Tests of isospin selection rules in exclusive and semi-inclusive decays of charged D mesons

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In the Cabibbo-allowed decays of the charged D meson, the standard model predicts that the isospin of the final state must be pure $T = 3/2$ for nonleptonic decays and $T = 1/2$ for semileptonic decays. We determine the consequences of these isospin rules for the ratios of K^- final states to $\bar{K}^0$ final states in both exclusive channels and in semileptonic ones. These ratios are bounded, and present data do not violate these bounds, but they are too sparse to make any definitive tests. Other predictions of the selection rules, such as the Dalitz plot for $\bar{K}\pi\pi$ modes, are also discussed.

I. INTRODUCTION

In two previous papers, we have discussed the difference in lifetimes between charged and neutral D mesons,[1] and what it implies for neutral-D-meson decay.[2] Here we turn our attention to Cabibbo-allowed decays of the charged D meson.

The principal issue in D^+ decay is not so much the dynamical mechanism behind the process as the isospin selection rules that it satisfies.[3] In the standard charm model of Glashow, Iliopoulos, and Maiani,[4] the effective Hamiltonian for Cabibbo-allowed, nonleptonic charm decay behaves as the $T_3=+1$ component of an isovector,[3] and so these decays will satisfy the selection rules $\Delta T_3 = \Delta T = 1$ to lowest order in the weak interaction, and to all orders in the strong, isospin-conserving quark-gluon interaction. Thus, unlike the $\Delta T = \frac{1}{2}$ rule for nonleptonic strange decay,[5] the $\Delta T = 1$ rule for nonleptonic charm decay provides us with a very clean test of the standard model.[6] The same is true of the semileptonic, Cabibbo-allowed decays of the D^+, which must conserve isospin to all orders of the weak and strong interactions.

For the charged D meson, the $\Delta T_3 = \Delta T = 1$ rule has the particularly simple consequence that the final state must have pure isospin $T_f = \frac{3}{2}$.[3] Similarly the conservation of isospin in semileptonic decays means that the hadrons in the final state must have a total isospin of $\frac{1}{2}$. In this paper, we propose to test these assignments for both exclusive and semi-inclusive decays of the D^+: We shall concentrate upon the ratio of decays into states containing a K^- mesons as compared with those containing a $\bar{K}^0$ meson, and we shall show that there are definite bounds upon this ratio resulting from the isospin of the final state. Although the present data[7,8] are too sparse to draw any definitive conclusions, they do appear to fall within the bounds. As we shall see, this leads to some interesting predictions about the structure of the final state, one example being the Dalitz plot for $D^+ \to \bar{K}^0\pi^+\pi^0$.

In the second section of the paper, we analyze the consequences of a $T_f = \frac{3}{2}$ final state for the Cabibbo-allowed, nonleptonic decays of the D^+, and we obtain bounds on the ratio R_H of K^- to $\bar{K}^0$ final states in purely hadronic decays. In the third section, we analyze the semileptonic decays and obtain bounds on R_L, the ratio of K^- to $\bar{K}^0$ in semileptonic decays. We then use this result to obtain bounds on R_X, the K^- to $\bar{K}^0$ ratio in all decays, leptonic and hadronic; these bounds depend on the semileptonic branching ratio of D^+. The final section is devoted to a comparison of theory and existing data.

II. NONLEPTONIC DECAYS

The only two-body final state of D^+ decay, namely $\bar{K}^0\pi^+$, automatically has $T_f = \frac{3}{2}$, and so we begin our analysis with the three-body final states $K^-\pi^+\pi^+$ and $\bar{K}^0\pi^+\pi^0$. In general the two-pion system can have isospin $T = 0$, 1, and 2, but in this case only the $T = 1$ and 2 states are allowed. Because of Bose statistics, the $T = 1$ state, which we denote by (P^+, P^0, P^-), must be antisymmetric under the exchange of spatial coordinates, and the $T = 2$ state, which we denote by $(Q^{++}, Q^{+0}, \ldots)$, must be symmetric. The $T = \frac{3}{2}$ final state can then be written as

$$q^{(2)}\left(\frac{2}{\sqrt{5}} K^- Q^{++} - \frac{1}{\sqrt{5}} \bar{K}^0 Q^{+0}\right) + p^{(2)} \bar{K}^0 P^+ , \qquad (1)$$

where $q^{(2)}$ is the amplitude for decay into the $T = 2$ two-pion system, and $p^{(2)}$ that for decay into the $T = 1$ system.

We can calculate the ratio of rates for the $K^-\pi^+\pi^+$ and $\bar{K}^0\pi^+\pi^0$ decay modes from Eq. (1). The states Q^{+0} and P^+ are orthogonal to one another because they have opposite spatial symmetries, and so they do not interfere coherently in the rate for $\bar{K}^0\pi^+\pi^0$. The ratio is then given by

$$R_2 \equiv \frac{B(K^-\pi^+\pi^+)}{B(\bar{K}^0\pi^+\pi^0)} = \frac{4|q^{(2)}|^2}{|q^{(2)}|^2+5|p^{(2)}|^2}, \tag{2}$$

where $B(K^\alpha\pi^\beta\pi^\gamma)$ denotes the branching ratio for the decay mode indicated by the superscripts α,β,γ. It is obvious that R_2 is bounded,

$$0 \leq R_2 \leq 4, \tag{3}$$

and that it will be less than 4 if and only if the amplitude $p^{(2)}$ is nonzero. Since $p^{(2)}$ refers to the spatially antisymmetric two-pion state, it follows that the Dalitz plot for $\bar{K}^0\pi^+\pi^0$ will show structure about the line of pion energies whenever $p^{(2)}$ does not vanish. Thus the $\Delta T=1$ rule predicts that if the ratio R_2 of Eq. (2) is less than 4, then the Dalitz plot for $K^0\pi^+\pi^0$ must show the above type of structure.

There are three possible modes of four-body decay: $K^-\pi^+\pi^+\pi^0$, $\bar{K}^0\pi^+\pi^+\pi^-$, and $\bar{K}^0\pi^+\pi^0\pi^0$. In general the three-pion system can have isospin ranging from 0 to 3, but the $T=\frac{3}{2}$ final-state requirement restricts it to an admixture of 1 and 2. Consequently the final state will have exactly the same form as in Eq. (1), except that we now interpret the $Q^{\alpha\beta}$ and P^α as the $T=2$ and $T=1$ states, respectively, of three pions.

One difference between the two-pion and three-pion decay modes shows up in the spatial symmetries of the $Q^{\alpha\beta}$ and P^γ states. The $T=2$ combination of three pions belongs to the two-dimensional, mixed-symmetry representation (2_M) of the permutation group S_3; thus the spatial wave function associated with it must also have mixed permutation symmetry to ensure that the complete wave function is totally symmetric under all permutations.[9] For $T=1$, there are two possible isospin wave functions, one totally symmetric under S_3 and the other of mixed symmetry; thus the spatial wave function of the P^α state will, in general, contain a totally symmetric component and a mixed symmetry one.[9] To the extent that it does contain a mixed symmetry component, the spatial wave function of the three-pion P^α state will not be orthogonal to that of the corresponding $Q^{\alpha\beta}$ state, in contrast to the situation for two pions.

What this result means in practice is that when we look at a particular decay mode, for example $\bar{K}^0\pi^+\pi^+\pi^-$, there can be coherent interference between the spatial wave function of Q^{+0} and the mixed symmetry component of P^+. However, when we sum over the decay rates of all modes with the same total charge on the three pions, e.g., $\bar{K}^0\pi^+\pi^+\pi^-$ and $\bar{K}^0\pi^+\pi^0\pi^0$, the interference terms drop out, and we are left with an incoherent sum over the Q^{+0} and P^+ states. Consequently, we can define a ratio R_3 for $D^+\to K^-(3\pi)^{++}$ versus $D^+\to K^0(3\pi)^+$ as

$$R_3 = \frac{B(K^-\pi^+\pi^+\pi^0)}{B(\bar{K}^0\pi^+\pi^+\pi^-)+B(\bar{K}^0\pi^+\pi^0\pi^0)}, \tag{4}$$

and by an analysis similar to that of the two-pion case, we find that

$$R_3 = \frac{4|q^{(3)}|^2}{|q^{(3)}|^2+5|p^{(3)}|^2}, \tag{5}$$

where $q^{(3)}$ is the three-pion amplitude corresponding to $q^{(2)}$ in Eq. (1), and $p^{(3)}$ is the amplitude corresponding to $p^{(2)}$. Again the limits on R_3 are

$$0 \leq R_3 \leq 4, \tag{6}$$

and a value of R less than 4 means that the amplitude $p^{(3)}$ is not zero.

We can now extend this analysis to final states with n pions, and thence to semi-inclusive decay rates. As in the case of the $\bar{K}(2\pi)$ and $\bar{K}(3\pi)$ decay modes, so in the case of $\bar{K}(n\pi)$ the $T=\frac{3}{2}$ final-state requirement limits the isospin of the $(n\pi)$ system to $T=1$ and $T=2$. Consequently the final state has exactly the same form as in Eq. (1) with $Q^{\alpha\beta}$ being the $T=2$ state of n pions, and P^α the corresponding $T=1$ state. The permutational properties of these states may be such that for a specific charge state, the spatial wave functions of $Q^{\alpha\beta}$ and P^α may interfere coherently with one another; however, the interference terms will drop out when we sum over all modes in which the n pions have the same total charge. Thus we define a ratio of rates for $D^+\to K^-(n\pi)^{++}$ versus $D^+\to\bar{K}^0(n\pi)^+$:

$$R_n = \frac{\sum_l B(K^-(n\pi)_l)}{\sum_m B(\bar{K}^0(n\pi)_m)}, \tag{7}$$

where the sum over l runs over all n-pion states with charge (+2), and that over m runs over corresponding states with charge (+1).

Using the same analysis as before, we find that

$$R_n = \frac{4|q^{(n)}|^2}{|q^{(n)}|^2+5|p^{(n)}|^2}, \tag{8}$$

where $q^{(n)}$ is the n-pion amplitude corresponding to $q^{(2)}$ in Eq. (1), and $p^{(n)}$ is the amplitude corresponding to $p^{(2)}$: The limits on R_n are

$$0 \leq R_n \leq 4. \tag{9}$$

A value of R_n less than 4 means that $p^{(n)}$ is different from zero, and that the n-pion system has a $T=1$ component.

To define a ratio for semi-inclusive hadronic decays $D^+\to K^-H^{++}$ versus $D^+\to\bar{K}^0H^+$, we must include the mode $D^+\to\bar{K}^0\pi^0$, whose amplitude we denote by $p^{(1)}$, and sum over the n-pion states from $n=2$ to the maximum N allowed by the decay energetics. The ratio is then given by

$$R_H = \frac{B(K^-H^{++})}{B(\overline{K}^0H^+)} = \frac{4\sum_{n=2}^{N}|q^{(n)}|^2}{\sum_{n=2}^{N}|q^{(n)}|^2 + 5\sum_{n=1}^{N}|p^{(n)}|^2}, \quad (10)$$

where the symbol H denotes all hadronic states of the appropriate charge. Again we find that R_H must lie in the range

$$0 \le R_H \le 4 \quad (11)$$

if the $\Delta T = 1$ rule is valid.

III. SEMILEPTONIC DECAYS

The semileptonic, Cabibbo-allowed decays of the D^+ conserve isospin, and so the hadrons in the final state must have charge zero, strangeness (−1), and isospin $\frac{1}{2}$. These are exactly the same quantum numbers as predicted by the pole model for nonleptonic decays of the D^0, and so our analysis will yield much the same results for ratios of K^- to $\overline{K}^0$ final states as occur in the pole model.[2] In particular the ratio will lie between 0 and 2.

Since the only three-body mode, namely $D^+ \to \overline{K}^0 l^+ \nu_l'$, automatically satisfies the selection rules, we begin with the four-body decay modes $D^+ \to \overline{K}^0\pi^0 l^+\nu$ and $K^-\pi^+ l^+ \nu_l^-$. From the $T = \frac{1}{2}$ requirement, the final state takes the form

$$L_p^{(1)}\left(\frac{\sqrt{2}}{\sqrt{3}}K^-\pi^+ - \frac{1}{\sqrt{3}}\overline{K}^0\pi^0\right)|l^+\nu_l\rangle, \quad (12)$$

where $L_p^{(1)}$ is the decay amplitude. From this we see immediately that the ratio of K^- to $\overline{K}^0$ final states is exactly equal to 2:

$$R_{l1} = \frac{B(K^-\pi^+l^+\nu_l)}{B(\overline{K}^0\pi^0l^+\nu_l)} = 2\,. \quad (13)$$

Next consider the five-body modes $D^+ \to \overline{K}(2\pi)l^+\nu$. Here the $T = \frac{1}{2}$ final-state requirement restricts the two-pion system to a spatially symmetric, isoscalar state S^0, and a spatially antisymmetric isovector (P^+, P^0, P^-). The final state can then be expressed in terms of an amplitude $L_S^{(2)}$ for the production of the S^0 state, and an amplitude $L_P^{(2)}$ for producing the P state

$$\left[L_S^{(2)}\overline{K}^0S^0 + L_P^{(2)}\left(\frac{\sqrt{2}}{\sqrt{3}}K^-P^+ - \frac{1}{\sqrt{3}}\overline{K}^0P^0\right)\right]|l^+\nu_l\rangle. \quad (14)$$

Because the spatial wave functions of the S^0 and P states are orthogonal to one another, they do not interfere in the rate for $D^+ \to K^0\pi^+\pi^-l^+\nu_l$. Thus the branching ratios for the various modes are

$$\begin{aligned} &B(K^-\pi^+\pi^0l^+\nu_l) = \tfrac{2}{3}\,|L_P^{(2)}|^2\rho\,,\\ &B(\overline{K}^0\pi^+\pi^-l^+\nu_l) = (\tfrac{1}{3}\,|L_P^{(2)}|^2 + \tfrac{2}{3}\,|L_S^{(2)}|^2)\rho\,, \quad (15)\\ &B(\overline{K}^0\pi^0\pi^0l^+\nu_l) = (\tfrac{1}{3}\,|L_S^{(2)}|^2)\rho\,, \end{aligned}$$

where ρ is a phase-space factor. From Eq. (15) we learn that

$$4B(\overline{K}^0\pi^0\pi^0l^+\nu) = 2B(\overline{K}^0\pi^+\pi^-l^+\nu) - B(K^-\pi^+\pi^0l\nu_l)\,. \quad (16)$$

In addition, the ratio of K^- to K^0 final states is

$$\begin{aligned} R_{l2} &\equiv \frac{B(K^-\pi^+\pi^0l^+\nu_l)}{B(\overline{K}^0\pi^+\pi^-l^+\nu) + B(\overline{K}^0\pi^0\pi^0l^+\nu_l)}\\ &= \frac{2\,|L_P^{(2)}|^2}{|L_p^{(2)}|^2 + 3\,|L_S^{(2)}|^2}\,. \quad (17) \end{aligned}$$

Obviously R_{l2} is bounded:

$$0 \le R_{l2} \le 2\,. \quad (18)$$

From here we can proceed directly to the n-pion and the semi-inclusive final states. The $T = \frac{1}{2}$ final-state requirement restricts the n-pion system to isospins $T = 0$ and 1, and so the structure of the final state is exactly as in Eq. (14). Consequently the ratio of K^- to K^0 final states is

$$\begin{aligned} R_{ln} &= \frac{\sum_i B(K^-(n\pi)_i l^+\nu_l)}{\sum_j B(\overline{K}^0(n\pi)_j l^+\nu_l)}\\ &= \frac{2\,|L_P^{(n)}|^2}{|L_P^{(n)}|^2 + 3\,|L_S^{(n)}|^2}\,, \quad (19) \end{aligned}$$

where the sums over i and j run over all n-pion states with total charges (+1) and zero respectively, and where $L_P^{(n)}$ and $L_S^{(n)}$ are the n-pion analogs of the amplitudes in Eq. (14). Again the limits on the ratio are

$$0 \le R_{ln} \le 2\,. \quad (20)$$

It is obvious that we can define a semi-inclusive ratio

$$R_L = \frac{B(K^-H^+l^+\nu_l)}{B(\overline{K}^0H^0l^+\nu_l)}, \quad (21)$$

where H represents all hadronic states of the appropriate charge, and that the bounds on this ratio are

$$0 \le R_L \le 2\,. \quad (22)$$

Using the general forms of the hadronic and semileptonic final states in Eqs. (1) and (14), respectively, we can combine the bounds on R_L in Eq. (22) with those on R_H in Eq. (11), to obtain a bound on the ratio of K^- to $\overline{K}^0$ final states in all decay modes, nonleptonic and semileptonic. We define the ratio as

$$R_X = \frac{B(K^- H^{++}) + B(K^- H^+ e^+ \nu_l) + B(K^- H^+ \mu^+ \nu_\mu)}{B(\bar{K}^0 H^+) + B(\bar{K}^0 H^0 e^+ \nu_e) + B(\bar{K}^0 H^0 \mu^+ \nu_\mu)}, \tag{23}$$

where H represents all hadronic states of the appropriate charge, and we find that R_X is bounded by the expression

$$0 \leq R_X \leq \frac{12 - 2B(\mathrm{SL})}{3 + 2B(\mathrm{SL})}, \tag{24}$$

where $B(\mathrm{SL})$ denotes the semileptonic branching ratio for D^+ decay:

$$B(\mathrm{SL}) = B(\bar{K}He^+\nu) + B(\bar{K}H\mu^+\nu). \tag{25}$$

The upper bound on R_X decreases monotonically from 4 to 2 as $B(\mathrm{SL})$ increases from 0 to 1.

IV. COMPARISON WITH EXPERIMENT

Present data on D^+ decays are rather sparse,[7,8] but we shall make comparisons wherever we can. Some exclusive $\bar{K}(n\pi)$ branching ratios are known, and so we can test some of the limits in Sec. II. Likewise the semi-inclusive branching ratios for K^- and $\bar{K}^0$ final states have been measured, and the first measurements of the semileptonic branching ratio have been made; thus we can test the bounds on R_X in Sec. III. In addition we can make some predictions about the structure of exclusive $\bar{K}(2\pi)$ final states; we shall, in fact, begin our discussion with these decay modes.

Experimentally, the branching ratio for $D^+ \to K^-\pi^+\pi^+$ is known quite well[7,8]:

$$B(K^-\pi^+\pi^+) = 4.5 \pm 0.8\%; \tag{26}$$

but that for $D^+ \to \bar{K}^0\pi^+\pi^0$ is subject to very large errors[8]:

$$B(\bar{K}^0\pi^+\pi^0) = 15.3 \pm 9\%. \tag{27}$$

Thus, while it is too early to draw any definite conclusion, it would appear that the ratio R_2 of Eqs. (2) and (3) will turn out to be less than 4—in fact it could well be much closer to $\frac{1}{4}$. This suggests that a $T=1$ two-pion system is present in the final state, and that the Dalitz plot for the $K^0\pi^+\pi^0$ mode will show definite structure with respect to the line of equal pion energies. At present there are too few events to test this prediction.

The only $\bar{K}(3\pi)$ decay mode seen up to now is $\bar{K}^0\pi^+\pi^+\pi^-$, but its branching ratio is quite large, namely[7] $(5.2 \pm 2.1)\%$. This suggests that its companion decay modes $\bar{K}^0\pi^+\pi^0\pi^0$ and $K^-\pi^+\pi^+\pi^0$ might occur at comparable levels; presumably they are hard to see because of the neutral pions in the final state. There is some evidence[7] for the $\bar{K}(4\pi)$ mode $K^-\pi^+\pi^+\pi^+\pi^-$ but the branching ratio has not been measured.

The K^- and $\bar{K}^0$ semi-inclusive branching ratios are known within rather large errors[7]:

$$\begin{aligned} B(K^-X^{++}) &= (17 \pm 4.5)\%, \\ B(\bar{K}^0X^+) &= (38 \pm 16)\%, \end{aligned} \tag{28}$$

and so the ratio R_X would appear to lie somewhere in the range

$$R_X \approx \tfrac{1}{2} - 1. \tag{29}$$

To compare this with the bound in Eq. (24), we note that the Mark II group[7] at SPEAR has recently found the electron semileptonic branching ratio to be $(15.8 \pm 5.8)\%$ while the DELCO group[8] has found it to be $(23 \pm 6)\%$. Taking the mean of 20% and doubling to include muonic decays, we find that $B(\mathrm{SL})$ is 40%. From Eq. (24), the upper bound on R_X is about 3, a value well above the most likely range in Eq. (29). Thus present data are consistent with the isospin selection rules of the standard model. More accurate tests are needed, however, before definite conclusions can be drawn.

It is amusing to note that we can gain some idea of the size of R_L, the K^- to $\bar{K}^0$ ratio in semileptonic decays [Eq. (21)] from the present data. It would appear that the ratio of $\bar{K}^0e^+\nu$ to $\bar{K}He^+\nu$ final states is 0.63 ± 0.2,[7,8] and that a large fraction of the $\bar{K}H$ state is $K^*(890)$.[7] If we take an idealized case in which the final state is an equal admixture of K^0 and $K^*(890)$, we set $L_S = L_P$ in the appropriate form of Eq. (14), and from Eqs. (17) or (19) we find that $R_L = \frac{1}{2}$, a value quite close to the value for R_X in Eq. (29).

In conclusion we would like to mention two points. Firstly, whenever a ratio of K^- to $\bar{K}^0$ final states falls below the upper bound, the multipion system in the final state will be an admixture of two different isospins; this is true for nonleptonic and semileptonic decays and can have importance for event distributions such as the Dalitz plot. Secondly, we have omitted any consideration of the decay mode $D^+ \to K^+\bar{K}^0\bar{K}^0$: It is Cabibbo-allowed, but is expected to be small partly because of phase space, and partly because it requires the creation of a strange quark-antiquark pair from the gluon field.

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