PHYSICAL REVIEW D VOLUME 21, NUMBER 9 1 MAY 1980

Absorbed Mueller-Regge model for backward inclusive proton production in pion-proton collisions

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(Received 26 January 1979)

We obtain fits to recent data for the backward inclusive reaction $\pi^- + p \to p + X$ at $p_{\rm lab} = 12$ GeV/c, using an absorbed baryon-exchange Mueller-Regge model in the normal Regge limit.

I. INTRODUCTION

A Regge approach has been applied in the past to two-body pion-nucleon backward-scattering processes, with the exchange of baryon trajectories, and absorption models have provided adequate fits to the data.[1-3] Recent experiments in the Omega spectrometer at CERN now provide an opportunity to examine a Mueller-Regge model for an inclusive reaction with baryon exchange. Data are available for the process $\pi^- + p \to p + X$, which involves only Δ_δ exchange (see Fig. 1), at $p_{\rm lab} = 12$ GeV/c, over a range $0.8 < M_X{}^2 < 8.0$ GeV^2 and $-u < 1.8$ (GeV/c)^2 (Refs. 4 and 5). We select a reduced set of data with $-u < 1.025$ (GeV/c)^2 and $M_X{}^2 > 4.2$ GeV^2 and apply an absorbed Mueller-Regge model in the normal Regge limit.

MacDowell symmetry[6] forces baryon trajectories to occur in pairs with opposite parity (parity doublets). The parity partner of the Δ_δ is not well observed experimentally and we note that a Carlitz-Kislinger cut formalism[7] forces the unwanted partner poles onto the unphysical sheet. We calculate Gottfried-Jackson-Sopkovich absorption corrections[8] to the basic pole model. This absorption prescription has previously been applied successfully to meson-exchange processes.[9]

II. FORMALISM

The Mueller generalized optical theorem[10] relates the inclusive cross section $a + b \to c + X$ to the $M_X{}^2$ discontinuity in the forward $3 \to 3$ amplitude (Fig. 2). Using s-channel helicity amplitudes, we have the following general expression for the unpolarized cross section:

$$\frac{s}{\pi}\frac{d^2\sigma}{du\,dM_X{}^2} = \frac{1}{64\pi^2k^2}\frac{1}{(2s_a+1)}\frac{1}{(2s_b+1)} \times \sum_{\lambda_a,\lambda_b,\lambda_c} H^{\lambda_a\lambda_b\lambda_c}_{\lambda_a\lambda_b\lambda_c}(s,u,M_X{}^2), \qquad (1)$$

where λ_i are helicity labels, s_i are spins, and k is the c.m. three-momentum. In this case ($s_a = 0$)

$$H^{\lambda_b\lambda_c}_{\lambda_b\lambda_c}(s,u,M_X{}^2) = \sum_\kappa J^{\lambda_c}_\mu \nabla^{\mu\nu}\Gamma^{\lambda_b\kappa}_\nu (J^{\lambda_c}_{\mu'}\nabla^{\mu'\nu'}\Gamma^{\lambda_b\kappa}_{\nu'})^\dagger , \qquad (2)$$

where $\nabla^{\mu\nu}$ is the Reggeized propagator, $J^{\lambda_c}_\mu$ is an on-shell current at the particle-particle-Reggeon vertex, κ is the helicity of the missing-mass state, and $\Gamma^{\lambda_b\kappa}_\nu$ is the structure function at the target-proton-Reggeon vertex. The spin-$\frac{3}{2}$ propagator is given by

$$\nabla^{\mu\nu} = D^{\mu\nu}(u-m^2)^{-1} ,$$

where m is the mass of the $\Delta_\delta(\frac{3}{2}^+)$ and

$$D_{\mu\nu} = (\not{P}+m)\left(-g_{\mu\nu} + \frac{2}{3m^2}P_\mu P_\nu + \tfrac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3m}(\gamma_\mu P_\nu + P_\mu\gamma_\nu)\right) - \frac{2}{3m^2}(P^2-m^2)[\gamma_\mu P_\nu - P_\mu\gamma_\nu + (\not{P}+m)\gamma_\mu\gamma_\nu] , \qquad (3)$$

with $P = p_a - p_c$. Reggeization is carried out by the standard replacement[11]

$$(u-m^2)^{-1} \to \tfrac{1}{2}\alpha'\Gamma(\tfrac{3}{2}-\alpha(u))(1+i\tau e^{-i\pi\alpha(u)})\left(\frac{s}{M_X{}^2}\right)^{\alpha(u)-3/2} ,$$

where we have used the Gell-Mann (nonsense) ghost-eliminating mechanism, α_0 and α' are the intercept and slope of the $I = \frac{3}{2}$, $p = +$, $\tau = -(\Delta_\delta)$

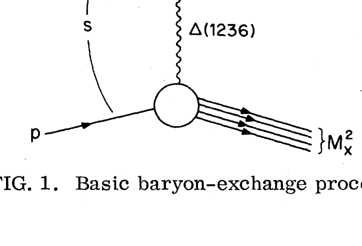

FIG. 1. Basic baryon-exchange process.

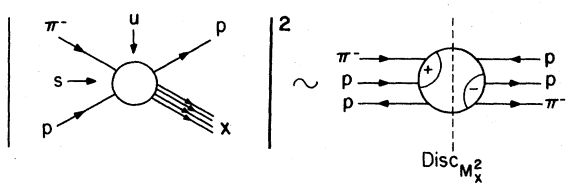

FIG. 2. Representation of Mueller generalized optical theorem.

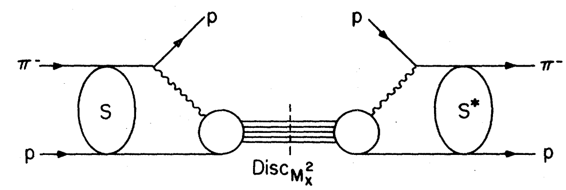

FIG. 3. Rescattering correction to helicity amplitudes.

baryon trajectory. The current is given by[12]

$$J_\mu^{\lambda_c}=g\bar{u}(p_c,\lambda_c)Q_\mu\ ,$$

where Q is the difference of the momenta of the incoming and outgoing baryons and u is the spin-$\frac{1}{2}$ wave function. The coupling constant g is connected to the on-shell coupling found from the Δ_δ width.[13]

To examine the behavior of the structure functions, we sum over κ, contract with Rarita–Schwinger spin-$\frac{3}{2}$ wave functions denoted by v_α, and apply the optical theorem:

$$\overline{\sum_{\lambda_b,r}}\ \sum_{\kappa}\bar{v}^\nu(P,r)\Gamma_\nu^{\lambda_b\kappa}\Gamma_{\nu'}^{\lambda_b\kappa *}v^{\nu'}(P,r)=M_X{}^2\sigma_{\text{tot}}(\bar{\Delta}p)\ . \tag{4}$$

We also obtain an alternative, completely general expression for the $\bar{\Delta}p$ total cross section. The elastic, no spin-flip $\bar{\Delta}p$ amplitude is given by[14]

$$T(\bar{\Delta}p)=\sum_j D_j^{\alpha\alpha'}X_{j\alpha\alpha'}\ , \tag{5}$$

where $D_j^{\alpha\alpha'}=A_jp_b^\alpha p_b^{\alpha'}-B_jg^{\alpha\alpha'}$ and X_j runs over scalar, tensor, axial-vector, vector, and pseudoscalar couplings. From this general ansatz, only vector and scalar contributions survive in the forward direction, so

$$T(\bar{\Delta}p)=(A_Sp_b^\alpha p_b^{\alpha'}-B_Sg^{\alpha\alpha'})\bar{v}_\alpha(P,r)v_{\alpha'}(P,r)\bar{u}(p_b,\lambda_b)u(p_b,\lambda_b)$$
$$+(A_Vp_b^\alpha p_b^{\alpha'}-B_Vg^{\alpha\alpha'})\bar{v}_\alpha(P,r)\gamma^\rho v_{\alpha'}(P,r)\bar{u}(p_b,\lambda_b)\gamma_\rho u(p_b,\lambda_b) \tag{6}$$

and

$$M_X{}^2\sigma_{\text{tot}}(\bar{\Delta}p)=\text{Im}\overline{\sum_{\lambda_b,r}}T(\bar{\Delta}p)\ . \tag{7}$$

Comparing Eqs. (7) and (4), and considering the overall s dependence of the cross section, we see that the dominant contribution will come from the term with form factor A_V. We have assumed that the $\bar{\Delta}p$ total cross section will approach a constant for large $M_X{}^2$ and then the term involving A_V in the inclusive cross section will be at least a factor of s larger than the remaining terms. So we take

$$\sum_{\lambda_b,\kappa}\Gamma_\nu^{\lambda_b\kappa}\Gamma_{\nu'}^{\lambda_b\kappa *}=\sum_{\lambda_b}2\,\text{Im}(A_V)p_{b_\nu}\not{p}_bp_{b_{\nu'}}\ . \tag{8}$$

We estimate the size of $\text{Im}(A_V)$ by assuming it is the only contribution to Eq. (7). In the limit $2p_b\cdot P\gg m^2m_b{}^2$ we have the approximation

$$p_b^\alpha\bar{v}_\alpha(P,r)\gamma^\mu v_{\alpha'}(P,r')p_b^{\alpha'}=\frac{4}{3m^2}(p_b\cdot P)^2P^\mu\delta_{rr'}\ ,$$

and get

$$\text{Im}(A_V)\sim\frac{3m^2}{M_X{}^4}\sigma_{\text{tot}}(\bar{\Delta}p)\ . \tag{9}$$

Since we have not taken the full expression for the $\bar{\Delta}p$ total cross section, we do not expect Eq. (9) to be an exact result, but consider that it will, at worst, provide sensible $M_X{}^2$ dependence and an order of magnitude estimate for $\text{Im}(A_V)$.

In the absorption prescription we employ, impact-parameter amplitudes are modified to take into account absorptive (unitarity) effects due to initial- and final-state rescatterings. The elastic scattering matrix $S(b)$, where b is the impact parameter, is introduced,

$$H_{\lambda_c\text{abs}}^{\lambda_c}(b',b)=S^{1/2*}(b')H_{\lambda_c}^{\lambda_c}(b',b)S^{1/2}(b)\ ,$$

with the usual Gaussian form

$$S(b)=1-C\,e^{-\lambda b^2}\ ,$$

where C is the opacity and $\lambda=R^{-2}$ (R = radius of interaction). Clearly, rescattering can be expected in both $ab(\bar{a}\bar{b})$ and $\bar{c}b(c\bar{b})$ channels, but we make the additional simplifying assumption that

$$S_{ab}^{1/2}S_{\bar{c}b}^{1/2}\simeq\frac{S_{ab}+S_{\bar{c}b}}{2}=S\ ,$$

and then we have (see Fig. 3)

$$H_{\lambda_c\text{abs}}^{\lambda_c}(s,u,M_X{}^2)=\int_0^\infty\tau'd\tau'\int_0^\infty\tau_1'd\tau_1'\int_0^\infty bdb\int_0^\infty b_1db_1J_\nu(b\tau)J_\nu(b\tau')S(b)J_{\nu'}(b_1\tau_1)J_{\nu'}(b_1\tau_1')S^*(b_1)H_{\lambda_c}^{\lambda_c}(s,\tau,\tau_1,M_X{}^2)\ ,$$

and finally

$$H^{\lambda c}_{\lambda_c \mathrm{abs}}(s,\tau,M_X{}^2)=\int_0^\infty \tau' d\tau' \int_0^\infty \tau_1' d\tau_1' H^{\lambda c}_{\lambda_c}(s,\tau',\tau_1',M_X{}^2)\left[\frac{1}{\tau}\delta(\tau-\tau')-\frac{C}{2\lambda}\exp\left(-\frac{\tau^2+\tau'^2}{4\lambda}\right)I_\nu\left(\frac{\tau\tau'}{2\lambda}\right)\right]$$
$$\times\left[\frac{1}{\tau}\delta(\tau-\tau_1')-\frac{C}{2\lambda}\exp\left(-\frac{\tau^2+\tau_1'^2}{4\lambda}\right)I_{\nu'}\left(\frac{\tau\tau_1'}{2\lambda}\right)\right],$$

with $\tau^2 q/k=u_{\min}-u$, where k and q are the initial and final c.m. three-momenta and ν and ν' are the total helicity flip on each side of the $M_X{}^2$ discontinuity. $J_\nu(z)$ is the Bessel function and $I_\nu(z)$ is the Bessel function of imaginary argument. We assume that the dominant configuration has no flip at the inclusive vertex. This can be justified using angular momentum arguments[15] and there is an absence of phenomenological evidence to the contrary. C and λ can be found from elastic scattering data, in this case $C\sim 0.7$ and $\lambda\sim 0.068$ $(\mathrm{GeV}/c)^{-2}$. For the evaluation of the absorbed amplitude, $\Gamma(\frac{3}{2}-\alpha(u))$ is approximated by a double exponential[16] $\sum_{j=1}^{2}A_j\exp(B_j u)$, with a conventional Δ_δ trajectory [$\alpha_0=0.05$ and $\alpha'=0.9$ $(\mathrm{GeV}/c)^{-2}$] $A_1=0.444\,798$, $A_2=0.440\,391$, $B_1=0.813\,327$ $(\mathrm{GeV}/c)^{-2}$, and $B_2=-0.825\,487$ $(\mathrm{GeV}/c)^{-2}$. We give full expressions for the amplitudes in the Appendix.

There is some ambiguity about the overall normalization. We have the traditional problem of extrapolating to baryon-exchange poles to determine the coupling constant.[17] The known Δ_δ width provides a coupling which is too high in parameter-free two-body Regge models.[2] Also, we do not expect $\sigma_{\mathrm{tot}}(\overline{\Delta}p)$, which appears in Eq. (9), to corres-

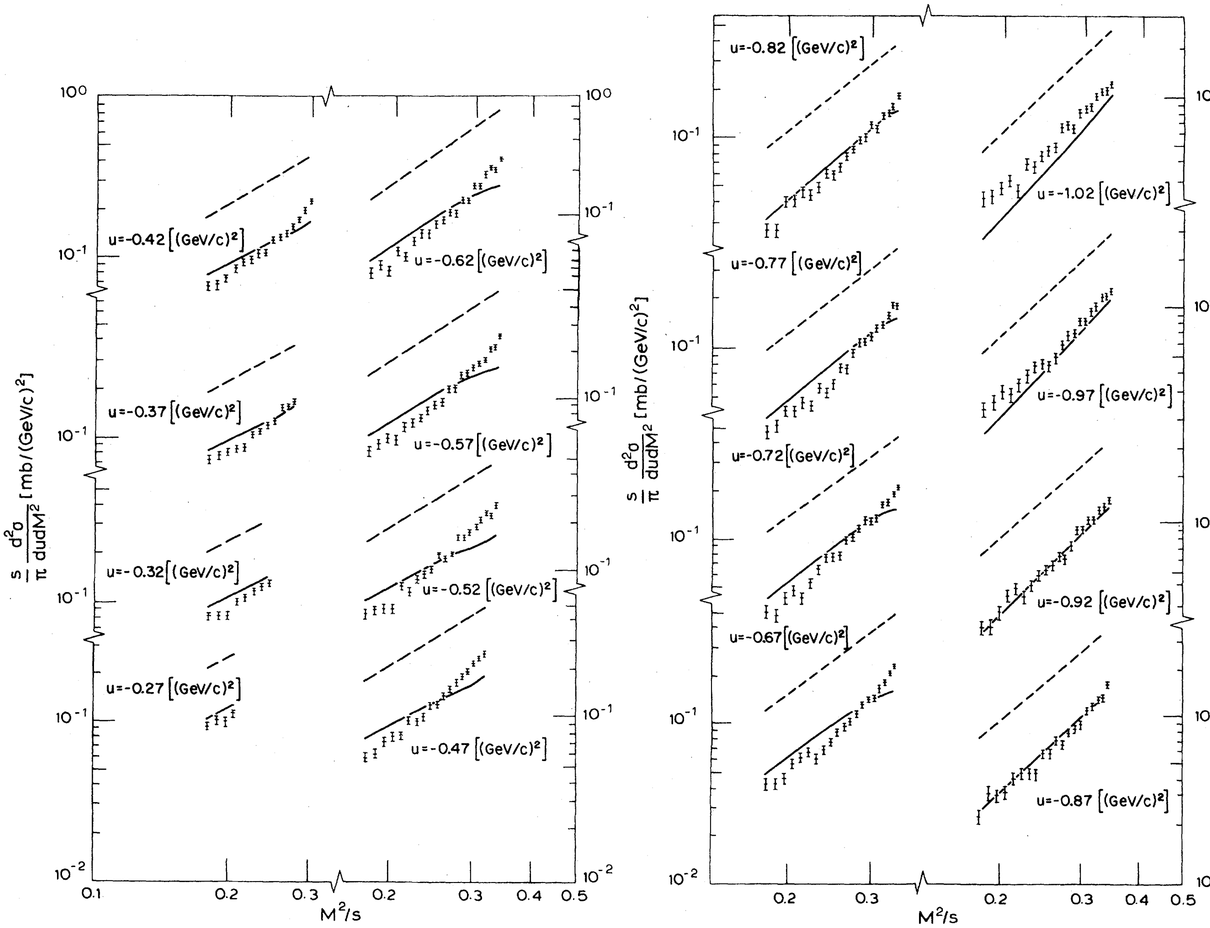

FIG. 4. Differential cross section plotted against $M_X{}^2/s$ for fixed values of u for $\pi^- + p \to p + X$ at $s=23.41$ GeV^2. (The dashed line represents the pole-only contribution while the continuous line represents the absorbed curve of the present model.)

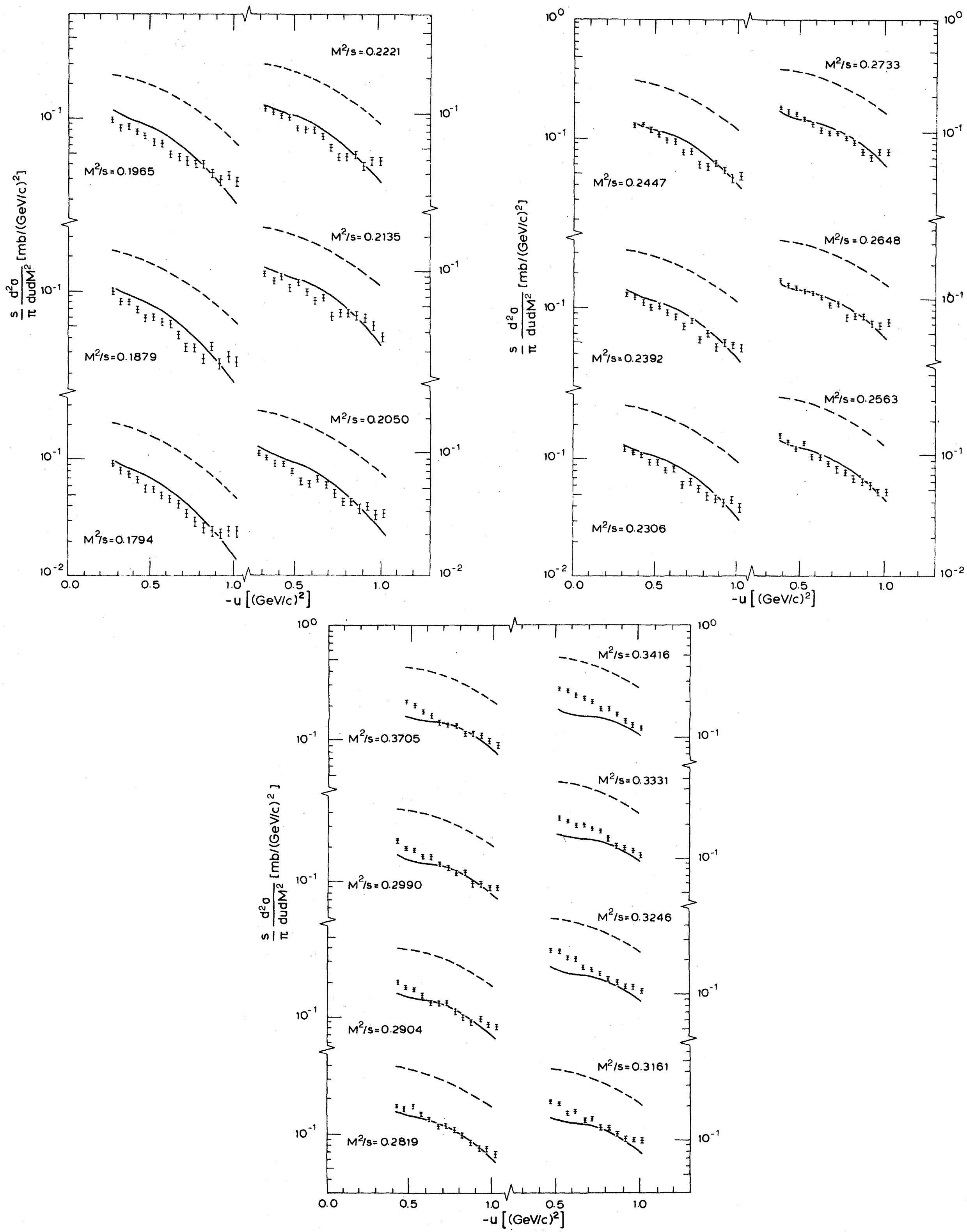

FIG. 5. Differential cross section plotted against u for fixed values of $M_X{}^2/s$ for $\pi^- + p \to p + X$ at $s = 23.41$ GeV^2. (The dashed line represents the pole-only contribution while the continuous line represents the absorbed curve of the present model.)

pond to the full asymptotic $\bar{\Delta}p$ total cross section. Although we might expect some resonance effects in the $M_X{}^2$ range under consideration, the data are for the most part smooth so we take $\sigma_{tot}(\bar{\Delta}p)\sim\sigma'$, a constant. Now we introduce a normalization N, and the overall adjustable normalization is $N\sigma'$.

The absorbed model is fitted to 277 data points with bin sizes 0.05 $(\text{GeV}/c)^2$ in u and 0.2 GeV^2 in $M_X{}^2$. The χ^2 function is minimized[18] by allowing the normalization ($N\sigma'$) and the trajectory parameters (α_0 and α') to vary. The result is shown by the solid line in Fig. 4 (fixed u) and Fig. 5 (fixed $M_X{}^2/s$). The values obtained are $N\sigma'=0.46$, $\alpha_0=0.0085$, and $\alpha'=0.97$ $(\text{GeV}/c)^{-2}$. For comparison, this normalization is then applied to a pole-only calculation with a conventional trajectory and the result corresponds to the dashed line on the figures. The graphs are computer plotted.[19,20] Errors shown are statistical and there is an additional 9% systematic error.

III. DISCUSSION

We see that for the reaction $\pi^-+p\to p+X$ in a normal Regge limit, both pole-only and absorbed models provide qualitatively good fits to the data. The fitted values for the trajectory parameters from the absorbed model correspond closely to the results expected from hadron spectroscopy. The absorbed model shows more structure and seems to indicate that more $M_X{}^2$ dependence is needed, but the assumption that σ' ~constant has oversimplified our $M_X{}^2$ behavior.

A previous analysis[4,5] of these data over the full available $M_X{}^2$ range used a simple triple-Regge model with Pomeron and baryon third-leg exchanges and found that a Δ_δ slope of ~0.5 $(\text{GeV}/c)^{-2}$ was obtained. We are satisfied that the nature of our fit allows us to conclude that the usual baryon trajectory is acceptable. Some discrepancy is expected since we understand that there is evidence that the data may be contaminated by production of $N\bar{N}$ pairs and $N^*\Delta$ resonances.

Our absorption model is factorizable and has identically zero momentum transfer at the bottom vertex and consequently predicts zero polarization, as of course does the pole-only case. As always in the inclusive Regge approach, measurement of the polarization will be of great interest.

Note added in proof. The model developed in the present paper has been extended to include the exchange of the $N_\alpha(938)$ baryon trajectory. The expression corresponding to Eq. (6) resulting from $N_\alpha(938)$ exchange is

$$\begin{aligned}4T(\bar{N}N)=&A_S\bar{v}(P,r)v(P,r')\bar{u}(P,\lambda_b)u(P,\lambda_b)+\tfrac{1}{2}A_T\bar{v}(P,r)\sigma^{\rho\delta}v(P,r')\bar{u}(P,\lambda_b)\sigma_{\rho\delta}u(P,\lambda_b)\\&+A_A\bar{v}(P,r)i\gamma_5\gamma^\rho v(P,r')\bar{u}(P,\lambda_b)i\gamma_5\gamma_\rho u(P,\lambda_b)+A_V\bar{v}(P,r)\gamma^\rho v(P,r')\bar{u}(P,\lambda_b)\gamma_\rho u(P,\lambda_b)\\&+A_P\bar{v}(P,r)\gamma_5 v(P,r')\bar{u}(P,\lambda_b)\gamma_5 u(P,\lambda_b)\,.\end{aligned}$$

If we sum over target spins, i.e., κ, then only A_S and A_V survive giving

$$T(\bar{N}N)=4mA_S+4Pp_aA_V\,.$$

Applying the optical theorem, this then gives

$$4m\,\text{Im}A_S+4Pp_a\,\text{Im}A_V=\Delta^{1/2}(M_X{}^2,m_N{}^2,m_N{}^2)\sigma_{\text{tot}}(\bar{N}N)\,,$$

which, in the triple-Regge limit, becomes

$$\text{Im}A_V+\frac{m}{p_a}\,\text{Im}A_S=\tfrac{1}{2}\sigma_{\text{tot}}(\bar{N}N)\,,$$

giving, at high energy, $\text{Im}A_V\sim\frac{1}{2}\sigma_{\text{tot}}(\bar{N}N)$.

This model has been applied to the data[21] on $K^-+p\to\Lambda+X$, which is mediated by $N_\alpha(938)$ baryon exchange. The $\Lambda\to p+K^-$ coupling is known. The results are encouraging[22] and indicate that, in addition to obtaining sensible parameters for the $N_\alpha(938)$ Regge trajectory, the absolute normalization of the single-particle-inclusive differential cross section is predictable. This is also the case[3] in two-body exclusive reactions mediated by $N_\alpha(938)$ baryon exchange.

ACKNOWLEDGMENTS

We wish to thank Professor G. Kramer for communicating to us the work of Dr. R. Tegen. We also wish to thank Professor J. Six and Dr. H. Yoshida for their cooperation and for providing the data in numerical form and Dr. J. H. Tabor for help with the computing. We gratefully acknowledge the encouragement of Professor H. G. Eggleston in this work. One of us (H.N.T.) wishes to thank the S.R.C. for financial support.

APPENDIX

The absorbed helicity amplitude in the c.m. is given by

$$\begin{aligned}H^{\lambda_c}_{\lambda_c\text{abs}}(\tau,\tau')=N'\{&C'[f(\tau)-f_{\text{abs}}(\tau)][f(\tau')-f_{\text{abs}}(\tau')]^*\\&+S[g(\tau)-g_{\text{abs}}(\tau)][g(\tau')-g_{\text{abs}}(\tau')]^*\}\,,\end{aligned}$$

with

$$N'=N\sigma'g^2\frac{6m^2}{M_X{}^4}(E_aE_b+k^2)^2\,,$$

$$C' = 2(E_bE_c - kq)[(m-m_c)^2 - m_a^2] + 4(E_aE_b + k^2)(E_aE_c + kq) + 4(m-m_c)m_c(E_aE_b + k^2)\,,$$

$$S = 2(E_bE_c + kq)[(m-m_c)^2 - m_a^2] + 4(E_aE_b + k^2)(E_aE_c - kq) + 4(m-m_c)m_c(E_aE_b + k^2)\,,$$

and defining

$$M_0 = \tfrac{1}{2}\alpha'(s/M^2)^{\alpha_0 + \alpha' u_{\min} - 1.5}\,,$$

$$\phi_j = -(q/k)[B_j + \alpha' \ln(s/M^2)]\,,$$

$$\xi_0 = -i\exp(-i\pi\alpha_0 - i\pi\alpha' u_{\min})\,,$$

$$\beta_0 = i\pi\alpha' q/k\,,$$

$$a_j = A_j \exp(B_j u_{\min})\,,$$

and

$$\psi_j = \phi_j + \beta_0\,,$$

then

$$f(\tau) = M_0 \sum_j a_j[\exp(\phi_j\tau^2) + \xi_0 \exp(\psi_j\tau^2)]\left(1 - \frac{\tau^2}{4k^2}\right)^{1/2}\,,$$

$$g(\tau) = M_0 \sum_j a_j[\exp(\phi_j\tau^2) + \xi_0 \exp(\psi_j\tau^2)]\frac{\tau}{2k}\,,$$

$$f_{\text{abs}}(\tau) = \frac{C}{2\lambda}\exp\left(-\frac{\tau^2}{4\lambda}\right)M_0 \sum_j a_j[F(\phi_j,\tau) + \xi_0 F(\psi_j,\tau)]\,,$$

and

$$g_{\text{abs}}(\tau) = \frac{C}{2\lambda}\exp\left(-\frac{\tau^2}{4\lambda}\right)M_0 \times \sum_j a_j[G(\phi_j,\tau) + \xi_0 G(\psi_j,\tau)]\,.$$

The functions $F(\alpha_j,\tau)$ and $G(\alpha_j,\tau)$ are given by

$$F(\alpha_j,\tau) = \frac{1}{4}\left(\frac{\tau}{\lambda}\right)^{1/2}\frac{\Gamma(1.25)}{\Gamma(1.5)}(-E_j)^{-1.25} \times \sum_{n=0}^{\infty} B(n)\left(\frac{1}{4k^2E_j}\right)^n \Phi(n+\tfrac{5}{4},\tfrac{3}{2};Z)\,,$$

$$G(\alpha_j,\tau) = \frac{1}{8k}\left(\frac{\tau}{\lambda}\right)^{1/2}\frac{\Gamma(1.75)}{\Gamma(1.5)}(-E_j)^{1.75}\,\Phi(\tfrac{7}{4},\tfrac{3}{2};Z)\,,$$

with

$$E_j = \alpha_j - \frac{1}{4\lambda}\,,$$

$$Z = -\frac{\tau^2}{16\lambda^2 E_j}\,,$$

$$B(n) = \binom{\frac{1}{2}}{n}(n-1+\tfrac{5}{4})(n-2+\tfrac{5}{4})\cdots(\tfrac{5}{4})\,,$$

and Φ is the degenerate hypergeometric function.

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