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Broken color symmetry and gluon masses in an SU(5) model of the electroweak-strong interaction

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In an SU(5) model of the electroweak-strong interaction, the shifts of vacuum expectation values of 24-plet Higgs fields break the color-SU(3) symmetry. The resulting masses of gluons and the other gauge bosons are obtained.

The success of the sequential $SU(2)\times U(1)$ model of the electroweak interaction encourages serious consideration of grand unification models of the electroweak-strong interaction. The simplest one, such as the SU(5) model,[1] is impressive in its numerical success for the masses of heavy negative quarks by the renormalization method. Unfortunately, there are some shortcomings of the SU(5) model, including the fact that it provides neither mass relations for positive quarks nor the understanding of the hierarchy problem, i.e., the mass of superheavy bosons $\gg$ the masses of weak bosons. A modified SU(5) model[2] with two different 24-plets and one 5-plet for Higgs particles gives interesting mass relations for both positive and negative quarks as well as for leptons, and generates all mixing angles of fermions.

Most grand-unification models, including the SU(5) model, assume exact color-SU(3) symmetry (quantum chromodynamics[3]) and the permanent confinement of quarks and gluons. The recent reports of the observation[4] of free fractional charges raise the possibility of nonexact color-SU(3) symmetry.

The purpose of the present work is to investigate the modified sequential SU(5) model[2] further to see whether the vacuum expectation values (VEV's) of Higgs fields can be shifted slightly by the interaction of 24-plet and 5-plet Higgs fields such that they can break spontaneously the color-SU(3) symmetry and, if so, obtain the resulting gluon masses and the masses of the other gauge bosons.

The model considered in Ref. 2 has two different 24-plets (Φ^i_j and Φ'^i_j) and one 5-plet (Ψ_i) of Higgs particles, where $i,j=1,2,\ldots,5$. The nonvanishing VEV's are $\langle\Phi^i_j\rangle=A_i a_{24}$, where $A_i=1$ for $i=1,2,3$ and $-\frac{3}{2}$ for $i=4,5$. The other 24-plet has $\langle\Phi'^i_i\rangle=A'_i a'_{24}$, where $A'_i=1$ for $i=1,\ldots,4$ and -4 for $i=5$. For the 5-plet Higgs fields, $\langle\Psi_i\rangle=0$ for $i=1,\ldots,4$ and a_5 for $i=5$.

We consider the following potentials for the Higgs fields:

$$V(\Phi)=-\tfrac{1}{2}\mu^2\,\mathrm{Tr}(\Phi^2)+\tfrac{1}{4}a[\mathrm{Tr}(\Phi^2)]^2+\tfrac{1}{2}b\,\mathrm{Tr}(\Phi^4)\,,$$

$$V(\Phi')=-\tfrac{1}{2}\mu'^2\,\mathrm{Tr}(\Phi'^2)+\tfrac{1}{4}a'[\mathrm{Tr}(\Phi'^2)]^2+\tfrac{1}{2}b'\,\mathrm{Tr}(\Phi'^4)\,,$$

$$V(\Psi)=\tfrac{1}{2}\nu^2(\Psi^\dagger\Psi)+\tfrac{1}{4}\lambda(\Psi^\dagger\Psi)^2\,. \tag{1}$$

The interaction of Φ and Ψ is given by

$$V(\Psi,\Phi)=\alpha(\Psi^\dagger\Psi)\,\mathrm{Tr}(\Phi^2)+\beta\Psi^\dagger\Phi^2\Psi\,. \tag{2}$$

The minimization condition of the total potential provides

$$\mu^2=aa_{24}{}^2\sum_i A_i{}^2+2ba_{24}{}^2\frac{\sum_i A_i{}^4}{\sum_i A_i{}^2}+2\alpha a_5{}^2+2\beta a_5{}^2\frac{A_5{}^2}{\sum_i A_i{}^2}\,. \tag{3}$$

The color-SU(3) symmetry can be broken slightly by shifting the VEV's of $\langle\Phi^i_i\rangle=A_i a_{24}$ to $(A_i+K_i\epsilon)a_{24}$, where $\epsilon\ll 1$. In order to produce a small symmetry breaking in the order of $\epsilon\sim a_5{}^2/a_{24}{}^2$, the following conditions have to be imposed in the minimization procedure for the potential in addition to the tracelessness conditions:

$$\sum_i A_i(K_i\epsilon)=0\,,\qquad \sum_i A_i{}^3(K_i\epsilon)=0\,. \tag{4}$$

Thus Eqs. (4) constraint the K_i's by

$$K_5=-K_4\,,\qquad K_3=-(K_1+K_2)\,. \tag{5}$$

It can be seen that a similar color-symmetry breaking in the order of $a_5{}^2/a'_{24}{}^2$ cannot be obtained for the case of Φ'. Therefore, we do not consider $V(\Psi,\Phi')$.

The new minimization of the total potential due to the VEV shifts can be used to obtain

$$\epsilon \simeq \frac{6\beta K^4 (a_5/a_{24})^2}{15a(K_1^2+K_2^2+K_1K_2+K_4^2)+b[17(K_1^2+K_2^2+K_1K_2)+47K_4^2]}. \tag{6}$$

The gluon (G) masses acquired by the VEV shifts are

$$\begin{aligned} m(G_{\rho^\pm}) &= \tfrac{1}{2} g_5 |K_1 - K_2| \epsilon a_{24}, \\ m(G_{K^{*\pm}}) &= \frac{|2K_1+K_2|}{|K_1-K_2|} m(G_{\rho^\pm}) \\ &= \frac{|2K_1+K_2|}{|K_1+2K_2|} m(G_{K^{*0}}), \\ m(G_{\rho^0}) &= m(G_{\omega^0}) = 0, \end{aligned} \tag{7}$$

where g_5 is the SU(5)-invariant coupling constant. The superheavy (or leptoquark) bosons X_i and Y_i have the masses

$$\begin{aligned} m^2(X_i) &= (\tfrac{1}{4} g_5^2 \tfrac{1}{2}) \left|\tfrac{5}{2} + (K_i + K_4)\epsilon\right|^2 a_{24}^2, \\ m^2(Y_i) &= (\tfrac{1}{4} g_5 \tfrac{1}{2})\left[\left|\tfrac{5}{2} + (K_i - K_4)\epsilon\right|^2 a_{24}^2 \right. \\ &\qquad \left. + 25 a_{24}'^2 + 2a_5^2\right], \end{aligned} \tag{8}$$

where $i = 1, 2, 3$ are the color indices. The mass degeneracy of different colors is removed.

As for the weak bosons,

$$\begin{aligned} m^2(W) &= (\tfrac{1}{4} g_5^2 \tfrac{1}{2})(|2K_4\epsilon|^2 a_{24}^2 + 25 a_{24}'^2 + 2a_5^2), \\ m^2(Z) &= (\tfrac{1}{4} g_5^2) a_5^2 \frac{1}{\cos^2\theta_W}. \end{aligned} \tag{9}$$

In order to make contact with the SU(2) × U(1) model[5] of the electroweak interaction in which $m(Z) = m(W)/\cos\theta_W$, $a_{24}' \ll a_5$ in Eq. (9). With $m(W) \simeq 84$ GeV and $m(X_i) \simeq 10^{14}$–10^{16} GeV, the color-symmetry-breaking constant $\epsilon \simeq 10^{-26}$–10^{-30} and the gluon masses[6] $m(G) \simeq 10^{-1}$–10^{-3} eV, if $K_i \simeq 1$ and $a \simeq b \simeq \beta$. The renormalization may alter the gluon masses.

The hierarchy of symmetry breaking, i.e., the fact that $m(X) \gg m(W)$, is intimately related to the spontaneous breaking of color symmetry in the present model, i.e., $\epsilon \propto [m(W)/m(X)]^2$. If there are free quarks which are the remnants of the primordial quarks in the early universe due to the broken color symmetry when hadrons are formed from hot quark and gluon soup, the collisions should produce the ratio R of the free quarks to the confined quarks in the order of $R|_{\text{th}} \simeq \epsilon \simeq 10^{-26}$–$10^{-30}$. This is a very crude estimate. The experimental observation[4] of two fractional charges in 9×10^{-5} gm of niobium ball indicates roughly $R|_{\text{exp}} \simeq 10^{-20}$, which is much greater than ϵ. Therefore, the observed fractional charges cannot be the remnants of the primordial quarks but the liberated quarks through the potential barrier deviated from the linear potential of the permanent confinement of quarks.

In summary, the present SU(5) model is capable of breaking color symmetry in the order of 10^{-26}–10^{-30}, and the resulting gluon masses are in the order of 10^{-1}–10^{-3} eV.

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[6]Compare this with $m(G) \simeq$ few–10 GeV in the $[\mathrm{Su}(4)]^4$ model. See J. C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973); Phys. Rev. Lett. 31, 661 (1973); Phys. Rev. D 10, 275 (1974).