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Scaling of differential cross section and prediction for pp scattering at higher energies

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Using extrapolation of the fit to all the available data on forward slope and the scaling of all the available data on the differential-cross-section ratio in the variable χ proposed recently using the convergent polynomial expansion, the cross-section ratio is predicted as a function of $|t|$ for higher energies.

I. INTRODUCTION

Experimental data on the differential cross section for pp scattering at extreme high energies will be available in the near future from accelerators, some of which are now under construction at various places in the world. It will be interesting if some predictions can be made before these experimental results come out. If agreement of the predicted results comes out to be at least reasonably good, there is a possibility that the method of prediction can be used to compute differential cross sections at asymptotic energies, depending upon the desired accuracy, as an alternative to highly expensive experiments with the accelerators.

Several scaling variables have been proposed both by geometrical models and in model-independent methods.[1] In no other variable has the scaling been shown to be exhibited by the cross-section-ratio data for diffraction scattering processes in such a remarkable fashion as in the variable χ proposed recently using Mandelstam analyticity and the convergent polynomial expansion (CPE).[1,2] Whereas the scaling variable proposed in Ref. 2 for all the elastic diffraction scattering processes introduces spurious cuts in the mapped plane and requires the information of at least one real zero on the physical region of the $x=\cos\theta$ plane, the variable proposed in Ref. 1 is much simpler; it neither introduces any spurious cut in the mapped plane when applied to pp scattering, nor requires any information on zeros.

As one of the important applications, it has been pointed out in Refs. 1 and 2 that it is possible to make partial-wave analysis at high energies, even in the diffraction region, with economy of computer time, using the phenomenon of scaling, demonstrated using the CPE.[1,2] In this paper, as a more important application of the method,[1] using scaling of the available data on the differential-cross-section ratio in the range $50 \leq P_{1ab} \leq 1500$ GeV/c in the variable χ proposed in Ref. 1, we predict the cross-section ratio for pp scattering at higher energies as a function of $|t|$. Our predictions are based on the extrapolation of the excellent fit to the forward-slope-parameter data[1] onto higher energies.

The paper is divided into four sections. In Sec. II a summary of the CPE approach to scaling, as described in Ref. 1, is presented. Section III contains the results of our predictions. Several aspects of the predicted results are discussed in Sec. IV.

II. CONVERGENT POLYNOMIAL EXPANSION AND SCALING IN pp SCATTERING

Neglecting pole contributions and using Mandelstam analyticity of the s and $\cos\theta$ planes, the method of approach to scaling in pp scattering by means of CPE, has been sufficiently well described in Ref. 1. In this section we summarize the method. Two different forms of CPE have been proposed[1] for the differential cross section for pp scattering:

$$\frac{d\sigma}{dt}=e^{-\alpha Z}\sum_n C_n(s)P_n(2\alpha Z) \tag{1}$$

valid for finite energies, and

$$\frac{d\sigma}{dt}=e^{-\alpha Z}\sum_n a_n(s)L_n(2\alpha Z) \tag{2}$$

valid for asymptotic energies, where

$$Z=(\sinh^{-1}\sqrt{w})^2 \tag{3}$$

with

$$w=\frac{1-x^2}{x_+{}^2-1}, \tag{4}$$

and

$$\alpha(s)=d_0+d_1\eta+d_2\eta^2 \tag{5}$$

with[3]

$$\eta(s)=\sinh^{-1}\left(\frac{s-4m^2}{4m_\pi{}^2}\right)^{1/2}. \tag{6}$$

In Eq. (4) x_+ $(-x_+)$ is the start of the right (left-) hand cut in the x plane, which is related to the boundary of the double spectral function ρ_{st} or

ρ_{su},

$$x_+ = 1 + t_R/2q^2, \tag{7a}$$

$$t_R = 4m_\pi{}^2 + \frac{4\lambda^4}{(s-4m^2)}, \tag{7b}$$

where the value $\lambda = m_\pi$ corresponds to the theoretical elastic boundary. An effective value of λ different from m_π, as determined by the slope-parameter data, has been termed as giving rise to an effective shape of spectral function.[1,2,4,5] By the conformal transformation Z, the cuts in the $x = \cos\theta$ plane are mapped onto the boundary and the entire domain of analyticity of the x plane onto the interior of a parabola with focus at the origin in the Z plane.[1,4]

At finite energies the physical region, $x \in [-1, +1]$, is mapped onto a finite segment of the right half of the ReZ axis, which determines the orthogonal polynomials for expansion to be $\{P_n(Z)\}$.[1] Since the length of this segment varies with energy, the nature of $\{P_n(Z)\}$ and the corresponding domain of convergence also vary. Further, since the domain of convergence of $\{P_n(Z)\}$ at any finite energy does not contain the whole interior of the parabola, the convergence of the expansion (1) is not maximum.[6-8] However, as $s \to \infty$, the image of the physical region spreads onto the entire semi-infinite line, $0 \le \mathrm{Re}Z \le \infty$, like $(\ln s)^2$, which is the correct physical region for Laguerre polynomials $\{L_n(Z)\}$, and the domain of convergence of expansion in terms of $\{L_n(Z)\}$ is the whole interior of the parabola. Thus at asymptotic energies the polynomials determined by the image of the physical region in the Z plane are uniquely $\{L_n(Z)\}$, and the rate of convergence of the expansion (2) is maximum.[6-8] As $s \to \infty$, $\{P_n(Z)\} \to \{L_n(Z)\}$ and CPE (1) approaches the optimized polynomial expansion (OPE) (3).

The conformal transformation (6) maps the left-hand cut of the s plane onto the boundary and the rest of the s plane onto the interior of a strip in the η plane. The physical unitarity cut is mapped onto the semi-infinite line $0 \le \mathrm{Re}\eta \le \infty$. The whole series expansion in (5) has been truncated only after three terms because of the restriction imposed by the unitarity upper bound on the forward slope parameter.[9,10] The formula for the forward slope parameter computed using Eq. (2) or (3) is

$$b(s) = \frac{\alpha(s)}{t_R}\left(1 - \frac{t_R}{4q^2 + t_R}\right) \tag{8}$$

valid for all energies. The asymptotic behavior of $b(s)$ is the same as that of $\alpha(s)$. This formula has given an excellent description of all the available data on $b(s)$ with the following parameters,[1]

$$d_0 = 0.659, \quad d_1 = 0.050, \quad \lambda = 0.424 \text{ GeV}, \tag{9}$$

which are consistent with $\ln s$ type of asymptotic behavior of the slope parameter. Defining

$$\chi(s,t) = \alpha(s)Z(s,t), \tag{10}$$

we obtain from (2)

$$f(s,t) = \frac{d\sigma}{dt}(s,t)\Big/\frac{d\sigma}{dt}(s,O) = e^{-\chi}\sum_n e_n L_n(2\chi), \tag{11}$$

where

$$e_n = \frac{a_n(s)}{\sum_n a_n(s) L_n(o)}. \tag{12}$$

Analogous expressions can be obtained from (1) in terms of $P_n(\chi)$.

Some important convergence properties of the polynomial expansion in the χ plane, for physical values of s, are noteworthy. Since, for $s \to \infty$, $\alpha(s) \to (\ln s)^M$ with $M = 0$, 1, and 2, the image of the physical region in the χ plane spreads onto the right half of the Reχ axis like $(\ln s)^{M+2}$. Thus, if $M > 0$, as the case is for pp scattering, the CPE in terms of $P_n(\chi)$ approaches the OPE (11) in terms of $L_n(\chi)$ faster in the energy scale, as $s \to \infty$ in the χ plane. Further, the images of all the singularities in the χ plane are pushed onto infinity like $(\ln s)^M$ as $s \to \infty$; this implies that $f(s,t)$ is an entire function of χ at asymptotic energies.

Certain attractive properties of χ, expressing its potentialities as a scaling variable, have been pointed out.[1] For high energies and all values of $|t|$, $\chi \sim b(s)Z$. For high energies and small values of $|t| \ll t_R = 0.078$ GeV^2, $\chi \sim tb(s) \to t(\ln s)^M$; but for larger $|t|$, with $|t| \gg t_R$ and $|t| \ll s$, the kinematical region where a large number of data points exist, $\chi \sim b(s)(\ln t)^2 \to (\ln s)^M(\ln t)^2$. The scaling variable $tb(s)$ has been obtained by model-independent method,[10] and also in geometrical models[11] for small $|t|$ and large s. The variable $t(\ln s)^2$ to which χ has the potentialities to reduce for $|t| \ll 0.078$ GeV^2, if the future data on $b(s)$ saturates the unitarity upper bound, has been obtained by Auberson, Kinoshita, and Martin.[12] It may be noted that except for small $|t| \ll 0.078$ GeV^2, the variable χ is completely different from other scaling variables.

In view of the uniqueness of the polynomials in (11) and the maximal convergence of the series, it may be possible that the same number of e_n's with the same values account for the t dependence of $f(s,t)$ at least in the peak region, for different

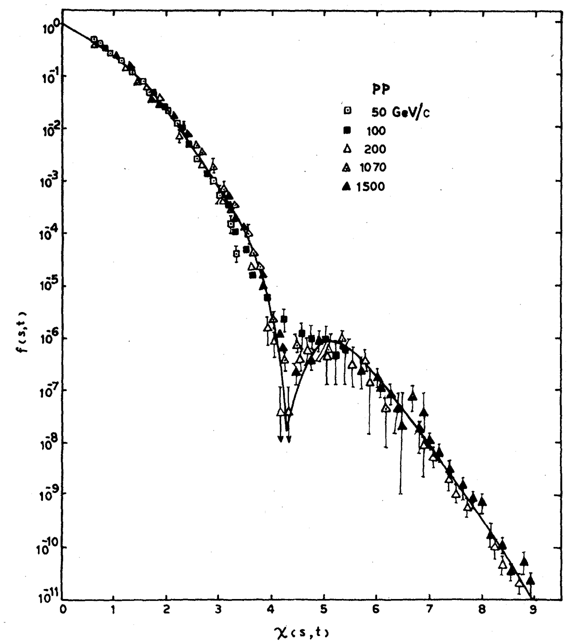

FIG. 1. Scaling of the available high-energy data on the differential-cross-section ratio. The solid line is the scaling curve obtained by a fit done by eye.

values of s in the asymptotic energy region. Further, because of the properties of χ, some of which are similar to those of known scaling variables, it is tempting to hypothesize scaling of $f(s,t)$ in χ; this is possible if the e_n's are independent of s. From (12) it is clear that one of the ways in which this can happen is that all "partial waves" possess the same asymptotic behavior. At present, although we do not know any theoretical proof as to why e_n's are independent of s, our hypothesis on scaling has been strongly supported by the existing data at high energies. With the parameters occurring in χ being known from the fit (9), all the available data on $f(s,t)$ for $P_{1ab} > 50$ GeV/c were plotted against χ as shown in Fig. 1. Scaling is surprisingly well exhibited even by those data points existing outside the forward peak, around the secondary maximum, and the region beyond it for $|t| \leq 10$ GeV^2. Such a scaling has not been observed in any other variable.

III. PREDICTIONS FOR HIGHER ENERGIES

In this section we predict $f(s,t)$ as a function of $|t|$ for several higher values of s at which experiments have not yet been performed. Although as has been mentioned in Ref. 1 the scaling curve can be obtained by fitting the data in Fig. 1 with the formula (11), it may not be possible to get the acceptable value of total χ^2 or a χ^2/DOF $\simeq 1$. Nevertheless, it is required to fit the data with (11) to know the scaling function. But here, without going to the complicacy of data fitting with the help of a computer, we take the scaling curve as that obtained by a fit to the eye, passing through the mean positions of the data points of Fig. 1, as shown by the solid curve. Since the spread in the data points in Fig. 1 is less (there is a little spread in the region of secondary maximum), the error committed in drawing such a curve by eye estimations is supposed to be less.

It has been observed[1] that scaling in χ improves for larger values of P_{1ab}. As energy increases the spread of the data points decreases and the limiting scaling curve is almost reached even for $P_{1ab} = 200$ GeV/c. For higher values of P_{1ab} upto 1500 GeV/c, the departure of the data points from the scaling curve is negligibly small. With the observation that almost the limiting scaling curve has been obtained as shown by the solid line of Fig. 1, it is safe to assume that all the future data on $f(s,t)$ for $P_{1ab} > 1500$ GeV/c will fall on this scaling curve.

Equation (3) can be rewritten for all values of s and t as

$$w = \frac{-t}{t_R}\,\frac{(s-4m^2+t)}{(s-4m^2+t_R)}. \qquad (13)$$

From Eq. (7b) we note that for high energies $t_R = 4m_\pi^2$. For large s and all values of $|t| \ll s$, Eq. (13) reduces to the simple form

$$w = -t/4m_\pi^2. \qquad (14)$$

Using Eqs. (3) and (14) we note that the resulting conformal transformation

$$Z = [\sinh^{-1}(-t/4m_\pi^2)^{1/2}]^2 \qquad (15)$$

is the one that would be obtained by mapping only the right-hand two-pion cut in the t plane onto the parabola and with the region $-\infty \leq \mathrm{Re}t \leq 0$ being mapped onto $0 \leq \mathrm{Re}Z \leq \infty$. In other words, the conformal mapping used in Ref. 1 and earlier ignores the presence of the left-hand cut at high

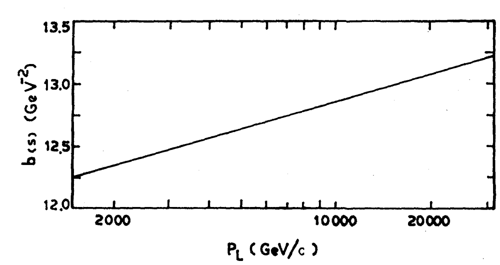

FIG. 2. Extrapolation of the forward slope parameter to higher energies using the formula described in the text.

energies. This is understandable since, for $s \to \infty$, the physical region in the t plane tends to be the entire left half of the Ret axis and the start of the left-hand cut is also pushed onto infinity, making its influence on scattering in the forward hemisphere negligible. At such energies the right-hand two-pion cut, being the nearest one, should have dominant contribution to scattering, at least in the forward hemisphere. Using (15) in (10), we obtain

$$|t| = 4m_\pi^2\{\sinh[\chi/\alpha(s)]^{1/2}\}^2 \qquad (16)$$

which is satisfied by all the points on the scaling curve. It may be noted that the above expression

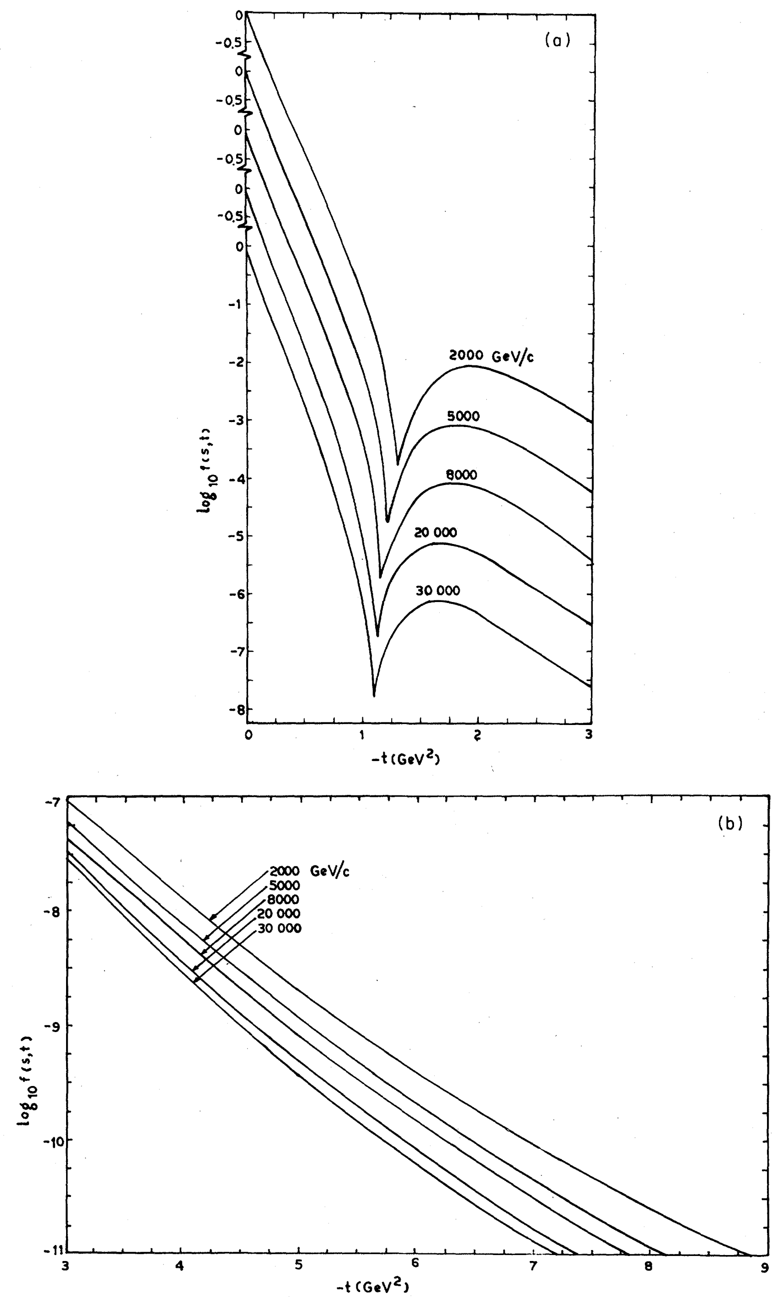

FIG. 3. (a) Prediction of the differential-cross-section ratio for different higher energies as a function of $|t|$ up to the region of the secondary maximum. (b) Prediction of the differential-cross-section ratio for different higher energies for larger-$|t|$ region.

for $|t|$ in terms of χ has been possible because of the simple nature of the transformation variable Z or χ. No such expression is possible with relatively more complicated variable of Ref. 2. However, recently scaling has been shown to be exhibited in terms of a simple variable for other diffractive[13] as well as nondiffractive processes,[14] in which case the relation (16) has also been obtained.

Using (16), the coordinates $f(s,t)$ and χ for different points on the scaling curve of Fig. 1 are noted. To know where a given value of $f(s,t)$, with a known χ, would fall in the $f(s,t)$ vs $|t|$ plot, for any higher value of s, it is necessary to know $\alpha(s)$ occurring in the right-hand side of (16). For this purpose we extrapolate the formula for $\alpha(s)$ to higher energies, retaining only the first two terms with the values of parameters given by the fit (9). This is equivalent to extrapolating the formula for the slope parameter to higher energies, as shown in Fig. 2. Since the formula for the slope parameter has been constructed using analyticity and the CPE, such an extrapolation, like many other extrapolated results used extensively in the literature[7,8] is expected to be stable. Of course objections might be raised to the fact that the pole contributions have not been explicitly retained in our analytically approximate formula for $b(s)$, but as has been already mentioned,[1] the theoretical upper bound on the absorptive part of differential cross section has been shown to saturate the data in the peak region[9,15] and the pole does not contribute to the absorptive part. In view of this, it makes almost no difference to neglect the pole contributions.

Using the extrapolated result on $\alpha(s)$ in Eq. (16), we compute the values of $|t|$ corresponding to the points on the scaling curve for a higher fixed value of s. This process is repeated for several fixed values of s to yield the expected curves in the $f(s,t)$ vs $|t|$ plot, as shown in Figs. 3(a) and 3(b). In Fig. 3(a) the predicted curves have been shown for $P_{1ab}=2000$, 5000, 8000, 20 000, and 30 000 GeV/c up to the region of the secondary maximum. In Fig. 3(b) the variation of $f(s,t)$ as a function of t has been predicted for the same energies beyond the secondary maximum region with $|t|<9$ GeV.[2] From these figures we note that, although energy dependence of $f(s,t)$ for smaller $|t|$ is slower, it becomes gradually prominent as we move away from the region of the forward peak. The energy dependence is quite remarkable for larger $|t|$ away from the secondary-maximum region. The positions of the dip and the secondary maximum approach the forward direction as the energy increases, although slowly, and the secondary maximum becomes

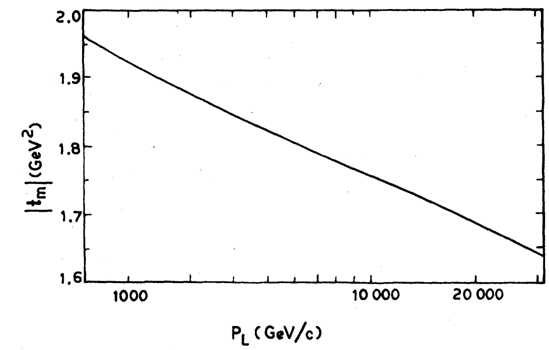

FIG. 4. Prediction of the energy dependence of the position of secondary maximum at higher energies.

sharper. Noting the positions of the secondary maximum in the scaling curve of Fig. 1 at $\chi_m=5.125$, we predict its position $t_m(s)$ to vary with energy as

$$|t_m(s)| = 4m_\pi^2\{\sinh[5.125/\alpha(s)]^{1/2}\}^2. \qquad (17)$$

This has been shown in Fig. 4. As a test of how reliable these predictions could be, the value of $\chi=\chi_0=4.35\pm0.05$ was computed[1] corresponding to the dip in the 200-GeV/c data of Akerlof *et al.*[16] The prediction of the dip done in this manner

$$|t_d(s)| = 4m_\pi^2\{\sinh[4.35\pm0.05/\alpha(s)]^{1/2}\}^2 \qquad (18)$$

was found to be in excellent agreement with all the available dip positions at higher and lower energies.[1] This fact enhances our confidence in the reliability of the predictions presented in Figs. 3(a), 3(b), and 4.

IV. DISCUSSION OF RESULTS

In this paper we have preditced the differential-cross-section ratio $f(s,t)$ as a function of $|t|$ for several higher energies. We have also predicted the energy dependence of the secondary-maximum position. The energy dependence of $f(s,t)$, although very slow near forward angles, gradually becomes prominent for longer $|t|$ values. The dip and the secondary maximum approach the forward direction with increasing energy, although slowly. The secondary maximum becomes sharper with energy. These predictions can be verified in the near future from the results of high-energy accelerators. If agreement of these results with experiment are found to be reasonably good, this method of computation of $f(s,t)$ may serve as an alternative to highly expensive experiments with accelerators. Our predictions are based on the observation of surprisingly good scaling of the available data on $f(s,t)$ for $50\leq P_{1ab}\leq 1500$ GeV/c, the fact that the data for high values of P_{1ab} fall on an almost limiting scaling curve, and the extrapolation of the excellent fit to all the available data on forward-slope parameters to higher energies.

One of the major contributions of the theory of analytic approximation proposed by Cutkosky and Deo,[6] and Ciulli[7] is its spectacular application to the extrapolation of physical quantities onto unknown regions which are either inaccessible to experiments or where experimental data do not exist. Let us examine how far our formula satisfies Mandelstam analyticity. As can be seen from Eq. (7b), for high energies there is negligible difference between cut positions of the real and the absorptive part. Therefore, a fractional part of the OPE (2) or (11) may be taken as the contribution of the real part and the other as that of the absorptive part. So far as the absorptive part is concerned, our construction satisfies its correct Mandelstam analyticity. So far as the real part is concerned, the pole contribution has not been taken into account. But it is well known by now, both theoretically[9,15] and experimentally,[17] that the absorptive part almost completely dominates high-energy diffraction scattering, at least near forward angles. In view of this, perhaps, we do not commit significant errors in not taking into account the pole contribution in the already negligible real part. A second argument leading to the fact that poles should not contribute at high energies is suggested from the excellent scaling of the data on $f(s,t)$ near forward angles. If the poles in the x plane contribute, one of them should have significant effects near forward angles. Using Mandelstam representation, one of the pole contributions in $d\sigma/dt$ is $g^2/(m^2-t)^2$, which is a function of t alone. The presence of this term in the cross section violates scaling. These arguments suggest that only cuts in the x plane are important for diffraction scattering.[18] Since our formula is consistent with the cut structure, the extrapolation of the slope parameter onto higher energies is supposed to be stable. Of course it is important to evaluate errors in the parameters in (9) to know how far the extrapolation is stable.

Because of the complicated nature of the scaling variable proposed in Ref. 2, it is not possible to write a simple relation like (16) using that variable. When the conformal mapping of the x plane, used in Ref. 1 and suggested for unsymmetrically cut x planes earlier, is applied for $\bar{p}p$, $\pi^{\pm}p$, and $K^{\pm}p$ scattering, it develops spurious cuts in the mapped plane. Further, the conformal mapping of Ref. 2 introduces spurious cuts for all processes. Because of this, it is not possible to carry out such predictions for other diffraction-scattering processes unambiguously. However, it has been demonstrated recently that scaling for both diffractive[13] as well as nondiffractive[14] processes can be exhibited in a remarkable fashion by means of a simpler variable which does not introduce any spurious cut or require any information on zero for its construction. Using the variable proposed in Refs. 13 and 14, it is possible to obtain a simple relation of the type (16) for any of the diffractive or nondiffractive processes possessing the unsymmetrically cut x plane of analyticity. Then using the similar method adopted here, predictions can also be made for any of the elastic diffractive processes such as $\pi^{\pm}p$, $K^{\pm}p$, and $\bar{p}p$ or the inelastic nondiffractive processes such as $\pi^-p\to\pi^0n, \eta n$, for which available data on $f(s,t)$ are sufficient to yield a well defined scaling curve. But since for high energies the formulas for the slope parameter and the conformal-mapping variables of the x plane for $|t|\ll s$, used in Refs. 1 and 13, are essentially the same, the scaling curve and the expected predictions for pp scattering with the variable of Ref. 13 would also be the same as obtained here.

In this work the differential cross section, normalized to its forward value. has been predicted. But the absolute value of $d\sigma/dt$ can be predicted if the available data at high energies on the forward differential cross section are fitted by using the technique of OPE and extrapolation is made onto higher energies. Although in that case the errors in the predicted values will magnify, such a problem needs attention.

ACKNOWLEDGMENT

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