - $s$  coupling were large instead, the above ratios would need to be multiplied by the sea ( $s$ ) to valence ratio which is  $\leq 0.1$ .

Looking at other possible  $b$ -quark final states may not provide such clean signals, with a few exceptions; one possibility is  $b \rightarrow s\bar{s}s$  leading to a three-kaon final state. Apart from possible non-leptonic enhancement effects this should occur with a rate

$$\Gamma_{3K} = \frac{G_F^2 M_b^5}{64\pi^3} (g_L^2 + g_R^2) \left( \frac{1}{4} - \frac{1}{3}x_W + \frac{2}{9}x_W^2 \right) \simeq 3.9 \times 10^{-2} \times (g_L^2 + g_R^2) \text{ eV} \quad (x_W \simeq 0.23). \quad (3.5)$$

Again, apart from the factor of  $(g_L^2 + g_R^2)$ , this rate is comparable to the lifetimes found in Tables I and II.

There are several important backgrounds to consider when looking for FCNC decays of  $b$  quarks; these result mainly from the decay chain

$$b \rightarrow c \rightarrow s. \quad (3.6)$$

Consider the FCNC decay  $b \rightarrow se^+e^-$ ; a substantial background will come from the decay chain

$$b \rightarrow ce^-\nu_e \rightarrow se^+\nu_e, \quad (3.7)$$

which will have a branching ratio of  $\sim 1\%$ . Similarly the decay  $b \rightarrow s\bar{s}s$  may be simulated by the process

$$b \rightarrow c\bar{c}s \rightarrow \bar{s} + x \rightarrow s + x. \quad (3.8)$$

We expect this branching ratio to be quite substantial ( $\sim 15\%$ ). Obviously, a good understanding of these backgrounds is necessary to observe a clean signal for the FCNC modes. Perhaps the best signal, if the  $b$ - $d$  FCNC is present, is to look for  $b \rightarrow d^*l^-\bar{l}$ . Here there will be no accompanying  $K$ 's and the backgrounds are dominated by electromagnetic effects only.

#### IV. CONSTRAINTS ON FCNC'S OF THE $\tau$

The present existing data on the  $\tau$  lepton is completely consistent with the expectations of the standard model.<sup>15</sup> One set of specific final-state modes which would signal  $\tau$  FCNC's is

$$\begin{aligned} \tau^- &\rightarrow e^-\mu^+\mu^-, \quad \tau^- \rightarrow e^-e^+e^-, \\ \tau^- &\rightarrow \mu^-\mu^+\mu^-, \quad \tau^- \rightarrow \mu^-e^+e^-. \end{aligned} \quad (4.1)$$

For these modes we would find the branching ratio

$$\begin{aligned} \frac{\Gamma(\tau \rightarrow 3l)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} &= \frac{1}{4}(g_L^2 + g_R^2)(1 - 4x_W + 8x_W^2) \\ &= 0.13(g_L^2 + g_R^2) \text{ for } x_W \simeq 0.23. \end{aligned} \quad (4.2)$$

Here  $g_L$  and  $g_R$  are the left- and right-handed coupling constants for the  $\tau$  FCNC. Given the present data,<sup>16</sup>

$$B(\tau \rightarrow 3l) < 0.04. \quad (4.3)$$

We find only the weak bound

$$\sum (g_L^2 + g_R^2) < 1.7, \quad (4.4)$$

which clearly does not rule out possible FCNC's involving the  $\tau$ . Here, the sum extends over the couplings for the processes (4.1).

As another measure of the possible lepton-number-violating FCNC's, we consider the deep-inelastic process  $(e, \mu)N \rightarrow \tau X$ . If we normalize this reaction to the charged-current reaction  $\nu_\mu N \rightarrow \mu X$  in the limit that only valence quarks contribute and neglect the phase-space suppression from the heavy  $\tau^-$  final state,<sup>17</sup> we find (assuming scaling as well)

$$\begin{aligned} T &= \frac{\sigma(e^-N \rightarrow \tau X)}{\sigma(\nu N \rightarrow \mu X)} \\ &\simeq (g_L^2 + \frac{1}{3}g_R^2)(u_L^2 + d_L^2) + (\frac{1}{3}g_L^2 + g_R^2)(u_R^2 + d_R^2), \end{aligned} \quad (4.5)$$

where

$$\begin{aligned} u_L &= \frac{1}{2} - \frac{2}{3}x_W, \quad u_R = -\frac{2}{3}x_W, \\ d_L &= -\frac{1}{2} + \frac{1}{3}x_W, \quad d_R = \frac{1}{3}x_W. \end{aligned} \quad (4.6)$$

For  $x_W \simeq 0.23$  this yields

$$T \simeq 0.31g_L^2 + 0.13g_R^2. \quad (4.7)$$

Hence,

$$\begin{aligned} \sigma(e^-N \rightarrow \tau X) &\simeq (0.31g_L^2 + 0.13g_R^2) \times 0.74 \times 10^{-38} \times E^e \text{ cm}^2, \end{aligned} \quad (4.8)$$

with  $E^e$  being the incident electron energy in GeV. A similar result would hold if the initial lepton

were a  $\mu^-$ . There have been many discussions of heavy-lepton production in  $\nu$  reactions<sup>18</sup>; the signals for  $\tau^-$  production by  $e^-$  ( $\mu^-$ ) would be similar to those discussed for  $\nu$ 's. Assuming an initially pure  $e^-$  ( $\mu^-$ ) beam, one would look for an outgoing  $\mu^-$  ( $e^-$ ) in the direction opposite to the hadronic jet with a large fraction of the total energy. This results from the leptonic decay of the  $\tau^-$  after it is produced at the leptonic vertex.

Another possible final state is back-to-back jets, one of which comes from the leptonic vertex; this results from the hadronic decay of the  $\tau$ . Such a signal, however, can have substantial background from other mechanisms such as associated production.

If FCNC's are present there are several  $\tau$  final states which would indicate their presence in addition to the pure leptonic decay discussed above; all of these final states would have to contain an  $e^-$  ( $\mu^-$ ). For example,

$$\begin{aligned} \tau^- &\rightarrow e^- \eta, \\ \tau^- &\rightarrow e^- \phi, \\ \tau^- &\rightarrow e^- \rho, \\ \tau^- &\rightarrow e^- \pi^0, \end{aligned} \quad (4.9)$$

and similarly for  $\tau^- \rightarrow \mu^-$ .

Limits on processes such as these have not been presented at this time and may be important in locating possible FCNC's of  $\tau$  leptons.

#### V. COMPARISON WITH PREVIOUS WORK

In order to look for FCNC's in heavy-quark systems we have had to examine the predictions of the KM model for the quantities  $\lambda$ ,  $\delta\lambda$ , and  $\delta m$  in both the  $B_d^0-\bar{B}_d^0$  and  $B_s^0-\bar{B}_s^0$  systems. In earlier work on the KM predictions for these same quantities, rough estimates were made by Ali and Aydin.<sup>11</sup> We have improved upon these calculations in the following ways.

First, we have made use of the newly obtained bounds on the KM mixing angles<sup>9</sup> (we have used the central values obtained by STW). Second, in calculating  $\delta m$ , we have kept the exact formula for  $A_{ij}$  [Eq. (2.17)] whereas only the lowest-order terms were kept by the above authors. Thirdly, in calculating both  $\lambda$  and  $\delta\lambda$  we have used a detailed calculation of the phase space for the various final states. Lastly, we have used a value of  $m_t$  ( $\approx 20$  GeV), since  $m_t \leq 15$  GeV seems to be ruled out by recent data from PETRA. [Furthermore, we have left  $B(f_B/f_\tau)^2$  as a free parameter.]

Our results for the KM predictions can be found in Tables I and II; these are to be compared with the predictions of Ali and Aydin (AA) found in their Table II. Note first that the values obtained here for  $\delta\lambda/\lambda$  are  $\sim 5$ - $10$  times larger than those of AA;

most of this can be traced to our value of  $\lambda$ , which is  $10$ - $20$  times smaller than that found by AA. This results from our different choice for mixing angles and our detailed phase-space considerations.

In calculating  $\delta m$ , AA have assumed  $B=1$  and  $(f_B/f_\tau)=3.85$  or  $2.31$ ; using these values we find our results for  $\delta m/\lambda$  in the  $B_d^0-\bar{B}_d^0$  system are larger roughly by a factor of  $10$ . This results, essentially, from the smaller value of  $\lambda$  obtained here. For the  $B_s^0-\bar{B}_s^0$  system, our results are again larger by a factor of  $5$ - $10$ , most of which is again traceable to our smaller  $\lambda$  value and to mixing-angle differences.

In conclusion, we find our results for these quantities to be in rough agreement with those of AA, except for the  $\lambda$  value by which we differ by roughly a factor of  $\sim 10$ , even though we have made the above improvements in the calculation.

We would like to point out that AA also consider  $CP$ -violation effects in the heavy-meson system; such a discussion is outside the scope of this paper.

#### VI. CONCLUSION

In this paper we have considered the possibility of the existence of flavor-changing neutral currents involving  $b$  quarks and  $\tau$  leptons. We have analyzed the  $B^0-\bar{B}^0$  system with and without these currents to see what kind of bounds future experiments can put on any FCNC couplings. We have examined several final states such as  $b \rightarrow se^+e^-$  which would signal  $b$  FCNC's although there is background of a substantial amount due to the ordinary charged-current decay chain

$$\begin{array}{l} b \rightarrow ce^-\bar{\nu}_e \\ \quad \searrow \\ \quad se^+\nu_e \end{array}$$

Similarly, we examined the possible limits on couplings which can be arrived at from  $\nu, \bar{\nu}$  production of  $b$ -flavored particles.

For  $\tau$  leptons, some limits already exist on possible FCNC's although they are quite weak [see Eq. (4.4)], and their existence is certainly not ruled out. One important mode is  $\tau$  decay into three charged leptons. We can also look for  $\tau$  production in  $e(\mu)N$  deep-inelastic interactions.

We conclude that the Glashow-Iliopoulos-Maiani mechanism has not been tested sufficiently to conclude that quarks (and leptons) heavier than  $u, d, s$ , and  $c$  ( $e, \mu$ ) do not participate in FCNC's. Much more experimental work is needed in this area.

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- <sup>1</sup>S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); Phys. Rev. D 5, 1412 (1972); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksells, Stockholm, 1968), p. 367.
- <sup>2</sup>S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).
- <sup>3</sup>M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
- <sup>4</sup>S. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977); E. A. Paschos, *ibid.* 15, 1966 (1977).
- <sup>5</sup>M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974).
- <sup>6</sup>V. Barger, W. F. Long, and S. Pakvasa, Phys. Rev. Lett. 42, 1585 (1979); R. E. Shrock, S. B. Treiman, and L. L. Wang, *ibid.* 42, 1589 (1979).
- <sup>7</sup>L. M. Seghal, Aachen Report No. PITHA 79/18, 1979 (unpublished); C. Y. Prescott *et al.*, Phys. Lett. 77B, 347 (1978); 84B, 524 (1979).
- <sup>8</sup>B. H. Wiik and G. Wolf, DESY Report No. DESY 78/23, 1978 (unpublished).
- <sup>9</sup>G. J. Feldman, SLAC Report No. SLAC-Pub-2230, 1978 (unpublished).
- <sup>10</sup>F. Buccella and L. Oliver, LBL Report No. LBL-9225, 1979 (unpublished).
- <sup>11</sup>See, for example, E. A. Paschos, Phys. Rev. Lett. 39, 858 (1977); J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. B131, 285 (1977); A. Ali and Z. Z. Aydin, *ibid.* B148, 165 (1979).
- <sup>12</sup>M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. 47, 277 (1975); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B100, 313 (1975); G. Altarelli, N. Cabibbo, and L. Maiani, Phys. Rev. Lett. 35, 635 (1975); Phys. Lett. 52B, 351 (1974).
- <sup>13</sup>S. Weinberg, in *Festschrift for I. I. Rabi*, edited by L. Motz (New York Academy of Sciences, New York, 1977); in *Neutrinos-78*, Proceedings of the International Conference on Neutrino Physics and Astrophysics, Purdue Univ., edited by E. C. Fowler (Purdue Univ. Press, W. Lafayette, Indiana, 1978).
- <sup>14</sup>See Ref. 5; see also R. E. Shrock and M. B. Voloshin, Princeton University report, 1979 (unpublished).
- <sup>15</sup>See Ref. 9.
- <sup>16</sup>M. L. Perl, in *Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977*, edited by F. Gutbrod (DESY, Hamburg, 1977), p. 145.
- <sup>17</sup>See, for example, C. H. Albright and C. Jarlskog, Nucl. Phys. B84, 467 (1975).
- <sup>18</sup>A. Pais and S. B. Treiman, Phys. Rev. Lett. 35, 1206 (1975); C. H. Albright and J. Smith, Phys. Lett. 77B, 94 (1978).