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Propagator momentum-transfer effects in expanded gauge models of neutral currents

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We generalize the Georgi-Weinberg discussion of the effective neutral-current Hamiltonian in expanded gauge models to include the four-momentum-transfer (q^2) effects due to neutral-gauge-boson propagators. We show that the condition that an $SU(2)\times U(1)\times G$ gauge theory reproduces the neutrino-quark interactions of the standard $SU(2)\times U(1)$ model at any $q^2\neq 0$ is that, in addition to satisfying the conditions of the Georgi-Weinberg theorem, the electromagnetic charge must decouple from the neutral generators of G. We also show that relationships among neutral-current parameters hold at any $q^2\neq 0$ if they hold at $q^2=0$. The quantity $(u_L^2+d_L^2-u_R^2-d_R^2)/(u_L^2+d_L^2+u_R^2+d_R^2)$ is studied in several expanded gauge theories using the $q^2\neq 0$ neutral-current formalism. Momentum-transfer and neutrino-energy (E_ν) dependences of this ratio put loose constraints on the parameters in several $SU(2)\times U(1)\times U(1)$ models. In the $SU_L(2)\times SU_R(2)\times U(1)$ models which we examine, the q^2 and E_ν dependence are essentially the same as that of the standard model for neutrino energies up to 2 TeV.

I. INTRODUCTION

Analyses of neutrino-hadron neutral-current couplings[1] and the measurements of the parity-violation asymmetry in polarized-electron scattering from deuterium[2] single out $SU(2)\times U(1)$ in the Weinberg-Salam standard form[3] as nature's choice of gauge theory of weak and electromagnetic interactions at currently available neutrino and electron energies. Further tests that this is indeed the correct theory can be made by measuring atomic parity violation in hydrogen and deuterium,[4] by measuring weak-electromagnetic interference effects in $e^+ + e^- \to \mu^+ + \mu^-$ (Ref. 5), and by improved statistics on ν-e, ν-hadron, and e-hadron weak-neutral-current amplitudes. In particular, the factorization among phenomenological neutral-current couplings which follows from the single Z-boson exchange characteristic of $SU(2)\times U(1)$ can be tested.[6]

Analyses of neutral currents that have been made to date and the tests which have been proposed for the future rely on effective four-fermion interaction parametrizations which ignore possible four-momentum dependence (g^2) due to neutral-gauge-boson propagators. Limits on the gauge-boson masses[7] certainly justify ignoring this dependence. In higher-energy neutrino and electron beams, including colliding beams, it might be necessary to include the q^2 dependence of the effective interaction due to vector-boson exchange. In this paper we develop a $q^2\neq 0$ neutral-current Hamiltonian formalism and apply it to general questions of neutral-current structure and to particular features of neutrino-hadron inclusive cross sections in several expanded gauge theories. In particular, we are interested in illustrating tests, independent of the $q^2=0$ factorization tests, of the single-Z-boson picture of neutral-current interactions.

In the following section we generalize the Georgi-Weinberg[8,9] analysis of the $q^2=0$, effective four-fermion neutral-current Hamiltonian in an arbitrary gauge group of weak and electromagnetic interactions to include the $q^2\neq 0$ neutral-vector-boson propagator effects. The condition under which the gauge theory $SU(2)\times U(1)\times G$ reproduces the neutrino-quark interactions of the standard $SU(2)\times U(1)$ model[3] at any $q^2\neq 0$ is shown to be that, in addition to satisfying the conditions of the Georgi-Weinberg theorem,[8] the electromagnetic charge decouples from the neutral generators of G. As a second application of the general formula, we show that relationships among neutral-current parameters such as those pointed out by Bernabêu and Jarlskog[10] hold at any q^2 if they hold at $q^2=0$.

In Sec. III we examine the quantity $(u_L{}^2+d_L{}^2-u_R{}^2-d_R{}^2)/(u_L{}^2+d_L{}^2+u_R{}^2+d_R{}^2)$ in several expanded gauge theories.[11] This quantity is independent of q^2 in theories with a single gauge boson such as the standard $SU(2)\times U(1)$ model. We show that the q^2 dependence and the E_ν dependence of this ratio put loose constraints on the parameters in several $SU(2)\times U(1)\times U(1)$ models of weak and electromagnetic interactions, constraints which go beyond those available from $q^2=0$ analyses alone. However, in the $SU_L(2)\times SU_R(2)\times U(1)$ gauge models which we examine, the q^2 dependence and E_ν dependence are constrained by low-energy phenomenology to be essentially identical to that of the standard model even at neutrino energies above 2 TeV.

In Sec. IV we summarize our conclusions and suggest further applications. In the Appendix are listed the fermion couplings to the gauge fields in models discussed in the text.

II. PROPAGATOR MOMENTUM-TRANSFER DEPENDENCE OF THE EFFECTIVE HAMILTONIAN FOR NEUTRAL CURRENT PROCESSES

Georgi and Weinberg[8,9] derived the necessary and sufficient conditions that must be met in order that a gauge theory of weak and electromagnetic interactions have the same zero four-momentum-transfer squared ($q^2=0$) neutral-current interactions for a neutral fermion f^0 as those of the standard Weinberg-Salam model.[3] Their principal result, on which discussion of $SU(2)\times U(1)$ is based, is stated as a theorem. We repeat the statement of the theorem for ease of reference. "Suppose that the effective gauge group of weak and electromagnetic interactions is a direct product $G_1\times G_2\times U(1)$, with G_1 and G_2 arbitrary gauge groups, not necessarily simple. Suppose that this gauge symmetry is broken by the vacuum expectation values of two sets of scalar fields ϕ_1 and ϕ_2. The ϕ_1 multiplet transforms nontrivially under G_1 and U(1) but is neutral under G_2. The ϕ_2 multiplet transforms nontrivially under G_2 and U(1) but is neutral under G_1. As usual, one linear combination of the neutral generators of G_1, G_2, and U(1) remains unbroken and is identified with the electric charge. ⋯ *At zero momentum transfer the neutral-current interactions of any fermion f^0 which is both electrically neutral and neutral under G_2 will be precisely the same as if the gauge group were just $G_1\times U(1)$ and broken only by $\langle\phi_1\rangle$.*"

We follow the notation of Georgi and Weinberg and extend their discussion to $q^2\neq 0$ effective interactions. *We find that if the electric charge operator contains no generators of G_2, their result obtains at arbitrary $q^2\neq 0$.*

Our development of the expression for the $q^2\neq 0$ neutral-current Hamiltonian in terms of a submatrix of the neutral-boson propagator matrix is patterned after the $q^2=0$ discussion.[8] Let G denote the local gauge group which is to be considered as a candidate for the theory of weak and electromagnetic interactions. In terms of the electrically neutral generators of G, to be referred to as T_α, the electric charge can be written

$$Q=\sum_\alpha c_\alpha T_\alpha . \tag{1}$$

The photon field is

$$A^\mu=\sum_\alpha p_\alpha A^\mu_\alpha , \tag{2}$$

where A^μ_α is the αth neutral gauge field of G, μ is its Lorentz index, and p_α are the components of the eigenvector which belongs to the zero eigenvalue of the neutral-gauge-boson mass matrix μ^2:

$$\sum_\beta \mu^2{}_{\alpha\beta}p_\beta=0,\quad \sum_\beta p_\beta{}^2=1 . \tag{3}$$

Likewise, the massive intermediate vector bosons Z^μ_i can be expressed as

$$Z^\mu_i=\sum_\alpha u_{i\alpha}A^\mu_\alpha , \tag{4}$$

where

$$\mu^2{}_{\alpha\beta}u_{i\beta}=m_i{}^2u_{i\beta} , \tag{5}$$

$$\sum_\beta u_{i\beta}u_{\beta j}=\delta_{ij} . \tag{6}$$

The full intermediate-vector-boson propagator $\Delta^{\mu\nu}_{\alpha\beta}(q^2)$ can be written as

$$\Delta^{\mu\nu}_{\alpha\beta}(q^2)=\Delta_{\alpha\beta}(q^2)g^{\mu\nu}+\Delta'_{\alpha\beta}(q^2)q^\mu q^\nu , \tag{7}$$

where g^μ are the components of the four-momentum carried by the propagator, $g^{\mu\nu}$ is the metric tensor, and α and β are gauge-group indices. The second term in Eq. (7) introduces factors of fermion mass in processes with external fermions, and its contribution will be negligible in any application of interest. We therefore consider the first term alone, and we have

$$\Delta_{\alpha\beta}(q^2)=\sum_i u_{\alpha i}u_{\beta i}(m_i{}^2-q^2)^{-1} . \tag{8}$$

Define

$$\zeta_{\alpha\beta}=\delta_{\alpha\beta}-p_\alpha p_\beta=\sum_i u_{\alpha i}u_{\beta i} . \tag{9}$$

Referring to Eq. (9), it is evident that

$$(\mu^2-q^2)_{\alpha\gamma}\Delta_{\gamma\beta}(q^2)=\zeta_{\alpha\beta} \tag{10a}$$

and

$$\sum_\alpha p_\alpha\Delta_{\alpha\beta}=\sum_\beta p_\beta\Delta_{\alpha\beta}=0 , \tag{10b}$$

so that one can write Eq. (10a) as

$$(\mu^2-q^2\zeta)_{\alpha\gamma}\Delta_{\gamma\beta}(q^2)=\zeta_{\alpha\beta} . \tag{11}$$

Next, suppose that there are m_1 electrically neutral generators of G_1 and m_2 electrically neutral generators of G_2. Let $(\mu^2-q^2\zeta)_{ij}$ denote a dimension $(m_1+m_2)\times(m_1+m_2)$ matrix which is obtained from the full matrix $(\mu^2-q^2\zeta)_{\alpha\beta}$ which is of dimension $(m_1+m_2+1)\times(m_1+m_2+1)$ by deleting the entries which refer to any one of the fields A^μ_α which contributes to the electric charge ($p_\alpha\neq 0$). Denote by T_0 the generator corresponding to the field chosen. At $q^2=0$, the form which we seek reduces to[8]

$$\Delta_{\alpha\beta}(q^2=0)=\sum_{ij}\zeta_{\alpha i}\zeta_{\beta j}[\mu^2]^{-1}{}_{ij} , \tag{12}$$

where $[\mu^2]^{-1}{}_{ij}$ stands for the elements of the inverse of the $(m_1+m_2)\times(m_1+m_2)$ submatrix of the mass matrix which is obtained by removing those entries of the full mass matrix which are associated with the generator T_0. The generalization of Eq. (12) to $q^2 \neq 0$ is[12]

$$\Delta_{\alpha\beta}(q^2)=\sum_{ij}\zeta_{\alpha i}\zeta_{j\beta}[\mu^2-q^2\zeta]^{-1}{}_{ij}\,, \qquad (13)$$

which obeys the defining properties (10a) and (10b), as one can readily verify.[13]

Turning to the effective four-fermion neutral-current Hamiltonian, we have

$$H_{\rm NC}=\tfrac{1}{2}\sum_{\alpha\beta}(\bar\psi\gamma^{\mu}t_\alpha\psi)(\bar\psi\gamma_\mu t_\beta\psi)g_\alpha g_\beta\Delta_{\alpha\beta}(q^2)$$
$$=\tfrac{1}{2}\sum_{ij}(\bar\psi\gamma^{\mu}n_i\psi)(\bar\psi\gamma_\mu n_j\psi)[\mu^2-q^2\zeta]^{-1}{}_{ij}\,, \qquad (14)$$

where

$$n_i=\sum_\alpha g_\alpha\zeta_{\alpha i}t_\alpha\,,$$

g_α is the canonically normalized coupling constant of the gauge field A_α to the current T_α, and t_α are the matrix representations of the generators T_α on the fermion fields. The i, j are indices of the submatrix described previously.

As a first application of this formula, we ask what conditions must be met in order that the theorem of Georgi and Weinberg generalize to $q^2\neq 0$. Consider the group $G_1\times G_2\times \mathrm{U}(1)$ and choose the generator of the U(1) invariant subgroup to be identified as T_0, the generator whose gauge field is removed in forming the submatrices $[\mu^2]_{ij}$ and $[\mu^2-q^2\zeta]_{ij}$, as discussed above Eq. (12). If a model which is based on the gauge group $G_1\times G_2\times \mathrm{U}(1)$ meets the conditions of the theorem due to Georgi and Weinberg as stated earlier, then the neutral-boson mass matrix and thus the $q^2=0$ effective neutral-current Hamiltonian can be written as the sums of two terms—one depending on $\langle\phi_1\rangle$ but not $\langle\phi_2\rangle$ and the other depending on $\langle\phi_2\rangle$ but not $\langle\phi_1\rangle$. This division of the Hamiltonian into two terms is necessary in order that the italicized phrase in the theorem follow. By inspection of Eqs. (13) and (14), we see that the corresponding division occurs in the $q^2\neq 0$ neutral-current effective Hamiltonian if, in addition to the $q^2=0$ conditions, one has $\zeta_{AB}=0$ for all A, B such that A is an index in G_1 and B is an index in G_2. Since $\zeta_{AB}=\delta_{AB}-p_Ap_B$, this means that either $p_A=0$ for all A in G_1 or $p_A=0$ for all A in G_2. In other words, the $q^2\neq 0$ effective neutral-current Hamiltonian is the sum of two separate pieces, one determined by $\langle\phi_1\rangle$ and the other determined by $\langle\phi_2\rangle$ if the $q^2=0$ conditions are satisfied, and either G_1 or G_2 has no generators which contribute to the electromagnetic current. In such a case, one must choose T_0 from among those generators which do contribute to the electric charge when the submatrices $\mu^2{}_{ij}$ and $(\mu^2-q^2\zeta)_{ij}$ are formed in the way described before Eq. (12). In applications of interest, T_0 is chosen to be the generator of U(1), so there is no difficulty in applying the stated condition.

As another simple application of Eq. (14), let us consider some relations among neutral-current parameters which hold in certain classes of models at $q^2=0$. Bernebéu and Jarlskog[10] noted that in some models[11]

$$4u_L+2d_L+g_V+g_A=0 \qquad (15a)$$

and

$$2u_L+4d_L+\rho=0\,, \qquad (15b)$$

while Sidhu[14] discussed cases where

$$4u_R-2d_R+g_V-g_A=0\,. \qquad (15c)$$

In addition, a condition between left- and right-hand parameters,

$$u_L+d_L-u_R-d_R=0\,, \qquad (15d)$$

holds in the Weinberg-Salam model,[3] for example. Mohapatra and Sidhu[15] have recently discussed the conditions under which relation (15a)–(15c) hold in $G_{1_L}\times G_{2_R}$ gauge theories at $q^2=0$. We can immediately generalize their results and a corresponding one for (15d) to arbitrary $q^2\neq 0$. We review the results and notations of Mohapatra and Sidhu. They show that

$$4u_L+2d_L+g_V+g_A=2\sum_{i=1}^{m_1}T^{\nu}_{iL}\sum_{j=1}^{m_1}[\mu^2]^{-1}{}_{ij}(2T^{u}_{jL}+T^{d}_{jL}+T^{e}_{jL})\,, \qquad (16a)$$

$$2u_L+4d_L+p=2\sum_{i=1}^{m_1}T^{\nu}_{iL}\sum_{j=1}^{m_1}[\mu^2]^{-1}{}_{ij}(T^{u}_{jL}+2T^{d}_{jL}+T^{\nu}_{jL})\,, \qquad (16b)$$

$$4u_R-2d_R+g_V-g_A=2\sum_{i=m_1+1}^{m_1+m_2}T^{\nu}_{iL}\sum_{i=m_1+1}^{m_1+m_2}[\mu^2]^{-1}{}_{ij}(2T^{u}_{jR}+T^{d}_{jR}+T^{e}_{jR})\,. \qquad (16c)$$

We add that

$$u_L+d_L-u_R-d_R=\sum_{i=1}^{m_1}T^{\nu}_{iL}\left\{\sum_{j=1}^{m_1}[\mu^2]^{-1}{}_{ij}(T^{u}_{jL}+T^{d}_{jL})-\sum_{j=m_1+1}^{m_1+m_2}[\mu^2]^{-1}{}_{ij}(T^{u}_{jR}+T^{d}_{jR})\right\}. \qquad (16d)$$

On the right-hand side of Eqs. (16) it is assumed that G_{1L} has a single U(1) factor labeled O, and that the remaining neutral generators are m_1 in number. The group G_{2R} has m_2 neutral generators. $T^{q(l)}_{iL(R)}$ denote the eigenvalues of the quark (q) or lepton (l) under the left (L) or right (R) generators. Evidently, the relations (15) are true if one has

$$2T^u_{jL}+T^d_{jL}+T^e_{jL}=0\,,\quad j=1,\ldots,m_1\,, \tag{17a}$$

$$T^u_{jL}+2T^d_{jL}+T^\nu_{jL}=0\,,\quad j=1,\ldots,m_1\,, \tag{17b}$$

$$2T^u_{jR}+2T^d_{jL}+T^e_{jR}=0\,,\quad j=m_1+1,\ldots,m_2+m_1\,, \tag{17c}$$

and, simultaneously,

$$T^u_{jR}+T^d_{jR}=0\,,\quad j=m_1+1\,,\ldots,m_1+m_2\,. \tag{17d}$$

Now the only change from the $q^2=0$ formulas which occurs at $q^2\neq 0$ is the replacement

$$[\mu^2]^{-1}{}_{ij}\to[\mu^2-q^2\zeta]^{-1}{}_{ij}\,.$$

Inspecting Eqs. (16), we see that Eqs. (17) ensure that Eqs. (15) are true at $q^2\neq 0$ if they are true at $q^2=0$. On the other hand, if (15d) is satisfied at $q^2=0$ by a cancellation between the two terms in brackets on the right-hand side of Eq. (16d), then (15d) will not be satisfied at $q^2\neq 0$. In any models we have considered, Eqs. (17d) are the cause of (15d) to be true.

In this section, we have derived Eqs. (13) and (14) and applied them to the study of general properties of $H^{\text{eff}}_{\text{NC}}$. In Sec. III we further illustrate Eqs. (13) and (14) by calculating u_R, d_R, u_L, and d_L in several models in which two massive neutral vector bosons appear. The propagator, Eq. (13), makes calculation extremely simple in these cases, and its advantage over diagonalization of the mass matrix and the effective Hamiltonian is evident.

III. NEUTRINO NEUTRAL CURRENTS IN SEVERAL EXPANDED GAUGE MODELS

In this section we examine the large-q^2 and large-energy behavior of inclusive neutrino-hadron neutral-current reactions in several models which have two neutral, massive vector bosons. We compare this behavior to that of the standard Weinberg-Salam model with its single-massive neutral vector boson. Our intention here is to illustrate the results of the previous section and to obtain a rough idea of the differences that show up when comparing models which have two neutral massive vector bosons to the standard model at neutrino beam energies (E_ν) up to about 2 TeV.

We study the ratio

$$\rho_{\text{NC}}(q^2)=\frac{[u_L{}^2+d_L{}^2-u_R{}^2-d_R{}^2]}{[u_L{}^2+d_L{}^2+u_R{}^2+d_R{}^2]} \tag{18}$$

as a function of q^2 at fixed neutrino energy E_ν and, by integrating over q^2, as a function of E_ν. The observables u_L, d_L, u_R, and d_R are defined in footnote 11, following one of the commonly used notations. The quantity $\rho_{\text{NC}}(q^2)$ has the virtue that it is a constant in models with only one massive neutral vector boson. Changes in this quantity as q^2 and/or E_ν are changed then illustrate effects which are not present in the standard model. In the standard Weinberg-Salam model, $\rho_{\text{NC}}(q^2)$ takes the value

$$\rho_{\text{NC}}=(\text{WS})=\frac{\frac{1}{2}-\sin^2\theta_W}{\frac{1}{2}-\sin^2\theta_W+\frac{10}{9}\sin^4\theta_W}\,. \tag{19}$$

We note that $\rho_{\text{NC}}(q^2)$ can be expressed in terms of differential cross sections if the contributions of strange and charmed quarks are neglected and if only one left-handed vector boson dominates the charged-current reactions. Under these conditions, one has

$$\rho_{\text{NC}}(q^2)=\frac{\left[\left(\frac{d^2\sigma^{\text{NC}}}{dxdy}-\frac{d^2\bar\sigma^{\text{NC}}}{dxdy}\right)\Big/\left(\frac{d^2\sigma^{\text{CC}}}{dxdy}-\frac{d^2\bar\sigma^{\text{CC}}}{dxdy}\right)\right]}{\left[\left(\frac{d^2\sigma^{\text{WC}}}{dxdy}+\frac{d^2\bar\sigma^{\text{NC}}}{dxdy}\right)\Big/\left(\frac{d^2\sigma^{\text{CC}}}{dxdy}+\frac{d^2\bar\sigma^{\text{CC}}}{dxdy}\right)\right]}\,, \tag{20}$$

where $d^2\sigma^{\text{NC}}/dx\,dy$ ($d^2\bar\sigma^{\text{NC}}/dx\,dy$) represent the ν ($\bar\nu$) neutral-current double differential inclusive cross section and similarly for charged currents (CC). The dimensionless variables x and y are defined as usual.[16] The equality between the bracketed numerators on the left- and right-hand sides of Eq. (20) [refer also to Eq. (18)] holds even when strange- and charmed-quark contributions are included.[17]

The expanded models

Both of the gauge groups SU(2) × U(1) × U(1) (Ref. 18) and $\text{SU}_L(2)\times\text{SU}_R(2)\times\text{U}(1)$ have two massive neutral vector bosons. Both have received considerable attention as uncomplicated expansions of the standard SU(2) × U(1) model. The neutrino neutral-current sector can be made to look identical or close to that of the standard model, while electron couplings can be made to be quite different.[19] The $\text{SU}_L(2)\times\text{SU}_R\times\text{U}(1)$ group[20] affords some interesting theoretical possibilities as well.

While the SU(2) × U(1) × U(1) group has three independent gauge coupling constants, the much stu-

TABLE I. Coefficients $q_{L,R}^{(0)}$, $q_{L,R}^{(1)}$, δ_1 and δ_2 as defined by Eq. (21) for the $SU(2)\times U(1)\times U(1)$ models which are treated in the text. The neutrino-quark scattering coefficients $q_{L,R}$ are defined in Ref. 11, following one of several commonly used notations.

	Four-quark model	Six-quark model[a]
$u_L^{(0)}$	$\frac{1}{2}-\frac{2}{3}\sin^2\theta$	$c-2b$
$d_L^{(0)}$	$-\frac{1}{2}+\frac{1}{3}\sin^2\theta$	$-c+b$
$u_R^{(0)}$	$\frac{1}{3}\sin^2\theta$	$-a-2b$
$d_R^{(0)}$	$-\frac{2}{3}\sin^2\theta$	$a+b$
$u_L^{(1)}$	$\sin^2\theta/(2\lambda q_1^2q_2^2)$	$c\sin^2\theta(\lambda q_1^2q_2^2)$
$d_L^{(1)}$	$-\sin^2\theta/(2\lambda q_1^2q_2^2)$	$-c\sin^2\theta/(\lambda q_1^2q_2^2)$
$u_R^{(1)}$	$-\sin^2\theta/(2\lambda q_1^2)$	$-c\sin^2\theta/(\lambda q_1^2)$
$d_R^{(1)}$	$\sin^2\theta/(2\lambda q_1^2)$	$c\sin^2\theta/(\lambda q_1^2)$
δ_1	$\left[\cos^2\theta+\frac{\sin^2\theta}{\lambda q_1^2q_2^2}(1+q_2^2)\right]$	$2c\left[\cos^2\theta+\frac{\sin^2\theta}{\lambda q_1^2q_2^2}(1+q_2^2)+\frac{\sin^2\alpha}{\lambda}\left(\cos^2\theta-\frac{2\sin^2\theta}{q_1^2}\right)\right]$
δ_2	$\frac{\sin^2\theta}{\lambda q_1^2q_2^2}$	$\frac{2c\sin^2\theta}{\lambda q_1^2q_2^2}$

[a]In terms of quantities defined in the text, $a=C\sin^2\theta/\lambda$, $b=(2c/3)[\sin^2\theta-\sin^2\theta q_1^2/(q_1^2+q_2^2+q_1^2q_2^2)\lambda]$, and $c=[2(1+\sin^2\theta\cos^2\theta/\lambda)]^{-1}$.

died right-left-symmetric version of $SU_L(2)\times SU_R(2)\times U(1)$ has only two. Because of this, the $SU_L(2)\times SU_R(2)\times U(1)$ models tend to be more tightly constrained by low-energy neutrino and electron data and these models show very little difference from the standard Weinberg-Salam model even at large q^2 and E_ν values.

SU(2)×U(1)×U(1)

We consider two models based on this group which have recently been discussed by McKay and Munczek.[9] The first version is a four-quark model, readily expandable to eight quarks, which identically reproduces the neutrino scattering and electron parity-violation formulas of the standard model at $q^2=0$. The second version is a six-quark model which incorporates an extra scalar doublet to give masses to two heavy quarks. Deviations from the standard model $q^2=0$ parametrization of neutral currents are proportional to the vacuum expectation value of this extra scalar field.

The relevant quantities which are necessary for evaluation of u_R, d_R, u_L, and d_L (see Ref. 11) are as follows:

(i) gauge couplings $\sqrt{2}g$, $q_1g/\sqrt{2}$, and $q_2g/\sqrt{2}$ of SU(2) and the two independent U(1) transformations, respectively,

(ii) the vacuum expectation values of doublets

$$\phi_1:(\tfrac{1}{2},0,-\tfrac{1}{2})\quad\langle\phi_1\rangle_0=\begin{pmatrix}\langle\phi_1^0\rangle_0\\0\end{pmatrix},$$
$$\phi_2:(\tfrac{1}{2},0,-\tfrac{1}{2})\quad\langle\phi_2\rangle_0=\begin{pmatrix}\langle\phi_2^0\rangle_0\\0\end{pmatrix},\qquad(21)$$
$$\phi_3:(\tfrac{1}{2},-\tfrac{1}{2},0)\quad\langle\phi_3\rangle_0=\begin{pmatrix}\langle\phi_3^0\rangle_0\\0\end{pmatrix},$$

and singlets

$$\eta:(0,\tfrac{1}{2},-\tfrac{1}{2})\quad\langle\eta_0\rangle\neq0\quad\text{neutral},$$
$$\chi:(0,\tfrac{1}{2},\tfrac{1}{2})\quad\langle\chi\rangle_0=0\quad\text{charged}$$

(the numbers in parentheses are the SU(2) and U(1) eigenvalues, respectively), and

(iii) parameters which are based on (i) and (ii) and defined to be

$$\begin{aligned}\sin^2\theta&=q_1^2q_2^2/(q_1^2+q_2^2+q_1^2q_2^2),\\ \lambda&=g^2\langle\eta\rangle_0^2/M_W^2,\\ \cos^2\alpha&=g^2(\langle\phi_1\rangle_0^2+\langle\phi_2\rangle_0^2)/M_W^2,\\ \sin^2\alpha&=g^2\langle\phi_3\rangle_0^2/M_W^2,\end{aligned}\qquad(22)$$

where $M_W^2=g^2(\langle\phi_1\rangle_0^2+\langle\phi_2\rangle_0^2+\langle\phi_3\rangle_0^2)$ is the charged-boson mass and $G_F/\sqrt{2}=g^2/(4M_W^2)$ is the effective weak four-fermion coupling constant. We refer the reader to the Appendix for specifica-

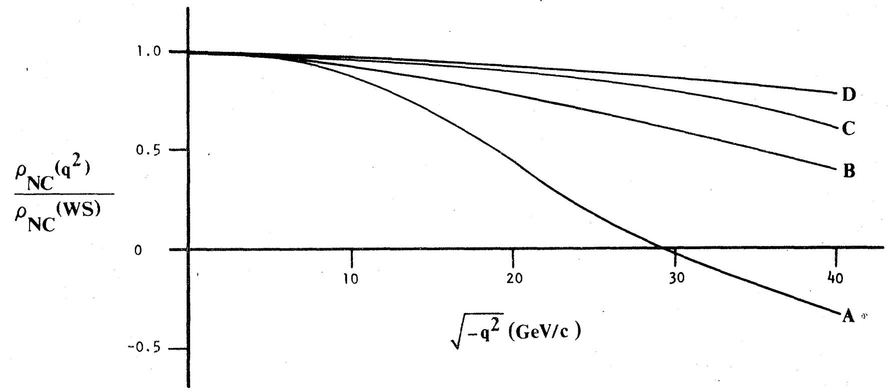

FIG. 1. $\rho_{NC}(q^2)$, Eq. (18), in the SU(2) × U(1) × U(1) four-quark model divided by ρ_{NC}(WS), Eq. (19), vs $\sqrt{-q^2}$ at E_ν = 0.8 TeV. The value $\sin^2\theta = 0.2$ is used. Curve A: $q_1^2 = 0.275$, $q_2^2 = 2.75$, $M_V^2/M_W^2 = 0.1$. Curve B: $q_1^2 = 0.35$, $q_2^2 = 0.875$, $M_V^2/M_W^2 = 0.1$. Curve C: $q_1^2 = 0.275$, $q_2^2 = 2.75$, $M_V^2/M_W^2 = 0.5$. Curve D: $q_1^2 = 0.5$, $q_2^2 = 0.5$, $M_V^2/M_W^2 = 0.1$.

tion of quark and lepton quantum numbers.

We can now express the neutrino neutral-current observables u_R, d_R, u_L, and d_L (Ref. 11) in the following way:

$$q_{L,R}(q^2) = \frac{q_{L,R}^{(0)} - (q^2/M_W^2)q_{L,R}^{(1)}}{1 + \delta_1(-q^2/M_W^2) + \delta_2(-q^2/M_W^2)^2}, \quad (23)$$

where $q = u$ or d, and the values for $u_L^{(0)}$, $d_L^{(0)}$, $u_R^{(0)}$, $d_R^{(0)}$, $u_L^{(1)}$, $d_L^{(1)}$, $u_R^{(1)}$, $d_R^{(1)}$, δ_1, and δ_2 are displayed in Table I for the two SU(2) × U(1) × U(1) models which we are considering.

In Fig. 1 we plot $\rho_{NC}(q^2)/\rho_{NC}$(WS) vs q^2 at $E_\nu = 10M_W = 800$ GeV for the four-quark models with $\sin^2\theta = 0.2$. Figure 1 shows curves for several choices of M_V^2/M_W^2, where M_V is the lighter of the two neutral bosons, and several choices of q_1^2 and q_2^2. As one expects, the results are sensitive to the values of M_V^2/M_W^2 and q_2, the strength of the vector boson which couples to the neutrino (see the Appendix). In Fig. 2 the corresponding plots of the quantity

$$\int_0^{2mE_\nu} d(-q^2)\rho_{NC}(q^2)/2mE_\nu\,\rho_{NC}(\mathrm{WS}) \equiv \frac{\rho_{NC}(E_\nu)}{E_\nu\,\rho_{NC}(\mathrm{WS})}$$

are shown for E_ν up to 1.6 TeV $\approx 20M_W$. Figures 1 and 2 show that the case $M_V = M_W/\sqrt{10} \approx 25$ GeV at $\sin^2\theta = 0.2$ with $q_2^2 = 2.75$ has a strong variation with E_ν and q^2. The shift from the standard model is about 10% at $E_\nu = 80$ GeV. Since present high-energy data from CERN and Fermilab are taken at approximately $\langle E_\nu \rangle = 100$ GeV (Ref. 21) and are

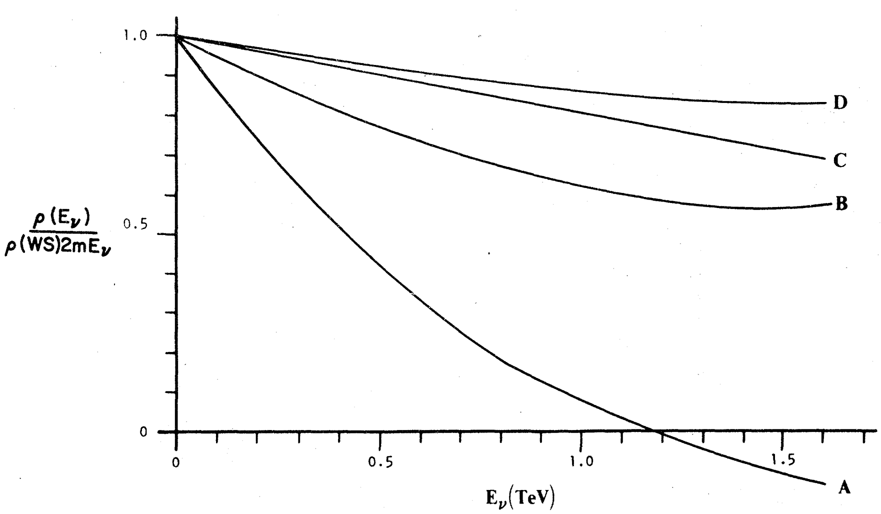

FIG. 2. The energy-dependent integrated ratio $\int_0^{2ME_\nu} d(-q^2)\,\rho_{NC}(q^2) = \rho_{NC}(E_\nu)$ divided by ρ_{NC}(WS)$2mE_\nu$ vs E_ν for the same model and parameter choices as in Fig. 1.

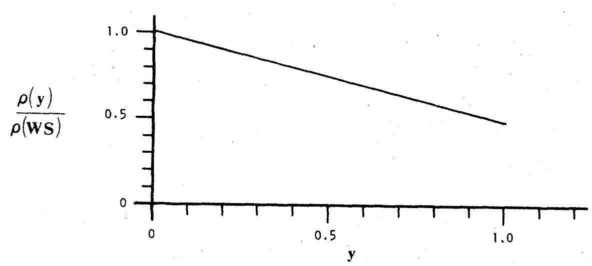

FIG. 3. The y dependence of $\rho_{NC}/\rho_{NC}(WS)$ at $E_\nu = 0.8$ TeV for case A of the same model as in Figs. 1 and 2.

compatible with the value of $\sin^2\theta = 0.2$ which is determined from lower-energy e-p asymmetry studies, the choice of $M_V \approx 25$ GeV and $q_2^{\,2} = 2.5$ in this $SU(2) \times U(1) \times U(1)$ model is barely acceptable. It is consistent with reported $e^+ + e^- \rightarrow \mu^+ + \mu^-$ asymmetry studies at $\langle E \rangle = 6.5$ GeV, but certainly the next higher energies which will be available at the energy-doubled Fermilab beam would make it possible to detect such a strong energy dependence. See also Fig. 3, where the y distribution for $E_\nu = 5 M_W \approx 400$ GeV is shown.

One can say then that $q_2^{\,2} \approx 3$ for $M_V = M_W/\sqrt{10}$ with the value $\sin^2\theta = 0.2$ are rough limits on parameters in this $SU(2) \times U(1) \times U(1)$ version. As M_V is allowed to rise, limits on $q_2^{\,2}$ are correspondingly allowed to rise. Of course, $q_2^{\,2} \approx 10$ would make the perturbation results suspect. In Fig. 4, curve A, we show the variation of the quantity $\rho(E_\nu)/\rho(0)E_\nu$ vs E_ν for the six-quark version of the $SU(2) \times U(1) \times U(1)$ model. Parameters are chosen so that $q^2 = 0$ results for ν and e weak neutral currents are in agreement with experiment within 1 standard deviation while the atomic parity violation in atomic Bi is nearly zero.[22] Using the same values of $q_1^{\,2}$, $q_2^{\,2}$, λ, and $\sin^2\theta$, we show in curve B, Fig. 4, the corresponding behavior in the four-quark model where the parity violation is the same as that of the standard model in which it is large. One clearly sees that the E_ν dependences of the two cases are indistinguishable from each other and from that of the standard model. In every case of the six-quark model for which atomic parity violation is small, there is no significant difference in q^2 or E_ν behavior.

Briefly summarizing the study of these two versions of $SU(2) \times U(1) \times U(1)$, we can say that the q^2 and E_ν dependence of neutrino neutral-current observables puts loose constraints on the choices of parameters, but that it may not be possible to clearly discriminate between different versions which have radically different electron neutral-current predictions for atomic parity violation. Cases which give almost no Bi parity violation have about the same E_ν and q^2 behavior as the standard model. Cases with large Bi parity violation are very similar to the corresponding four-quark $SU(2) \times U(1) \times U(1)$ cases (ones with similar values of $q_1^{\,2}$, $q_2^{\,2}$, and λ). For example, compare curve C, Fig. 4, with curve A, Fig. 2.

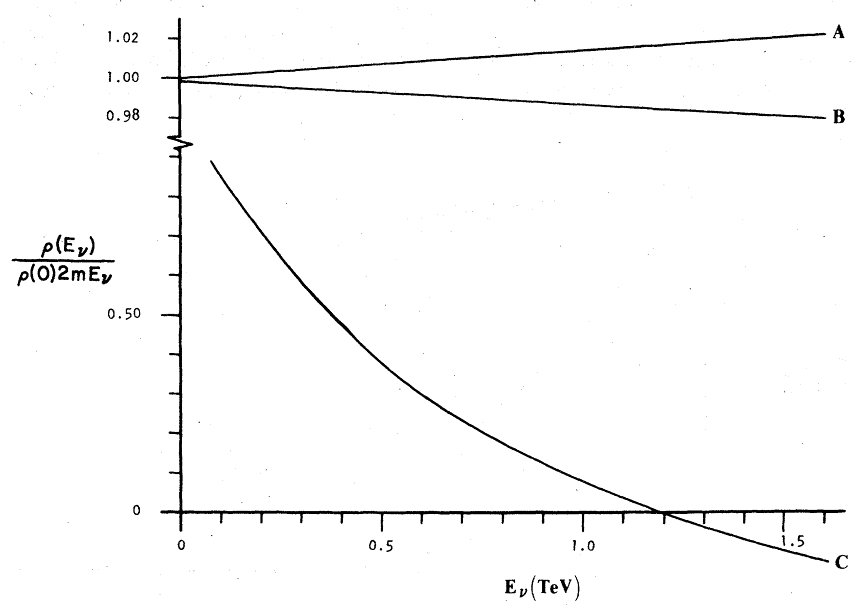

FIG. 4. The same energy-dependent ratio as in Fig. 2 but for the six-quark $SU(2) \times U(1) \times U(1)$ model. Parameter choices are such that the measure of atomic Bi parity violation Q_W takes the values $Q_W = +3$ (curve A), $Q_W = -110$ (curve B), and $Q_W = -102$ (curve C). In all three cases, the neutral-current neutrino and electron scattering parameters are in agreement with low-energy experimental values.

$SU_L(2)\times SU_R(2)\times U(1)$

Electron-scattering asymmetry measurements[2] have ruled out the left-right-symmetric versions of this class of models which have natural parity conservation in the weak neutral currents.[23] However, there are versions in which the electron-scattering asymmetry is correctly described,[23,24] and these are of great interest because relationships between fermion masses and weak-current mixing angles (e.g., the Cabibbo angle) have been discovered in models based on this group. The left-right symmetry and spontaneous breaking of parity might also provide a natural escape from the instanton-generated strong CP violation,[25] in addition to providing an origin for weak parity violation which is more to the liking of some authors than that of the standard model.

We study the neutral-current propagator effects in those versions of the model which have the following scalar particles: (1) at least one multiplet of the type

$$\phi_1:(\tfrac{1}{2},\tfrac{1}{2},0)\quad \langle\phi_1\rangle_0=\begin{bmatrix}\kappa e^{i\alpha} & 0\\ 0 & \kappa' e^{i\alpha'}\end{bmatrix},$$

where the values of T_L, T_R, and Y, the eigenvalues of total left spin, right spin, and the singlet generator, are listed in the parenthesis, (2) two multiplets of types

$$x_L:(\tfrac{1}{2},0,\tfrac{1}{2})\quad\langle x_L\rangle_0=(\lambda_L^0),$$
$$x_R:(0,\tfrac{1}{2},\tfrac{1}{2})\quad\langle x_R\rangle_0=(\lambda_R^0),$$

and (3) two multiplets of the types

$$\delta_L:(1,0,0)\quad\langle\delta_L\rangle_0=0,$$
$$\delta_R:(0,1,0)\quad\langle\delta_R\rangle_0\neq 0.$$

These $\delta_{L,R}$ fields are introduced so that the mass of the charged vector bosons $W_R^{\pm}$, which couple to right-handed currents, can be made arbitrarily large compared to the mass of $W_L^{\pm}$. Imposing this condition enables us to maintain the approximate identity [Eq. (20] and makes our discussion of the various models more uniform.

The fermion quantum numbers which we need to evaluate H_{NC}^{eff} are listed in the Appendix, and the expressions for u_L, d_L, u_R, and d_R, defined in the same way as in Eq. (21), can be obtained from the entries of Table II. By choosing $\lambda_L=\lambda_R$ in the entries of Table II, we can reproduce the results of those models which have natural-parity conservation in the neutral current and which have a scalar field

$$\rho:(\tfrac{1}{2},\tfrac{1}{2},-1)\quad\langle\rho\rangle=\begin{bmatrix}0 & 0\\ a & 0\end{bmatrix}.$$

Of course the relation $\lambda_L=\lambda_R$ is not a natural one, but it yields the same expressions as the case in

TABLE II. Coefficients $q_{L,R}^{(0)}$, $q_{L,R}^{(1)}$, δ_1, and δ_2 as defined by Eq. (21) for the $SU_L(2)\times SU_R(2)\times U(1)$ models which are treated in the text. The quantities ϵ and δ^2 are defined in the text in terms of vacuum expectation values.

${u_L}^{(0)}$	$\left(\frac{1}{2}-\frac{\sin^2\theta}{3}-\epsilon\frac{\sin^2\theta}{3}\right)F(\epsilon,\delta)$ [a]
${d_L}^{(0)}$	$\left(-\frac{1}{2}+\frac{\sin^2\theta}{6}+\epsilon\frac{\sin^2\theta}{6}\right)F(\epsilon,\delta)$
${u_R}^{(0)}$	$\left[-\frac{\sin^2\theta}{3}+\epsilon\left(\frac{1}{2}-\frac{\sin^2\theta}{3}\right)\right]F(\epsilon,\delta)$
${d_R}^{(0)}$	$\left[\frac{\sin^2\theta}{6}+\epsilon\left(-\frac{1}{2}+\frac{\sin^2\theta}{6}\right)\right]F(\epsilon,\delta)$
${u_L}^{(1)}$	$(-\frac{1}{2}+\frac{5}{12}\sin^2\theta)\left[\epsilon+\delta^2\frac{(1-\epsilon)}{\epsilon}\right]F(\epsilon,\delta)$
${d_L}^{(1)}$	$(\frac{1}{2}-\frac{7}{12}\sin^2\theta)\left[\epsilon+\delta^2\frac{(1-\epsilon)}{\epsilon}\right]F(\epsilon,\delta)$
${u_R}^{(1)}$	$-\left[\epsilon+\delta^2\frac{(1-\epsilon)}{\epsilon}\right]F(\epsilon,\delta)/12$
${d_R}^{(1)}$	$-\left[\epsilon+\delta^2\frac{(1-\epsilon)}{\epsilon}\right]F(\epsilon,\delta)/12$
δ_1	$\epsilon\left\{2+\frac{(1-\epsilon)}{\epsilon}(1+\delta^2)-\frac{\sin^2\theta}{2}\left[4+\frac{(1-\epsilon)}{\epsilon}(1+\delta^2)\right]\right\}F(\epsilon,\delta)$
δ_2	$\cos^2\theta[\epsilon+\delta^2(1-\epsilon)]F(\epsilon,\delta)$

[a] $F(\epsilon,\delta)=\dfrac{\epsilon+\delta^2(1-\epsilon)}{(1-\epsilon)(\epsilon+\delta^2)}$.

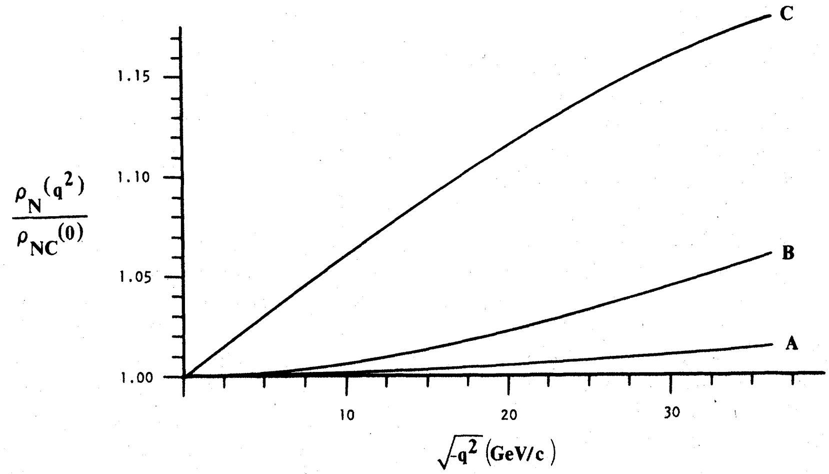

FIG. 5. $\rho_{NC}(q^2)/\rho_{NC}(0)$ vs $\sqrt{-q^2}$ in the $SU_L(2)\times SU_R(2)\times U(1)$ model for parameter choices as explained in the text. Note that the scale is different from that of Figs. 1 and 2.

which parity is conserved naturally.

Deshpande[26] has derived a constraint on the parameters of $SU_L(2)\times SU_R(2)\times U(1)$ models which is imposed by the measured electron-scattering asymmetry. For the case

$$(\kappa^2+\kappa')/\lambda_L{}^2=(\kappa^2+\kappa'^2)/\lambda_R{}^2=0,$$

it reduces to the statement that

$$\delta^2\equiv\lambda_L{}^2/\lambda_R{}^2<0.18$$

when $\sin^2\theta=2/(2+g^2/g'^2)=0.5\pm0.1$. Here g is the (common) gauge coupling constant of the SU(2)'s and g' is the U(1) coupling constant. We use this constraint as a guide in choosing values of parameters. As introduced in the literature[23,24] the quantity

$$\epsilon=(\kappa^2+\kappa'^2)/(\kappa^2+\kappa'^2+\lambda_R{}^2)$$

is useful in simplifying expressions for u_L, d_L, u_R, and d_R.

In Fig. 5 we show the behavior of $\rho_{NC}(q^2)/\rho_{NC}(0)$ as a function of q^2 for $E_\nu=10M_W\sim0.7$ TeV in the cases of $\epsilon=0.1$, $\sin^2\theta=0.6$ (for which $M_W\simeq68$ GeV) and $\lambda_L{}^2/\lambda_R{}^2=0.1$, 1, and 10, shown as curves A, B, and C, respectively. The case in which $\lambda_L{}^2/\lambda_R{}^2=0.1$ is consistent with all know neutral-current data except for those experiments which find little or no parity violation in Bi. Figure 6 shows the plots of $\rho_{NC}(E_\nu)/\rho_{NC}(0)E_\nu$ vs E_ν for $0<E_\nu<1.4$ TeV for the same parameter choices as in Fig. 5. We see from these figures that the cases $\lambda_L{}^2/\lambda_R{}^2=0.1$ and $\lambda_L{}^2/\lambda_R{}^2=1$ cannot be distinguished from the standard model. The latter case is independently eliminated by the electron deep-inelastic scattering asymmetry, of course. We show this case here to indicate that the test we are discussing is a very weak one in the $SU_L(2)\times SU_R(2)\times U(1)$ models. The case $\lambda_L{}^2/\lambda_R{}^2=10$,

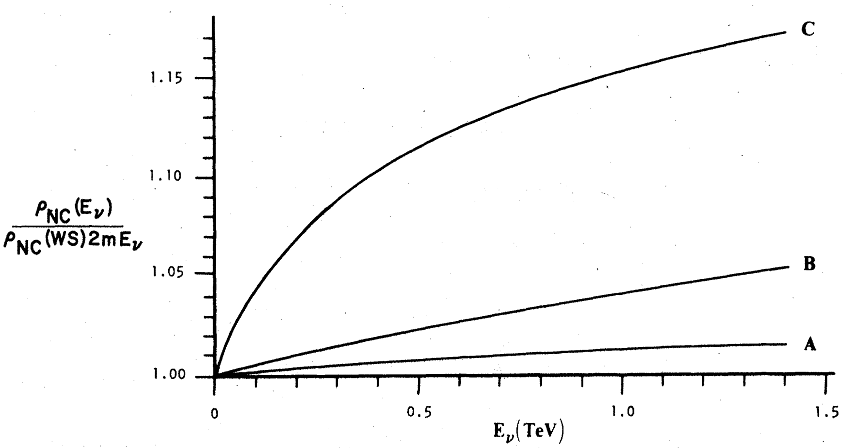

FIG. 6. The energy dependence for the integrated ratio $\rho_{NC}(E_\nu)$ divided by $\rho_{NC}(0)$ $(2ME_\nu)$ for the same model and parameter choices as in Fig. 5.

also ruled out by electron-scattering asymmetry (the measured value is negative while the $\lambda_L{}^2/\lambda_R{}^2 = 10$ yields a large positive value since a light, right-hand-coupled vector meson is present), is shown since it is on the margin of being distinguishable from the standard model at $E_\nu \simeq 10 M_W \simeq 700$ GeV and illustrates the kind of test which one could apply to a model that successfully described a (hypothetical) large positive electron asymmetry.

Recapping the situation in $SU_L(2) \times SU_R(2) \times U(1)$ models, we find that those versions of this class of models which are consistent with low-energy neutral-current data will continue to have essentially the same neutral-current parametrization as the standard model up to neutrino energies of at least 2 TeV. Though we have shown only $\sin^2\theta = 0.6$ and $\epsilon = 0.1$ in Figs. 5 and 6, this conclusion is the same for $\sin^2\theta < 0.6$ and for $-0.1 < \epsilon < 0.1$.[28]

IV. CONCLUDING REMARKS

We have generalized the consideration of neutral-current phenomena at $q^2 = 0$ in expanded gauge models[8,9] to include $q^2 \neq 0$ effects.[12] An expression for the effective neutral-current Hamiltonian was derived in Sec. II. The form, like the $q^2 = 0$ form due to Georgi and Weinberg,[8] does not require diagonalization of the propagator matrix. Only a submatrix of dimension one less than the full dimension of the propagator is needed, and the form lends itself readily to the study of general properties of the effective Hamiltonian and to calculation of neutral-current predictions in specific models.

Using the result Eq. (14) of Sec. II, we argued that the neutrino interactions of a given model would be identical to those of the standard model[3] if the conditions of the Georgi-Weinberg theorem were met and, in addition, the electromagnetic current coupled only to the neutral generators of SU(2) and U(1). We also used our expression for the effective neutral-current Hamiltonian to show that several relationships among neutral-current parameters[10,11,14,15] hold at any $q^2 \neq 0$ if they hold at $q^2 = 0$.

In Sec. III we compared the q^2 dependence of neutrino neutral-current deep-inelastic processes in several expanded gauge models to the q^2 dependence in the standard model. The quantity[11]

$$(u_L{}^2 + d_L{}^2 - u_R{}^2 - d_R{}^2)/(u_L{}^2 + d_L{}^2 + u_R{}^2 + d_R{}^2)$$

was used for this purpose, since it is a constant in the standard model, and since scale-violating effects are weak in such a ratio (see also comments in footnote 17). We found that measureable differences[27] from the standard model begin to show up in several versions of an $SU(2) \times U(1) \times U(1)$ scheme[9] at $E_\nu \gtrsim 100$ GeV for a range of values of the coupling constants and neutral-boson masses which gives satisfactory description of all lower-energy phenomena. Versions of a six-quark scheme with parameters adjusted to ensure little or no atomic Bi parity violation were found to be indistinguishable from the standard model for neutrino reactions up to 2 TeV. On one hand, this is discouraging because future experimental large-E_ν agreement with the standard model would not independently exclude the possibility of having no atomic parity violation in Bi. On the other hand, rapid q^2 and E_ν dependence of TeV-region neutrino neutral-current reactions would rule out these models which have small Bi parity violation. On the whole, we conclude that $SU(2) \times U(1) \times U(1)$ schemes which are identical to or within experimental limits of the standard-model predictions for all $q^2 = 0$ phenomena, including atomic Bi predictions, can be quite different from or nearly the same as the standard model at energies of 1–2 TeV (see Figs. 1–4), depending upon the specific values of parameters in the model.

Turning to $SU_L(2) \times SU_R(2) \times U(1)$ as a second illustration, we discovered that those versions with relatively simple scalar-field choices, as discussed in the literature,[23,24] will have essentially the same high-energy neutrino predictions as the standard model does due to the restrictions imposed on masses and couplings by the low-energy neutral-current data. Figures 5 and 6 summarize this observation.

We have compared in some detail the neutrino neutral-current predictions in several expanded gauge theories to those of the standard model. One could also make a detailed study of the other neutral-current processes which will be of interest at high energy and widen the survey of expanded gauge models. If e-p colliding beams become available, the study of q^2 and energy dependence of parity-violating asymmetries will be of special interest as a further tool in testing theories of weak and electromagnetic interactions. Likewise, discovery of asymmetry in $e^+ + e^- \to \mu^+ + \mu^-$ and measurement of its energy dependence would uncover more about the gauge boson or bosons which mediate the neutral current. The $q^2 \neq 0$ formalism developed in Sec. II as a generalization of Ref. 8 (see Ref. 12) will lend itself readily to computation of those processes in any model of interest. Those models which have two massive gauge bosons are especially easy to handle since, without diagonalization, one need only deal with two-by-two propagator matrices. These and other questions we leave to future work.

ACKNOWLEDGMENT

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APPENDIX

The gauge coupling constants, photon-field expression, and relevant fermion representation assignments are summarized here for the models discussed in the text.

$SU(2)\times U_b(1)\times U_c(1)$

We studied two models having this gauge symmetry group. The first one is a four-quark model and it reduces to the Weinberg-Salam in the $q^2=0$ limit. The second is a six-quark version with a $q^2=0$ limit slightly different from the Weinberg-Salam model. These models can be described as follows.

(a) Coupling constants. There are three coupling constants $g\sqrt{2}$, $(g\sqrt{2})\times\frac{1}{2}q_1$, and $(g\sqrt{2})\times\frac{1}{2}q_2$, associated with SU(2), $U_b(1)$, and $U_c(1)$, respectively.

(b) Fermion representations. Quantum numbers are given in the form (a,b,c) where a, b, and c refer to the weak isospin T_3 and the two hypercharged Y_b and Y_c, respectively. The following relation is satisfied:

$$Q=T_3+Y_b+Y_c\,.$$

The assignments are as follows:

Fermions	Quantum numbers
Leptons	
E_L, M_L, T_L	$(\frac{1}{2},0,-\frac{1}{2})$
e_R, μ_R, τ_R	$(0,0,-1)$
Quarks	
D_1, D_2, D_3	$(\frac{1}{2},\frac{1}{6},0)$
μ_R, c_R, t_R	$(0,\frac{1}{6},\frac{1}{2})$
d_R, s_R, b_R	$(0,\frac{1}{6},-\frac{1}{2})$

where

$$E_L=\begin{pmatrix}\nu_e\\ e'^-\end{pmatrix}_L,\quad M_L=\begin{pmatrix}\nu_\mu\\ \mu'^-\end{pmatrix}_L,\quad T_L=T_L=\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L,$$

$$D_1=\begin{pmatrix}\mu\\ d'\end{pmatrix}_L,\quad D_2=\begin{pmatrix}c\\ s'\end{pmatrix}_L,\quad D_3=\begin{pmatrix}t\\ b\end{pmatrix}.$$

The primed fields are linear combinations of the physical fields with the same charge. The first two columns correspond to the first model (four quarks). The second model includes the third column.

(c) Photon field and electric charge. These are given by

$$A_\mu=\frac{q_2B_\mu+q_1C_\mu+q_1q_2V_{3\mu}}{(q_1{}^2+q_2{}^2+q_1{}^2q_2{}^2)^{1/2}}\,,$$

$$e=q\sqrt{2}\sin\theta\,,\quad \sin\theta=\frac{q_1q_2}{(q_1{}^2+q_2{}^2+q_1{}^2q_2{}^2)^{1/2}}\,.$$

$SU_L(2)\times SU_R(2)\times U(1)$

Two models were analyzed, one of them with natural-parity conservation. Their structure is the following:

(a) Coupling constants. There are only two coupling constants: g and g' corresponding to SU(2) (both of them) and U(1), respectively.

(b) Fermion representations. Quantum numbers are given in the form (a,b,c) where a, b, and c refer to the two weak isospins (T_{3L}, T_{3R}) and the hypercharge, respectively. The charge relation

$$Q=T_{3L}+T_{3R}+Y$$

is satisfied. The assignments are as follows:

Fermions	Quantum numbers
Leptons	
$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L,\ \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L$	$(\frac{1}{2},0,-\frac{1}{2})$
$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_R,\ \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_R$	$(0,\frac{1}{2},-\frac{1}{2})$
Quarks	
$\begin{pmatrix}u\\ d\end{pmatrix}_L,\begin{pmatrix}c\\ s\end{pmatrix}_L,$	$(\frac{1}{2},0,1\sqrt{6})$
$\begin{pmatrix}u\\ d\end{pmatrix}_R,\begin{pmatrix}c\\ s\end{pmatrix}_R,$	$(0,\frac{1}{2},1\sqrt{6})$

(c) Photon field and electric charge. These are given by

$$A_\mu=\frac{\sin\theta}{\sqrt{2}}\,(W^3_{L\mu}+W^3_{R\mu})+(\cos\theta)B_\mu\,,$$

$$e=(g/\sqrt{2})\sin\theta\,.$$

[1]L. Q. Hung and J. J. Sakurai, Phys. Lett. 72B, 208 (1977); L. M. Sehgal, *ibid.* 71B, 99 (1977); P. N. Scharbach, Nucl. Phys. B82, 155 (1974); L. F. Abbott and R. M. Barnett, Phys. Rev. Lett. 40, 1303 (1978); Phys. Rev. D 18, 3214 (1978); P. Langacker and D. Sidhu, Phys. Rev. Lett. 41, 732 (1978).

[2]C. Y. Prescott *et al.*, Phys. Lett. 77B, 347 (1978); SLAC Report No. Pub. 2319, 1979 (unpublished). An analysis of this reaction was given prior to the measurements by R. N. Cahn and F. J. Gilman, Phys. Rev. D 17, 1313 (1978).

[3]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1971); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity, Nobel Symposium No. 8*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367. The sequential six-leptons-and-six-quarks generalization with the $\Delta S=1$ neutral-current cancellation mechanism of S. L. Glashow, J. Iliopoulos, and L. Maiani [Phys. Rev. D 2, 1285 (1970)] will be referred to as the standard model.

[4]For a description of these experiments see, for example, E. Hinds, in VIth International Workshop on Weak Interactions with Very High Energy Beams, transparency compilation by K. E. Lassila and B. L. Young (unpublished), p. 150; R. R. Lewis, *ibid.* p. 206.

[5]T. Himel *et al.*, Phys. Rev. Lett. 41, 449 (1978). This experiment finds $M_Z \geq 53 g_a/e$ GeV, assuming that $M_Z \geq 30$ GeV and where g_a is the axial-vector electron-muon coupling.

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[7]T. Himel *et al.* (Ref. 5); B. C. Barish, Phys. Rep. 39C, 279 (1978).

[8]H. Georgi and S. Weinberg, Phys. Rev. D 17, 275 (1978).

[9]D. McKay and H. Munczek [Phys. Rev. D 20, 794 (1979)] point out a generaliztion of Ref. 8 which covers the electron parity-violating neutral-current interaction.

[10]J. Bernabéu and C. Jarlskog, Phys. Lett. 69B, 71 (1977).

[11]d_R, d_L, u_R, u_L, g_V, g_A, and ρ, the neutral-current parameters which we use, are summarized in the following expression:

$$H_{NC}^{\text{eff}} = \frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\lambda (1-\gamma_5)\nu_\mu \; [u_R \bar{u} \gamma^\lambda (1+\gamma_5) u + d_R \bar{d} \gamma^\lambda (1+\gamma_5) d + u_L \bar{u} \gamma^\lambda (1-\gamma_5) u + d_L \bar{d} \gamma^\lambda (1-\gamma_5) d + \bar{e} \gamma^\lambda (g_\nu + g_A \gamma_5) e] + \frac{G_F}{2\sqrt{2}} \rho \bar{\nu}_\mu \gamma_\lambda (1-\gamma_5) \nu_\mu \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5) \nu_\mu .$$

[12]After completing the generalization of the theorem of Georgi and Weinberg to $q^2 \neq 0$ which we present in this section and making a preliminary study of $SU(2)\times U(1) \times U(1)$ models as reported in the next section, we received a report by G. Costa, M. D'Anna, and P. Marcolungo [Nuovo Cimento 50A, 177 (1979)] in which Eqs. (13) and (14) of this section are derived. These authors estimate sizes of the inverse mass submatrix elements and discuss order of magnitude of effects to be expected in $SU_L(2)\times SU_R(2)\times U_{L+R}(1)$ and $SU_L(2)\times S_R(1) \times U_L(1)\times U_R(1)$ models.

[13]One can readily verify the alternative forms for the propagator $\Delta_{\alpha\beta} = [1-q^2\Delta^0]^{-1}_{\alpha\beta'} \Delta^0_{\beta'\beta} = \Delta^0_{\alpha\beta'} [1-q^2\Delta^0]^{-1}_{\beta'\beta}$, where Δ^0 refers to the $q^2=0$ propagator which is discussed in the work of Georgi and Weinberg (Ref. 8).

[14]D. P. Sidhu, Phys. Lett. 87B, 67 (1979).

[15]R. N. Mohapatra and D. P. Sidhu, Report No. BNL-25866 (unpublished).

[16]For definitions see, for example, P. Roy, *Theory of Lepton Hadron Processes at High Energies* (Clarendon, Oxford, 1975).

[17]We have compared the q^2 and E_ν behavior of the numerator of Eq. (20) between expanded models and the Weinberg-Salam model in the cases described later in this section. The comparisons are almost identical to the comparisons between Eq. (20) and Eq. (19). Since ρ_{NC} is a constant [Eq. (19)] in the standard Weinberg-Salam model, we have chosen to use this ratio in our discussion even though it is more model dependent than the numerator alone.

[18]Versions of the weak-electromagnetic gauge group $SU(2)\times U(1)\times U(1)$ date back to the beginning of the intense activity on such theories. Examples are: J. Schechter and Y. Ueda, Phys. Rev. D 2, 736 (1970); H. Munczek, University of Kansas report, 1973 (unpublished); H. Fritzsch and P. Minkowski, Nucl. Phys. B 103, 61 (1976).

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[21]H. Deden *et al.*, Nucl. Phys. B149, 1 (1979); P. Wanderer *et al.*, Phys. Rev. D 17, 1679 (1978); M. Holder *et al.*, Phys. Lett. 72B, 254 (1977); F. S. Merritt *et al.*, Phys. Rev. D 17, 2199 (1978).

[22]We have not shown plots of ρ_{NC} for different sets of q_1, q_2 and λ choices in the six-quark case because the pattern follows that of the four-quark case illustrated in Figs. 1–3.

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[26]N. G. Deshpande, Oregon Report No. OITS-105, 1978 (unpublished).

[27]As a rough guideline, we say that two curves are indistinguishable if they differ by less than 10% for a q^2 or E_ν range. This guideline is suggested by errors on currently available ν reaction data.

[28]In the form presented, $\epsilon < 0$ is allowed only for the model with natural parity conservation in neutral currents. One can imagine more complicated scalar-field combinations where χ, ρ, ϕ, and δ are all present and $\epsilon < 0$ is allowed with $\chi_L \neq \chi_R$.