New mass relations and mixing angles in an SU(5) model of the electroweak-strong interaction

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(Received 14 December 1979)

In an SU(5) model of the electroweak-strong interaction, a mass relation $m_{u,c,t}(M \simeq 10^{14}\ \mathrm{GeV}/c^2) = 2\sqrt{2} m_{d,s,b}(M)$, and others are obtained by the Higgs mechanism consisting of $10_F \times 10_F \times 5_H^*$, $5_F^* \times 5_F \times 24_H$, and $5_F^* \times 10_F \times 5_H$ couplings. The renormalization corrections to the mass relation give $m_{u,c,t}(\mu^2 = 10\ \mathrm{GeV}^2/c^4) = 3.072 m_{d,s,b}(\mu)$, which is in good agreement for the heavy quarks. The Kobayashi-Maskawa mixing angles for (d,s,b), (u,c,t), and (e,μ,τ) are predicted to be approximately the same if the effect due to the mass differences is neglected.

The sequential standard SU(2)×U(1) gauge theory of the electroweak interaction[1] has been successful in accounting for the charmed quark,[2] the weak neutral current,[3] the b quark,[4] the heavy lepton[5] τ^-, and parity violation in inelastic electron scattering.[6] With the growing confidence in the sequential SU(2)×U(1) model, the interest in the grand unification[7,8] of the electromagnetic, weak, and strong interactions (hereafter called electroweak-strong interaction) has increased recently. Among the candidate models of the electroweak-strong interaction, the simplest and the smallest group which contain color SU(3) for quantum chromodynamics and SU(2)×U(1) for quantum flavor dynamics is SU(5). The SU(5) model[7] has been able to predict the mass of heavy quark b by considering the corrections due to the renormalization of fermion masses below the grand-unification mass.

In the SU(5) model of Ref. 7, there exist mass relations of the negative quarks ($q^-=d$, s, or b) and the negative leptons ($l^-=e$, u, τ). The shortcoming of the model is that there are no predictions on the masses of the positive quarks ($q^+=u$, c, or t). There are attempts[9] to derive mass relations between q^+ and q^- quarks. The purpose of the present work is to present a Higgs mechanism which is able to predict the masses of q^+ quarks through the mass formulas connecting q^+, q^-, and l^- particles.

The sequential SU(5) model consists of the three families

$$L_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & -u_1 & -d_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & -u_2 & -d_2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L , \quad R_1 = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \\ \bar{\nu}_e \end{pmatrix}_R , \tag{1}$$

and similarly L_2 (L_3) and R_2 (R_3) by $u \to c$ (t), $d \to s$ (b), $e^+ \to \mu^+$ (τ^+), and $\bar{\nu}_e \to \bar{\nu}_\mu$ ($\bar{\nu}_\tau$). Higgs particles consist of a 5-plet (Ψ_a) and two 24-plets (Φ_b^a and $\Phi_b'^a$) whose vacuum expectation values (VEV's) are given by

$$\langle \Psi_a \rangle = a_5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} ,$$

$$\langle \Phi_b^a \rangle = a_{24} \begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & A & \\ 0 & & & & A \end{pmatrix} , \tag{2}$$

$$\langle \Phi_b'^a \rangle = a_{24}' \begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & C & \\ 0 & & & & D \end{pmatrix} ,$$

where $a,b = 1,2,\ldots,5$, $A = -\frac{3}{2}$, $C = 1$, and $D = -4$. $\langle \Phi_b^a \rangle$ and $\langle \Phi_b'^a \rangle$ are the only two possible VEV's of 24-plets due to the minimization conditions[10] imposed on the Higgs potential.

In order to create a rich fermion mass spectrum, we adopt the following Lagrangians relevant to the fermion masses:

$$\mathcal{L}_f^{(-)} = \sum_{i,j=1}^{3} \frac{G_{ij}^{(-)}}{\sqrt{2}} (\bar{R}_i \Psi L_j + \bar{L}_j \Psi^\dagger R_i) \tag{3}$$

by $5_F^* \times 10_F \times 5_H$ couplings for the mass of the negative quarks,

$$\mathcal{L}_f^{(+)} = \sum_{i,j=1}^{3} \frac{G_{ij}^{(+)}}{\sqrt{2}} (\bar{L}_i^c \Psi L_j + \bar{L}_j \Psi^\dagger L_i^c) \quad (4)$$

by $10_F \times 10_F \times 5_H^*$ couplings for the mass of the positive quarks, and

$$\mathcal{L}_f^{(0,-)} = \sum_{i,j=1}^{3} \frac{g_{ij}}{\sqrt{2}} (\bar{R}_i^c \Phi R_j + \bar{R}_j \Phi^\dagger R_i^c) + \sum_{i,j=1}^{3} \frac{g'_{ij}}{\sqrt{2}} (\bar{R}_i^c \Phi' R_j + \bar{R}_j \Phi'^\dagger R_i^c) \quad (5)$$

by $5_F^* \times 5_F \times 24_H$ couplings for the mass of the negative and neutral fermions. In Eq. (4), the charge-conjugate field is denoted by L_i^c, and $\bar{L}_i^c \Psi L_j = \bar{L}_i^{ab} \gamma_0 L_j^{cd} \epsilon_{abcde} \Psi^e$, where $a,b,c,d,e = 1,2,\ldots,5$ and ϵ_{abcde} is the antisymmetric tensor. The coupling constants in Eqs. (3), (4), and (5) are symmetric under the indices.

The 3×3 matrices for the negative quarks q^- (d, s, b), the negative leptons l^- (e, μ, τ), the neutral leptons l^0 $(\nu_e, \nu_\mu, \nu_\tau)$, and the positive quarks q^+ (u, c, t) are, respectively,

$$\begin{aligned} M_{ij}(q^-) &= \frac{a_5}{2} G_{ij}^{(-)} + \frac{a_{24}}{\sqrt{2}} g_{ij} + \frac{a'_{24}}{\sqrt{2}} g'_{ij}, \\ M_{ij}(l^-) &= \frac{a_5}{2} G_{ij}^{(-)} + A\frac{a_{24}}{\sqrt{2}} g_{ij} + C\frac{a'_{24}}{\sqrt{2}} g'_{ij}, \\ M_{ij}(l^0) &= A\frac{a_{24}}{\sqrt{2}} g_{ij} + D\frac{a'_{24}}{\sqrt{2}} g'_{ij}, \\ M_{ij}(q^+) &= 2\sqrt{2}\, \frac{a_5}{2} G_{ij}^{(+)}, \end{aligned} \quad (6)$$

where $i, j = 1, 2, 3$. It is possible to choose a_{24}, a'_{24}, q_{ij}, and g'_{ij} such that a_5 and G_{ij} contribute to the fermion masses predominantly. In order to give weak gauge bosons proper masses, $a'_{24} \ll a_5$.[12] We have no natural explanations why these values should have such diverse magnitudes, as much as there is understanding on the fact that a_{24} should be much greater than a_5 to give superheavy bosons proper masses in a standard SU(5) model. This problem of hierarchy, which arises in any grand-unification model, is not well understood.

Thus we get a new mass formula at the level of the grand-unification mass[7] $M \simeq 10^{14}$ GeV,

$$\sqrt{2} \sum m_{q^+}(M) - [3 \sum m_{q^-}(M) + \sum M_{l^-}(M)] = \sum m_{l^0}(M), \quad (7)$$

where $\sum m_{q^+}(M) \equiv m_u(M) + m_c(M) + m_t(M)$, etc., and in the case of $G_{ij}^{(+)} \simeq G_{ij}^{(-)} \equiv G_{ij}$. Even though there is no compelling reason for this approximate equality, the possibility is not ruled out. The correction factors due to the renormalization of masses below the grand-unification mass modify Eq. (7) to

$$\sqrt{2} R_{q^+} \sum m_{q^+}(\mu) = 3R_{q^-} \sum m_{q^-}(\mu) + R_{l^-} \sum m_{l^-}(\mu), \quad (8)$$

if $\sum m_{l^0}(M) \simeq 0$, or $a'_{24} g'_{ij} = -\frac{3}{8} a_{24} g_{ij}$, and the correction factors are

$$\begin{aligned} &R_{q^+} \equiv \frac{\gamma_1^{\,2}}{\gamma_3}, \quad R_{q^-} \equiv \frac{1}{\gamma_1 \gamma_3}, \quad R_{l^-} \equiv \gamma_1^{\,9}, \\ &\gamma_1 \equiv \left(\frac{\alpha_1(\mu)}{\alpha_G}\right)^{3/20}, \quad \gamma_2 \equiv \left(\frac{\alpha_2(\mu)}{\alpha_G}\right)^{27/(88-8f)}, \\ &\gamma_3 \equiv \left(\frac{\alpha_s(\mu)}{\alpha_G}\right)^{4/(11-2f/3)}. \end{aligned} \quad (9)$$

In Eq. (9), $\alpha_1 = 1/137$, α_2, $\alpha_5(\mu^2 = 10\ \text{GeV}^2/c^4) = 0.19-0.32$, and $\alpha_G = 0.022$ are, respectively, the coupling constants associated with U(1), SU(2), SU(3), and SU(6), and the number of flavors $f = 6$.

At the level of $\mu^2 = 10\ \text{GeV}^2/c^4$ and with the input of $m_u(\mu) \simeq m_d(\mu) = 0.39$, $m_s(\mu) = 0.50$, $m_c(\mu) = 1.55$, $m_b(\mu) = 4.73$, $m_e(\mu) = 5\times10^{-3}$, $m_\mu(\mu) = 0.105$, $m_\tau(\mu) = 1.782$ all in the units of GeV/c^2, Eq. (8) predicts $m_t(\mu) = 15.44 \pm 0.65\ \text{GeV}/c^2$, which is comparable with the result of the 0(10) model.[9] If g_{ij} (or g'_{ij}) $\ll G_{ij}$, i.e., $m_{l0}(\mu) \simeq 0$, Eq. (6) reduces to the mass relations of Ref. 7 and a new mass formula for q^+ quarks:

$$R_{q^-} m_{d,s,b}(\mu) = R_{l^-} m_{e,\mu,\tau}(\mu) = \frac{1}{2\sqrt{2}} R_{q^+} m_{u,c,t}(\mu) \quad (10)$$

and predicts

$$m_{u,c,t}(\mu) = 2\sqrt{2}\, \frac{R_{q^-}}{R_{q^+}}\, m_{d,s,b}(\mu), \quad (11)$$

where $2\sqrt{2}(R_{q^-}/R_{q^+}) = 2(\sqrt{2}/\gamma_1^{\,3}) = 3.072$. Thus the masses of heavy positive quarks are predicted to be $m_c(\mu) = 1.54\ \text{GeV}/c^2$ and $m_t(\mu) = 14.53\ \text{GeV}/c^2$ by this approximation. Equation (10) does not give good numerical predictions for light quarks, and this is understood by the fact that the light quarks acquire the substantial part of their masses from the chiral symmetry breaking (or dynamically), and the lowest-order perturbation is not reliable because of large α_s.

From the mass matrices in Eq. (6), the generalized Euler's (Kobayashi-Maskawa[1]) mixing angles are approximately

$$\theta_i(l^-) \gtrsim \theta_i(q^-) \gtrsim \theta_i(q^+),\ \theta_i(l^0) \simeq 0,\ i = 1, 2, 3 \quad (12)$$

if $g_{ij}, g'_{ij} \ll G_{ij}$. For the case of q^- quarks, the experimental analysis[11] gives

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 0.97 & 0.22 & 0.068 \\ -0.22 & 0.85 & 0.48 \\ -0.046 & 0.48 & -0.88 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (13)$$

if the CP-violating phase is neglected. Our prediction is that the mixings of (u, c, t) and (e, μ, τ) are approximately the same as Eq. (13) if we neglect the effect due to the mass differences. If the effect of the mass difference in Eq. (6) is considered, the mixings of (u, c, t) and (e, μ, τ) are, respectively, greater and less than the one for (d, s, b).

Since the diagonalized mass matrix at the level of M is only affected by a common correction factor due to the renormalization, no additional rotation is needed at the level of μ. Thus the mixing angles θ_i are independent of μ.

In summary, a set of new mass formulas predict $m_c = 1.54$ GeV/c^2, $m_t = 15.44 \pm 0.65$ GeV/c^2, and the approximately same Kobayashi-Maskawa mixing angles for (d, s, b), (u, c, t), and (e, μ, τ).

It is a pleasure to acknowledge the hospitality of the Theoretical Physics Department at Fermi National Accelerator Laboratory where part of the work was carried out.

[1] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367; S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970); H. Georgi and S. L. Glashow, *ibid.* 7, 561 (1973); M. Kobayashi and Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[2] J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, *ibid.* 33, 1406 (1974); C. Bacci *et al.*, *ibid.* 33, 1408 (1974); T. Appelquist and H. D. Politzer, *ibid.* 34, 43 (1975); A. De Rújula and S. L. Glashow, *ibid.* 34, 46 (1975).

[3] L. F. Abbott and R. M. Barnett, Phys. Rev. Lett. 40, 1303 (1978) and references therein.

[4] S. W. Herb *et al.*, Phys. Rev. Lett. 39, 252 (1977).

[5] M. L. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975); Phys. Lett. 63B, 466 (1976).

[6] C. Y. Prescott *et al.*, Phys. Lett. 77B, 347 (1978).

[7] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B135, 66 (1978).

[8] K. Inoue, A. Kakuto, and Y. Nakano, Prog. Theor. Phys. 58, 630 (1977); S. K. Yun, Phys. Rev. D 18, 3472 (1978).

[9] H. Georgi and D. V. Nanopoulos, Phys. Lett. 82B, 392 (1979).

[10] L.-F. Li, Phys. Rev. D 9, 1723 (1974).

[11] R. E. Shrock, S. B. Treiman, and L.-L. Wang, Phys. Rev. Lett. 42, 1589 (1979).

[12] For the masses of various gauge bosons in this model and other related features, see S. K. Yun, following paper, Phys. Rev. D 21, 2690 (1980).