Particle ratios of jets and quark statistics

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Ratios of hadrons in the jets produced by the annihilation of e^+e^- are calculated in a quark "statistical" model. Both two-jet and three-jet events are considered.

In the recent experiments of e^+e^- annihilation at DESY,[1] the jet formation of final-state hadrons was observed and analyzed. According to the naive quark model, it is generally believed that the formation of the two jets is due to the disintegration of a pair of $q,\bar{q}$ quarks created by the virtual photon. This interpretation is convincing, since the polar-angular distribution of the jet axis with respect to the beam is consistent with the form $(1+\cos^2\theta)$, in agreement with the hypothesis that the primary constituents are a pair of spin-$\frac{1}{2}$ quarks. In addition to this basic two-jet structure quantum chromodynamics (QCD) (Ref. 2) predicts that at very high energy one of the outgoing quarks may emit a hard gluon g which then turns into a "gluon jet," thus resulting in a three-jet final state of hadrons. In QCD, the probability of emitting a single gluon is proportional to $\alpha_s(s)$, two-gluons proportional to $\alpha_s^2(s)$, etc. At high energy, since $\alpha_s(s) \ll 1$, the emission of several gluons can be neglected in comparison to a single-gluon emission.

In this paper we use the quark statistics (i.e., the standard quark assumption together with Chao-Yang statistics) that we developed in our previous paper[3] to calculate the particle ratios of the "quark jet" and the "gluon jet." This statistical approach, without any dynamical assumption, has been quite successful[3,4] in calculating the particle ratios in hadronic production at large transverse momentum and in calculating the pair-production spectrum of oppositely charged hadrons.

Let us briefly recapitulate the formulas we need for our calculation. Consider a collection of l quarks of types u, d, and s and their associated antiquarks $\bar{u}$, $\bar{d}$, and $\bar{s}$. The quantum state of the collection is given by (a,b,c), which is equivalent to a quantum state of a u quarks, b d quarks, and c s quarks. The distribution function $N^l_{a,b,c}$, which is the number of possible ways of distributing the quantum number (a,b,c) over the l quarks, follows the binomial distribution with the suppression factor γ for the strange quark:

$$\left(x+y+\gamma z+\frac{1}{x}+\frac{1}{y}+\gamma\frac{1}{z}\right)^l=\sum_{a,b,c,} N^l_{a,b,c,}x^a y^b z^c . \quad (1)$$

For a large value of l, the asymptotic expansion of $N^l_{a,b,c,}$ with a, b, and c fixed is

$$\begin{aligned}N^l_{a,b,c,}=(4+2\gamma)^l\left(\frac{2+\gamma}{2\pi l}\right)^{3/2}\frac{1}{\sqrt{\gamma}}\Bigg(1-\frac{1}{4l}\left[(4+2\gamma)\left(a^2+b^2+\frac{c^2}{\gamma}\right)-\frac{\gamma^2-5\gamma+1}{\gamma}\right]\\+\frac{1}{128l^2}\left[(2+\gamma)^2\left(20+\frac{9}{\gamma^2}+\frac{4}{\gamma}\right)-70(2+\gamma)\left(2+\frac{1}{\gamma}\right)+385\right]\\+\frac{1}{48l^2}\Bigg\{(2+\gamma)^2\left[6\left(a^2+b^2+\frac{c^2}{\gamma}\right)^2-3\left(6a^2+6b^2+\frac{5c^2}{\gamma^2}+\frac{a^2+b^2+2c^2}{\gamma}\right)\right]\\+105(2+\gamma)\left(a^2+b^2+\frac{c^2}{\gamma}\right)\Bigg\}\Bigg). \quad (2)\end{aligned}$$

The probability of finding k quarks in the final state is assumed to be

$$P_{q_1,q_1,\ldots,q_k}(l)=\frac{N^{l-k}_{a-\alpha,b-\beta,c-\delta}}{N^l_{a,b,c}}, \quad (3)$$

where (α,β,δ) is the quantum state of the collection of k quarks with quantum numbers equivalent to α u quarks, β d quarks, and δ s quarks, and (a,b,c) is the initial state.

We now consider the e^+e^- annihilation into two hadronic jets through the formation of a $q\bar{q}$ pair via the virtual photon. To find a particular type of hadron in the jet, the only reasonable assumption that can be made is that the probability of producing the outgoing hadron h (quark content $q_1q_2,\ldots,q_k$) in the jet is proportional to the quark correlated probability $P_{q_1,q_2,\ldots,q_k}$. Furthermore, if we assume that the creation of any $q\bar{q}$ pair in the e^+e^- annihilation is equally probable, we then have to take the average of all three kinds of quarks (or antiquarks) which turn into hadronic jets.

For $q \to \pi^\pm$, $K^\pm$, p, and $\bar{p}$, we have

$$\pi^+ \propto \left(\frac{N^{l-2}_{0,1,0}}{N^l_{1,0,0}} + \frac{N^{l-2}_{-1,2,0}}{N^l_{0,1,0}} + \frac{N^{l-2}_{-1,1,1}}{N^l_{0,0,1}}\right), \quad \pi^- \propto \left(\frac{N^{l-2}_{2,-1,0}}{N^l_{1,0,0}} + \frac{N^{l-2}_{1,0,0}}{N^l_{0,1,0}} + \frac{N^{l-2}_{1,-1,1}}{N^l_{0,0,1}}\right),$$
$$K^+ \propto \left(\frac{N^{l-2}_{0,0,1}}{N^l_{1,0,0}} + \frac{N^{l-2}_{-1,1,1}}{N^l_{0,1,0}} + \frac{N^{l-2}_{-1,0,2}}{N^l_{0,0,1}}\right), \quad K^- \propto \left(\frac{N^{l-2}_{2,0,-1}}{N^l_{1,0,0}} + \frac{N^{l-2}_{1,1,-1}}{N^l_{0,1,0}} + \frac{N^{l-2}_{1,0,0}}{N^l_{0,0,1}}\right), \tag{4}$$
$$p \propto \left(\frac{N^{l-3}_{-1,-1,0}}{N^l_{1,0,0}} + \frac{N^{l-3}_{-2,0,0}}{N^l_{0,1,0}} + \frac{N^{l-3}_{-2,-1,1}}{N^l_{0,0,1}}\right), \quad \bar{p} \propto \left(\frac{N^{l-3}_{3,1,0}}{N^l_{1,0,0}} + \frac{N^{l-3}_{2,2,0}}{N^l_{0,1,0}} + \frac{N^{l-3}_{2,1,1}}{N^l_{0,0,1}}\right).$$

For $\bar{q} \to \pi^\pm$, $K^\pm$, p, and $\bar{p}$, we have

$$\pi^+ \propto \left(\frac{N^{l-2}_{-2,1,0}}{N^l_{-1,0,0}} + \frac{N^{l-2}_{-1,0,0}}{N^l_{0,-1,0}} + \frac{N^{l-2}_{-1,1,-1}}{N^l_{0,0,-1}}\right), \quad \pi^- \propto \left(\frac{N^{l-2}_{0,-1,0}}{N^l_{-1,0,0}} + \frac{N^{l-2}_{1,-2,0}}{N^l_{0,-1,0}} + \frac{N^{l-2}_{1,-1,-1}}{N^l_{0,0,-1}}\right),$$
$$K^+ \propto \left(\frac{N^{l-2}_{-2,0,1}}{N^l_{-1,0,0}} + \frac{N^{l-2}_{-1,-1,1}}{N^l_{0,-1,0}} + \frac{N^{l-2}_{-1,0,0}}{N^l_{0,0,-1}}\right), \quad K^- \propto \left(\frac{N^{l-2}_{0,0,-1}}{N^l_{-1,0,0}} + \frac{N^{l-2}_{1,-1,-1}}{N^l_{0,-1,0}} + \frac{N^{l-2}_{1,0,-2}}{N^l_{0,0,-1}}\right), \tag{5}$$
$$p \propto \left(\frac{N^{l-3}_{-3,-1,0}}{N^l_{-1,0,0}} + \frac{N^{l-3}_{-2,-2,0}}{N^l_{0,-1,0}} + \frac{N^{l-3}_{-2,-1,-1}}{N^l_{0,0,-1}}\right), \quad \bar{p} \propto \left(\frac{N^{l-3}_{1,1,0}}{N^l_{-1,0,0}} + \frac{N^{l-3}_{2,0,0}}{N^l_{0,-1,0}} + \frac{N^{l-3}_{2,1,-1}}{N^l_{0,0,-1}}\right).$$

For large values of l, we neglect, as a first approximation, the l^{-2} and all the higher-order terms, and we assume the suppression factor $\gamma = 0.1$ for each strange quark created in the final state. In this case Eq. (4) becomes

$$\pi^+ \propto 1 - \frac{2.1}{l}, \quad \pi^- \propto 1 - \frac{2.1}{l},$$
$$K^+ \propto 1 - \frac{17.8}{l}, \quad K^- \propto 1 - \frac{5.3}{l}, \tag{6}$$
$$p \propto 1 - \frac{3.2}{l}, \quad \bar{p} \propto 1 - \frac{7.4}{l}.$$

The above equations indicate that

$$\pi^+ = \pi^- > K^- > K^+, \quad p > \bar{p}, \tag{7}$$

and the particle ratios are

$$\frac{K^+}{\pi^+} \sim 1 - \frac{15.7}{l},$$
$$\frac{K^-}{\pi^-} \sim 1 - \frac{3.2}{l}, \tag{8}$$
$$\frac{p}{\pi^+} \sim C\left(1 - \frac{1.1}{l}\right),$$
$$\frac{\bar{p}}{\pi^-} \sim C\left(1 - \frac{5.3}{l}\right).$$

The constant C in the above equations is due to the different proportionality constants in producing baryons and mesons in Eq. (6). It is interesting to note that our results have the following relations:

$$\frac{K^-}{\pi^-} > \frac{K^+}{\pi^+}, \quad \frac{p}{\pi^+} > \frac{\bar{p}}{\pi^-}, \tag{9}$$

which are to be tested by future experiments.

Similarly, Eq. (5) gives

$$\pi^+ \propto 1 - \frac{2.1}{l}, \quad \pi^- \propto 1 - \frac{2.1}{l},$$
$$K^+ \propto 1 - \frac{5.3}{l}, \quad K^- \propto 1 - \frac{17.8}{l}, \tag{10}$$
$$p \propto C\left(1 - \frac{7.4}{l}\right), \quad \bar{p} \propto C\left(1 - \frac{3.2}{l}\right),$$

which shows that

$$\pi^+ = \pi^- > K^+ > K^-, \quad \bar{p} > p, \tag{11}$$

and the particle ratios

$$\frac{K^-}{\pi^-} < \frac{K^+}{\pi^+}, \quad \frac{p}{\pi^+} < \frac{\bar{p}}{\pi^-}. \tag{12}$$

So far we have considered only the two-jet structures. However, there are evidences that three jets are observed in e^+e^- annihilation. According to QCD, two possible types of three-jet structures can be obtained, namely, (i) a hard gluon is emitted either by the quark or by the antiquark and the final state consists of two quark jets originating from the $q\bar{q}$ pair and a gluon jet from the hard gluon, and (ii) three gluon jets are emitted through the decay of heavy quarkonium states such as ψ/J or Υ particles. (In the QCD approach, the direct hadronic decay of vector mesons may be considered as the annihilation of q and $\bar{q}$ into three gluons which then combine together to form hadrons. If this picture is correct, we may also conjecture that the three hadronic jets are originated from the three gluons emitted through the annihilation of $Q\bar{Q}$ vector mesons.)

Let us first consider the two quark jets and a gluon jet. The particle ratios for the two quark jets are similar to the previous cases. However, the gluon jet would be different. In the quark model, gluons can be considered as a $q\bar{q}$ pair which carries a quark color but not quark flavor. Using this conjecture, we are able to calculate the outgoing hadrons in the gluon jet as follows:

$$\pi^+ \propto 1 - \frac{0.83}{l}\,, \quad \pi^- \propto 1 - \frac{0.83}{l}\,,$$
$$K^+ \propto 1 - \frac{10.3}{l}\,, \quad K^- \propto 1 - \frac{10.3}{l}\,, \tag{13}$$
$$p \propto 1 - \frac{4.0}{l}\,, \quad \bar{p} \propto 1 - \frac{4.0}{l}\,.$$

From the above equations, we have for the gluon jet

$$\pi^+ = \pi^-\,, \quad K^+ = K^-\,, \quad p = \bar{p}\,,$$
$$\pi^\pm > K^\pm\,, \quad \frac{K^+}{\pi^+} = \frac{K^-}{\pi^-}\,, \quad \text{and} \quad \frac{p}{\pi^+} = \frac{\bar{p}}{\pi^-}\,. \tag{14}$$

For the case of three gluon jets emitted through the annihilation of heavy quarkonium states, the distribution and the particle ratios are similar to the gluon jet radiated away from a quark or antiquark.

We conclude that the distribution of the outgoing hadrons of jets in the e^+e^- annihilation can be determined by our quark statistical approach and comparison with experiment in the near future would be extremely interesting. Further work on extending our model to determine the particle ratios of jets in hadron-hadron collisions is in progress and the results will be published in a separate paper.

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