Model based on gauge symmetry group $G = G_{wk} \times [SU(3) \times SU(3)]_c$

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We formulate a gauge model for basic interactions based on the symmetry group $G = G_{wk} \times G_c$, where G_c is the chiral symmetry group $[SU(3) \times SU(3)]_c$ in color space. G_{wk} is taken to be the left-right-symmetric model $SU_L(2) \times SU_R(2) \times U(1)$. The chiral color symmetry is spontaneously broken in such a way that quarks acquire a common mass and an octet of axial-vector gluons become massive but an octet of vector gluons remain massless. In this way quark mass arises from spontaneous color-chiral-symmetry breaking. The experimental consequences of the left-right-symmetric model are discussed and it is shown that one version of this model gives results similar to the Salam-Weinberg model for the currently available energies. There is also another version, where the results are again similar to the Salam-Weinberg model except that the y dependence for the asymmetry parameter for the deep-inelastic scattering of polarized electrons is completely different although its value at $y = 0.21$ is compatible with experiment.

I. INTRODUCTION

There is now convincing evidence that neutral weak currents, as predicted by the unified gauge theory of weak and electromagnetic interactions, exist in nature. The unification of weak and electromagnetic interactions based on the gauge group $SU_L(2) \times U(1)$ seems to be well established.[1]

It is now believed that quarks carry color and that fractionally charged quarks are confined in colorless hadrons. The interactions between colored quarks are mediated by massless vector colored gluons. The underlying theory for these interactions is a gauge theory called quantum chromodynamics (QCD). It is believed that QCD is the best candidate for a theory of the strong interactions.

Each quark carries three colors, blue (b), yellow (y), and red (r). Thus the color gauge group is $SU_c(3)$ and QCD has a non-Abelian character in contrast to QED, which is Abelian. If the gauge group of QCD is an exact local symmetry, the color-triplet quarks and color-octet gluons of the theory are not expected to exist as real particles—only color-singlet hadrons exist. QCD has nothing to say about the origin of masses of the quarks.

It is believed that the underlying theory for the three basic interactions, strong, electromagnetic, and weak, is a gauge theory based on some symmetry group G. The symmetry group G contains $G_{wk} \times G_c$. It seems that G_{wk} contains the group $SU_L(2) \times U(1)$. For G_c, it is common belief that $G_c = SU_c(3)$.

In this paper, we take the color gauge group G_c to be a chiral group[2] $[SU(3) \times SU(3)]_c$. The electroweak gauge group G_{wk} is taken to be $SU_L(2) \times SU_R(2) \times U(1)$. This group has been extensively studied by many authors.[3] In order to give masses to weak vector bosons, the electroweak gauge group $G_{wk} = SU_L(2) \times SU_R(2) \times U(1)$ is spontaneously broken by introducing three Higgs scalars, ϕ, ϕ', and ϕ_R, belonging to representations (2, 1), (1, 2), and (1, 3) of the group $SU_L(2) \times SU_R(2)$. So far, charged weak vector bosons $W^{\pm}_{L\mu}$ and $W^{\pm}_{R\mu}$ are eigenstates of mass matrix and there is no mixing between them.

The quarks are also massless. In order to give mass to quarks, the Higgs scalar Σ belonging to the representation (2, 2) of G_{wk} must be introduced. The quarks then acquire mass, and this in general leads to mixing between $W^{\pm}_{L\mu}$ and $W^{\pm}_{R\mu}$. There is no way to estimate the parameter $\langle\Sigma\rangle/\langle\phi\rangle$ in order to see whether the mixing between $W^{\pm}_{L\mu}$ and $W^{\pm}_{R\mu}$ is negligible in order to have μ-decay and β-decay universality. In this approach, the symmetry of the strong-interaction Lagrangian cannot be chiral because the quark mass matrix explicitly breaks this symmetry.

Our approach is different in three aspects from these authors.

(1) We take neutrinos to be left-handed so that they are massless and remain massless even after radiative corrections. With this restriction, we must assign leptons[4] to the gauge group $SU_L(2) \times SU_R(2)$ in the following way: (ν_e, l_L), etc., belong to representation (2, 1) and e_R, etc., are singlets. The left-handed and right-handed quarks, however, belong to representations $(2, 1, \overline{3}, 1)$ and $(1, 2, 1, \overline{3})$ of groups $[SU_L(2) \times SU_R(2)]_W \times G_c$.

(2) Leptons acquire masses by spontaneous breaking of the gauge group G_{wk}; in particular, by giving a nonzero expectation value to the neutral component of the Higgs scalar ϕ.

(3) In order to give masses to quarks, we break the symmetry group G in two steps. For this purpose, we introduce scalar fields $\mathfrak{M}$ belonging to representation $(2, 2, \overline{3}, 3+3, \overline{3})$ of G. The symmetry is now spontaneously broken in such a way that $\mathfrak{M} \to (1/\sqrt{2})\, I\times M$, where I is a unit matrix in the space of the group $SU_L(2)\times SU_R(2)$ and matrix M represents a nonet of scalar and pseudoscalar mesons belonging to the representation $(\overline{3}, 3)+(3, \overline{3})$ in color space. In this way, we can have a chiral-invariant strong-interaction Lagrangian. The chiral symmetry is then broken spontaneously in such a way that the octet of vector gluons remains massless, but the octet of axial-vector gluons acquires a common mass. Before symmetry breaking the quarks were massless, and quarks acquire their common mass after symmetry breaking. It is shown that by means of a chiral transformation, the nonet of colored scalars and pseudoscalar mesons can be decoupled from quarks. The extension of the color gauge group $SU_c(3)$ to the chiral group $[\,SU(3)\times SU(3)]_c$ is attractive in the sense that quarks are massless before symmetry breaking, but they acquire their mass by breaking local color symmetry. Our point of view is that lepton mass arises from spontaneous breaking of G_{wk}, but quark mass arises also from spontaneous breaking of chiral color gauge symmetry. We realize that we still do not understand how each quark flavor acquires a different mass. This may arise due to breaking of the flavor $SU(n)$ symmetry $(n=3, 4, \ldots)$.

Another advantage of the octet of axial-vector gluons is that it may be helpful in understanding the hyperfine mass splitting of charmonium states. It is well known that the effective potential obtained from a single vector-gluon exchange cannot explain hyperfine splitting of charmonium states (especially the mass difference between $^3S_1$ and $^1S_0$ states).

It may, however, be noted that by breaking the symmetry in this way, there is no mixing between flavor space and color space. But weak vector bosons $W_{L\mu}$ and $W_{R\mu}$ acquire an additional term in their masses and it also introduces mixing between $W_{L\mu}$ and $W_{R\mu}$. It is shown that this contribution to the weak vector-boson matrix can safely be neglected. The advantage of our approach is that the mixing parameter turns out to be naturally small.

Finally we show that the spontaneous breaking of the electroweak gauge group $SU_L(2)\times SU_R(2)\times U(1)$ through Higgs scalars can be adjusted in such a way that it reproduces all the results in the standard model of Salam and Weinberg[5] (SW) for the currently available energies. A version of the spontaneous symmetry breaking can also be found where *only* the asymmetry $A=(\sigma_R-\sigma_L)/(\sigma_R+\sigma_L)$ for the scattering of polarized electrons on the deuteron or the proton is different in its dependence on y from that in the SW model, although the value of A is compatible with its experimental value at $y=0.21$.

II. THE FERMI LAGRANGIAN

The local gauge symmetry group we consider is a direct product of G_{wk} and G_c:

$$G_{wk}\times G_c \equiv [SU_L(2)\times SU_R(2)\times U(1)]_W \times [SU(3)\times SU(3)]_c\,.$$

Under the above group, quarks transform as (as far as the non-Abelian part of the group is concerned)

$$q^{(1)}_{L,R}=\frac{1\pm\gamma_5}{2}\begin{pmatrix} u_b & u_y & u_r\\ d'_b & d'_y & d'_r\end{pmatrix}\equiv\begin{cases}(2,1,\overline{3},1)\\ (1,2,1,\overline{3}),\\ Y=1/3\end{cases}$$

$$q^{(2)}_{L,R}=\frac{1\pm\gamma_5}{2}\begin{pmatrix} c_b & c_y & c_r\\ s'_b & s'_y & s'_r\end{pmatrix}\equiv\begin{cases}(2,1,\overline{3},1)\\ (1,2,1,\overline{3}),\\ Y=1/3\end{cases}$$

where $d'=d\cos\theta_C+s\sin\theta_C$, $s'=d\sin\theta_C+s\cos\theta_C$. The leptons are assigned to the following representation of the above group:

$$L_l=\begin{pmatrix}\nu_l\\ l_L^-\end{pmatrix}\equiv(2,1,1,1),\quad Y=-1\,,$$

$$R_l=l_L^-\equiv(1,1,1,1),\quad Y=-2\,,$$

$$l=e,\mu,\tau,\ldots,$$

$$l_{L,R}=\frac{1\pm\gamma_5}{2}l\,.$$

For quarks, the gauge-invariant Lagrangain is given by

$$\mathcal{L}_q=-\sum_{a=1,2}\mathrm{Tr}(\overline{q}^{(a)}_L\gamma_\mu\nabla_{L\mu}q^{(a)}_L+\overline{q}^{(a)}_R\gamma_\mu\nabla_{R\mu}q^{(a)}_R)\,, \tag{1a}$$

where

$$\nabla_{L\mu}=\partial_\mu-igW_{L\mu}-\tfrac{1}{2}i\tfrac{1}{3}g'B_\mu+\frac{i}{\sqrt{2}}g_sG_{L\mu}\,, \tag{1b}$$

$$\nabla_{R\mu}=\partial_\mu-igW_{R\mu}-\tfrac{1}{2}i\tfrac{1}{3}g'B_\mu+\frac{i}{\sqrt{2}}g_sG_{R\mu}\,. \tag{1c}$$

Here g is the weak coupling constant and $W_{L\mu}$ and

$W_{R\mu}$ are weak vector bosons

$$W_{L\mu}\equiv\tfrac{1}{2}\vec{\tau}\cdot\vec{W}_{L\mu}$$

$$=\tfrac{1}{2}\begin{pmatrix} W_{3L\mu} & \sqrt{2}\,W^-_{L\mu} \\ \sqrt{2}\,W^+_{L\mu} & -W_{3L\mu}\end{pmatrix}, \tag{2a}$$

$$W_{R\mu}\equiv\tfrac{1}{2}\vec{\tau}\cdot\vec{W}_{R\mu}, \tag{2b}$$

g_s is the coupling constant associated with the color group G_c and $G_{L\mu}$ and $G_{R\mu}$ are color-carrying gluons

$$G_{L\mu}=\frac{1}{\sqrt{2}}\vec{\lambda}^c\cdot\vec{G}_{L\mu}$$

$$=\frac{1}{\sqrt{2}}\vec{\lambda}^c\cdot\left(\frac{\vec{G}^V_\mu+\vec{G}^A_\mu}{2}\right)=\frac{G^V_\mu+G^A_\mu}{2}, \tag{3a}$$

$$G_{R\mu}=\tfrac{1}{2}\vec{\lambda}^c\cdot\vec{G}_{R\mu}\equiv\frac{G^V_\mu-G^A_\mu}{2}, \tag{3b}$$

where $\vec{\lambda}^c$ are the usual eight Gell-Mann λ matrices in color space. G^V_μ and G^A_μ are octets of vector and axial-vector gluons, respectively, written as 3×3 matrices.

The gauge-invariant Lagrangians for weak vector bosons and gluons are given by

$$\mathcal{L}_0^{\rm WVB}=-\tfrac{1}{4}\vec{W}_{L\mu\nu}\cdot\vec{W}_{L\mu\nu}-\tfrac{1}{4}\vec{W}_{R\mu\nu}\cdot\vec{W}_{R\mu\nu}-\tfrac{1}{4}B_{\mu\nu}B_{\mu\nu}, \tag{4a}$$

where

$$\vec{W}_{L\mu\nu}\equiv\partial_\nu\vec{W}_{L\mu}-\partial_\mu\vec{W}_{L\nu}-g\vec{W}_{L\mu}\times\vec{W}_{L\nu}, \tag{4b}$$

$$\vec{W}_{R\mu\nu}\equiv\partial_\nu\vec{W}_{R\mu}-\partial_\mu\vec{W}_{R\nu}-g\vec{W}_{R\mu}\times\vec{W}_{R\nu}, \tag{4c}$$

$$B_{\mu\nu}\equiv\partial_\nu B_\mu-\partial_\mu B_\nu, \tag{4d}$$

and

$$\mathcal{L}_0^G-\tfrac{1}{4}\mathrm{Tr}(G^V_{\mu\nu}G^V_{\mu\nu}+G^A_{\mu\nu}G^A_{\mu\nu}), \tag{5a}$$

where

$$G^V_{\mu\nu}=\partial_\nu G^V_\mu-\partial_\mu G^V_\nu+\frac{i}{\sqrt{2}}g_s[G^V_\mu,G^V_\nu]+\frac{i}{\sqrt{2}}g_s[G^A_\mu,G^A_\nu], \tag{5b}$$

$$G^A_{\mu\nu}=\partial_\nu G^A_\mu-\partial_\mu G^A_\nu+\frac{i}{\sqrt{2}}g_s[G^V_\mu,G^A_\nu]-\frac{i}{\sqrt{2}}g_s[G^V_\nu,G^A_\mu]. \tag{5c}$$

In order to break the gauge symmetry of group G spontaneously, we first introduce three sets of Higgs scalars

$$\phi=\begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix},\quad \phi'=\begin{bmatrix}\phi^{+\prime}\\ \phi^{0\prime}\end{bmatrix},$$

$$\phi_R=\begin{bmatrix}\phi_R^+\\ \phi_R^0\\ \phi_R^-\end{bmatrix}, \tag{6}$$

belonging, respectively, to representations (2, 1), (1, 2), and (1, 3) of the group $G_{\rm wk}$, but they are singlets under the chiral color group. Note that only φ is coupled to leptons. ϕ, ϕ', and ϕ_R are not coupled to quarks. Thus we introduce another set of scalar fields $\mathfrak{M}$ belonging to the representation $(2,\bar{2},\bar{3},3+3,\bar{3})$ of the group G. We shall not display the gauge-invariant Lagrangians involving ϕ, ϕ' and ϕ_R as they are well known. We display below the gauge-invariant coupling of $\mathfrak{M}$ to weak vector bosons, gluons, and quarks

$$\mathcal{L}_{\mathfrak{M}}=-2g_q\sum_{a=1,2}\mathrm{Tr}[\bar{q}_L^{(a)}\mathfrak{M}q_R^{(a)}]+\text{H.c.}-\tfrac{1}{2}\mathrm{Tr}[D_\mu\mathfrak{M}^\dagger D_\mu\mathfrak{M}], \tag{7a}$$

where

$$D_\mu\mathfrak{M}=\partial_\mu\mathfrak{M}-igW_{L\mu}\mathfrak{M}+ig\mathfrak{M}W_{R\mu}-\frac{i}{\sqrt{2}}g_s(G^V_\mu-\gamma_5G^A_\mu)\mathfrak{M}$$
$$+\frac{i}{\sqrt{2}}g_s\mathfrak{M}(G^V_\mu+\gamma_5G^A_\mu). \tag{7b}$$

To give masses to weak vector bosons and leptons, i.e., to break $G_{\rm wk}$ spontaneously to the $[\mathrm{U}(1)]_{\rm em}$ level or G to $[\mathrm{U}(1)]_{\rm em}\times[\mathrm{SU}(3)\times\mathrm{SU}(3)]_c$, we now give nonzero vacuum expectation values to scalar fields

$$\langle\phi\rangle\to\begin{bmatrix}0\\ \lambda\end{bmatrix},\quad \langle\phi'\rangle\to\begin{bmatrix}0\\ \lambda'\end{bmatrix},$$

$$\langle\phi_R\rangle\to\begin{bmatrix}0\\ \nu\\ 0\end{bmatrix}, \tag{8}$$

and

$$\mathfrak{M}\to\frac{1}{\sqrt{2}}I\times M, \tag{9}$$

where I is a unit 2×2 matrix in the space of $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$ and $M\equiv S+i\gamma_5P$ is a 3×3 matrix in color space. S_i and P_i, $i=0,1,\ldots,8$ are nonets of scalars and pseudoscalar fields belonging to the representation $[(\bar{3},3)+(3,\bar{3})]$ in color space. Note that

$$S=\frac{1}{\sqrt{2}}\sum_i\lambda_i^cS_i,\quad P=\frac{1}{\sqrt{2}}\sum_i\lambda_i^cP_i.$$

The weak vector bosons now acquire masses and part of this Lagrangian due to breaking envisaged in Eqs. (8) is

$$\mathcal{L}^W_{\rm mass}=-\tfrac{1}{8}\lambda^2(2g^2W^-_{L\mu}W^+_{L\mu}+g^2W_{3L\mu}{}^2-2gg'W_{3L\mu}B_\mu+g'^2B_\mu{}^2)$$
$$-\tfrac{1}{8}\lambda'^2(2g^2W^-_{R\mu}W^+_{R\mu}+g^2W_{3R\mu}{}^2-2gg'W_{3R\mu}B_\mu+g'^2B_\mu{}^2)$$
$$-\tfrac{1}{8}\nu^2(2g^2W^-_{R\mu}W^+_{R\mu}). \tag{10}$$

Thus we see that in this part of the Lagrangian, there is no mixing between $W^{\pm}_{L\mu}$ and $W^{\pm}_{R\mu}$. The Lagrangian (7) due to symmetry breaking as implied in Eq. (9) now takes the form

$$\mathcal{L}_{\mathfrak{M}} \to -\sqrt{2}\,g_q \bar{\zeta}_\alpha M \zeta_\alpha - \tfrac{1}{2}\,\mathrm{Tr}(D_\mu M^\dagger D_\mu M) - \tfrac{1}{4} g^2\,\mathrm{Tr}[(W_{L\mu} - W_{R\mu})(W_{L\mu} - W_{R\mu})] \times \mathrm{Tr}[M^\dagger M], \tag{11a}$$

where

$$\zeta_\alpha = \begin{bmatrix} \alpha_b \\ \alpha_r \\ \alpha_y \end{bmatrix}, \quad \alpha = c, u, d, s \tag{11b}$$

and

$$D_\mu M = \partial_\mu M - \frac{i}{\sqrt{2}} g_s (G^V_\mu - \gamma_5 G^A_\mu) M + \frac{i}{\sqrt{2}} M (G^V_\mu + \gamma_5 G^A_\mu). \tag{11c}$$

Note that (11a) is invariant under the chiral color group. We shall come back to the third term in (11a) in the next section.

III. THE STRONG-INTERACTION (COLOR) LAGRANGIAN

From Eqs. (1), (7), and (11) we have

$$\mathcal{L}^c = \sum_{\alpha = c, u, d, s} \frac{1}{\sqrt{2}} g_s (\bar{\zeta}_\alpha \gamma_\mu G^V_\mu \zeta_\alpha + \bar{\zeta}_\alpha \gamma_\mu \gamma_5 G^A_\mu \zeta_\alpha) - \sqrt{2}\,g_q \bar{\zeta}_\alpha M \zeta_\alpha - \tfrac{1}{2}\,\mathrm{Tr}(D_\mu M^\dagger D_\mu M). \tag{12a}$$

We note that

$$-\tfrac{1}{2}\,\mathrm{Tr}(D_\mu M^\dagger D_\mu M) = -\tfrac{1}{2}\,\mathrm{Tr}(D_\mu S D_\mu S + D_\mu P D_\mu P), \tag{12b}$$

where

$$D_\mu S = \partial_\mu S - \frac{i}{\sqrt{2}} g_s [G^V_\mu, S] - \frac{1}{\sqrt{2}} g_s \{G^A_\mu, P\}, \tag{12c}$$

$$D_\mu P = \partial_\mu P - \frac{i}{\sqrt{2}} g_s [G^V_\mu, P] + \frac{1}{\sqrt{2}} g_s \{G^A_\mu, S\}. \tag{12d}$$

To the Lagrangian (12a) we must add a term

$$V(S, P) = -\tfrac{1}{2}\mu^2\,\mathrm{Tr}(mm^\dagger) - f_1[\mathrm{Tr}(mm^\dagger)]^2 - f_2\,\mathrm{Tr}(mm^\dagger mm^\dagger) - f_3(\det m + \det m^\dagger), \tag{12e}$$

where

$$m = S + iP. \tag{12f}$$

Although only $\mathrm{Tr}(mm^\dagger)$ is in general chiral invariant, but as has been emphasized in Ref. 6, a realization of the chiral symmetry can be obtained by assuming that

$$mm^\dagger = m^\dagger m, \tag{13}$$

which implies

$$S = \sum C_n P^n, \text{ or } [S, P] = 0,$$

and we may set

$$mm^\dagger = m^\dagger m = S^2 + P^2 = R^2, \tag{14}$$

where R is constant and chiral invariant. Under these circumstances, and noting that $[(1 - i\gamma_5\xi)/(1 + i\gamma_5\xi)]$ transforms in the same way as $S + i\gamma_5 P$, we can, by means of chiral transformation, transform[7]

$$\bar{\zeta}_\alpha (S + i\gamma_5 P)\zeta_\alpha \to R\bar{\zeta}'_\alpha \zeta'_\alpha, \tag{15a}$$

where

$$\zeta_\alpha \to \frac{1 + i\gamma_5\xi}{(1 + \xi^2)^{1/2}} \zeta'_\alpha, \tag{15b}$$

$$\frac{1 + i\gamma_5\xi}{(1 + \xi^2)^{1/2}} (S + i\gamma_5 P) \frac{1 + i\gamma_5\xi}{(1 + \xi^2)^{1/2}} = R, \tag{15c}$$

$$\xi = \frac{1}{\sqrt{2}} \sum_i \lambda^c_i \xi_i, \quad \xi^2 = \tfrac{1}{2}\{\xi, \xi\}. \tag{15d}$$

From this it follows that

$$S + i\gamma_5 P = R[(1 - i\gamma_5\xi)/(1 + i\gamma_5\xi)], \tag{16a}$$

$$S = R\frac{1 - \xi^2}{1 + \xi^2}, \tag{16b}$$

$$P = -R\frac{2\xi}{1 + \xi^2}, \tag{16c}$$

$$-\bar{\zeta}_\alpha \gamma_\mu \partial_\mu \zeta_\alpha + \frac{i}{\sqrt{2}} g_s \bar{\zeta}_\alpha \gamma_\mu (G^V_\mu + \gamma_5 G^A_\mu)\zeta_\alpha \to \bar{\zeta}'_\alpha \gamma_\mu \partial_\mu \zeta'_\alpha + \frac{i}{\sqrt{2}} g_s \bar{\zeta}'_\alpha \gamma_\mu (G'^V_\mu + \gamma_5 G'^A_\mu)\zeta'_\alpha, \tag{17a}$$

where

$$G'^V_\mu + \gamma_5 G'^A_\mu = U(G^V_\mu + \gamma_5 G^A_\mu + \sqrt{2}\,\frac{i}{g_s}\partial_\mu)U^\dagger, \tag{17b}$$

$$U = \frac{1 - i\gamma_5\xi}{(1 + \xi^2)^{1/2}}, \quad U^\dagger = \frac{1 + i\gamma_5\xi}{(1 + \xi^2)^{1/2}}, \tag{17c}$$

$$UU^\dagger = 1. \tag{17d}$$

From Eqs. (16a) and (17c) we have

$$S + i\gamma_5 P = RU^2, \tag{18a}$$

$$S - i\gamma_5 P = RU^{2\dagger}. \tag{18b}$$

In terms of new fields,

$$-\tfrac{1}{2}\,\mathrm{Tr}(D_\mu S D_\mu S + D_\mu P D_\mu P) + V(S, P) \to -\tfrac{1}{2} R^2\,\mathrm{Tr}(D_\mu U^2 D_\mu U^{2\dagger}) + V(\xi), \tag{19a}$$

where

$$D_\mu U^2 = \partial_\mu U^2 + i\sqrt{2}\,g_s U\gamma_5 G'^A_\mu U - (\partial_\mu U)U + U^2(\partial_\mu U^\dagger)U, \tag{19b}$$

$$D_\mu U^{2\dagger} = \partial_\mu U^{2\dagger} + i\sqrt{2} g_s U^\dagger \gamma_5 G_\mu'^A U^\dagger - (\partial_\mu U^\dagger) U^\dagger + U^{2\dagger}(\partial_\mu U) U^\dagger , \quad (19c)$$

$$V(\xi) = -\tfrac{1}{2}\mu^2(3R) - 9f_1R^2 - 3f_2R^2 - f_3R[\det(1-i\xi)/(1+i\xi) + \det(1+i\xi)/(1-i\xi)] . \quad (19d)$$

Now dropping the prime, we have from Eqs. (17a) and (19) the strong- (color) interaction Lagrangian

$$\mathcal{L}_q^c = -\sum_\alpha \left(\bar{\zeta}_\alpha \gamma_\mu \partial_\mu \zeta_\alpha + \frac{i}{\sqrt{2}} g_s \bar{\zeta}_\alpha \gamma_\mu (G_\mu^V + \gamma_5 G_\mu^A)\zeta_\alpha\right) - \sqrt{2} g_q R \bar{\zeta}_\alpha \zeta_\alpha - \tfrac{1}{2}R^2 \operatorname{Tr}(D_\mu U^2 D_\mu U^{2\dagger}) + V(\xi) . \quad (20)$$

It is now clear from Eqs. (20), (19b), and (19c) that quarks have acquired a common mass $\sqrt{2} g_q R$, vector gluons G_μ^V remain massless, and axial-vector gluons G_μ^A have acquired a common mass

$$m_{G^A}{}^2 = 2g_s{}^2 R^2 . \quad (21)$$

We have obtained the remarkable result that the color-carrying nonets of scalar and pseudoscalar mesons are decoupled from the quarks; the quarks are massive and asymptotically free[8] as the gauge coupling constant g_s becomes smaller and smaller at short distances. The asymptotic freedom also leads to the fact that axial-vector gluons become lighter and lighter at short distances. For large distances, since vector gluons are massless, the multi-vector-gluon exchange can supply the necessary confining potential. The axial-vector gluon can give an effective spin-dependent potential which may be helpful in explaining the hyperfine splitting of charmonium states, which cannot be understood from the vector-gluon exchange alone.

Alternatively, one can understand the confinement of quarks[9] in a hadron in that R may be small inside a hadron so that quarks are light inside a hadron. R may become large as the distance increases, so that quarks become very massive when the distance becomes large as compared with the Compton wavelength of a hadron. It is conceivable that both mechanisms are not mutually exclusive and may exist together.

Finally we note that the last term in (11a) now takes the form

$$= -\frac{1}{2}\frac{3g^2}{4}R^2[2(W^+_{L\mu} - W^+_{R\mu})(W^-_{L\mu} - W^-_{R\mu}) + (W_{3L\mu} - W_{3R\mu})(W_{3L\mu} - W_{3R\mu})] . \quad (22)$$

From this term we see that the weak-vector-boson mass matrix has acquired a value $(3g^2/4)R^2$. We note that the mass of the axial-vector gluon is

$$m_{G^A}{}^2 = 2\alpha_s 4\pi R^2 . \quad (23a)$$

Taking $\alpha_s \approx 0.2$ and $m_{G^A}{}^2 \leq 100$ GeV^2, we have

$$R^2 \leq 21 \text{ GeV}^2 . \quad (23b)$$

In comparison, we note that

$$\lambda^2 \sim (\sqrt{2}\, G_F)^{-1} \sim \frac{1}{\sqrt{2}} 10^5 \text{ GeV}^2 .$$

Hence we see that $R^2/\lambda^2 \ll 1$. This is true even if m_{G^A} is as large as 100 GeV. Hence we can safely neglect the contribution given in Eq. (22) for weak vector bosons.

IV. THE WEAK-INTERACTION LAGRANGIAN

From Eq. (1) it is clear that the weak-interaction Lagrangian for quarks can be written as

$$\mathcal{L}_q^{W,em} = -\sum_n \sum_{a=1,2} [\bar{\psi}_{Ln}^{(a)} \gamma_\mu(-igW_{L\mu} - \tfrac{1}{6} ig' B_\mu)\psi_{Ln}^{(a)} + \bar{\psi}_{Rn}^{(a)} \gamma_\mu(-igW_{R\mu} - \tfrac{1}{6} ig' B_\mu)\psi_{Rn}^{(a)}] , \quad (24a)$$

where

$$\psi_n^{(1)} = \begin{pmatrix} u_n \\ d_n' \end{pmatrix}, \quad \psi_n^{(2)} = \begin{pmatrix} c_n \\ s_n' \end{pmatrix}, \quad n = b, r, y. \quad (24b)$$

From Eqs. (24a) and (24b) we have

$$\mathcal{L}_q^{W,em} = \frac{g}{2\sqrt{2}}(J_{L\mu}^W W^-_{L\mu} + J_{R\mu}^W W^-_{R\mu} + \text{H.c.}) + \tfrac{1}{4}g(J_{L\mu}^{W^0} W_{3L\mu} + J_{R\mu}^{W^0} W_{3R\mu}) + \tfrac{1}{6}g' V_\mu^I B_\mu , \quad (25a)$$

where

$$J_{L,R\mu}^W = \sum_n i\bar{q}_n \gamma_\mu (1 \pm \gamma_5) W q_n , \quad (25b)$$

$$J_{L,R\mu}^{W^0} = \sum_n i\bar{q}_n \gamma_\mu (1 \pm \gamma_5) W^0 q_n , \quad (25c)$$

$$V_\mu^I = \sum_n i\bar{q}_n \gamma_\mu q_n . \quad (25d)$$

Here

$$q = \begin{bmatrix} c \\ u \\ d \\ s \end{bmatrix}, \quad (26a)$$

$$W = \begin{pmatrix} 0 & \omega \\ 0 & 0 \end{pmatrix}, \quad \omega = \begin{pmatrix} -\sin\theta_C & \cos\theta_C \\ \cos\theta_C & \sin\theta_C \end{pmatrix}, \quad (26b)$$

$$W^0 = [W, W^\dagger] . \quad (26c)$$

We now consider the weak-interaction Lagrangian for leptons. For the representation content of leptons as given previously,

$$\mathcal{L}_l^{W,\mathrm{em}} = -\overline{L}_l\gamma_\mu(-igW_{L\mu}+\tfrac{1}{2}ig'B_\mu)L_l - \overline{R}_l\gamma_\mu(ig'B_\mu)R_l$$
$$= \frac{ig}{2\sqrt{2}}[\overline{\nu}_l\gamma_\mu(1+\gamma_5)lW^-_{L\mu} + \mathrm{H.c.}] + \tfrac{1}{4}ig[\overline{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l - \overline{l}\gamma_\mu(1+\gamma_5)l]W_{3L\mu}$$
$$-\tfrac{1}{4}ig'[\overline{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l + \overline{l}\gamma_\mu(1+\gamma_5)l]B_\mu - \tfrac{1}{2}ig'[l\gamma_\mu(1-\gamma_5)l]B_\mu . \quad (27)$$

By neglecting the term (22), as discussed in the previous section, the Lagrangian for the masses of weak vector bosons is given in Eq. (10). Accordingly,

$$m_{W_L}{}^2 = \tfrac{1}{4}g^2\lambda^2 ,$$
$$m_{W_R}{}^2 = \tfrac{1}{4}g^2(\lambda'^2 + \nu^2) , \quad (28a)$$

while for the neutral vector bosons, the mass matrix is given by

$$M^{\mathrm{neutral}} = \frac{1}{4}\begin{bmatrix} g^2\lambda^2 & 0 & -gg'\lambda^2 \\ 0 & g^2\lambda'^2 & -gg'\lambda'^2 \\ -gg'\lambda^2 & -gg'\lambda'^2 & g'^2(\lambda^2+\lambda'^2)\end{bmatrix} . \quad (28b)$$

Let us denote the physical neutral vector bosons by A_μ, $Z_{1\mu}$, and $Z_{2\mu}$:

$$A_\mu = \cos\alpha Z_{V\mu} - \sin\alpha B_\mu ,$$
$$Z_{1\mu} = \cos\beta(\sin\alpha Z_{V\mu} + \cos\alpha B_\mu) + \sin\beta Z_{A\mu} , \quad (29a)$$
$$Z_{2\mu} = -\sin\beta(\sin\alpha Z_{V\mu} + \cos\alpha B_\mu) + \cos\beta Z_{A\mu} ,$$

where

$$Z_{V\mu} = \frac{W_{3L\mu}+W_{3R\mu}}{\sqrt{2}}, \quad Z_{A\mu} = \frac{W_{3L\mu}-W_{3R\mu}}{\sqrt{2}}. \quad (29b)$$

The diagonalization gives

$$m_A{}^2 = 0 , \quad (30a)$$

$$m_{Z_2,Z_1}{}^2 = \frac{g^2}{16}(\lambda^2+\lambda'^2) \times\left[\frac{1+\sin^2\alpha}{\sin^2\alpha} \mp \frac{\cos^2\alpha}{\sin^2\alpha}(1+4\rho^2)^{1/2}\right] , \quad (30b)$$

where

$$\tan\beta = \frac{2\rho}{1+(1+4\rho^2)^{1/2}} \quad (30c)$$

and

$$\rho = -\frac{\sin\alpha}{\cos^2\alpha}\eta , \quad (30d)$$

with

$$\eta = \frac{\lambda'^2-\lambda^2}{\lambda'^2+\lambda^2} . \quad (30e)$$

Thus the vector boson A_μ can be identified with the photon. The electric charge e is then given by

$$e = \frac{gg'}{(g^2+2g'^2)^{1/2}} , \quad (31a)$$

with

$$\tan\alpha = -\frac{g}{\sqrt{2}g'} . \quad (31b)$$

In terms of physical vector bosons, the weak- and electromagnetic-interaction Lagrangian for quarks is given by

$$\mathcal{L}_q^{W,\mathrm{em}} = \frac{2}{2\sqrt{2}}(J^W_{L\mu}W^-_{L\mu} + J^W_{R\mu}W^-_{R\mu} + \mathrm{H.c.}) + eJ^{\mathrm{em}}_\mu A_\mu$$
$$+\frac{1}{4\sqrt{2}}\{-(g^2+2g'^2)^{1/2}\cos\beta[(J^{W0}_{L\mu}+J^{W0}_{R\mu}) - 4\cos^2\alpha J^{\mathrm{em}}_\mu] + g\sin\beta[(J^{W0}_{L\mu}-J^{W0}_{R\mu})]\}Z_{1\mu}$$
$$+\frac{1}{4\sqrt{2}}\{(g^2+2g'^2)^{1/2}\sin\beta[(J^{W0}_{L\mu}+J^{W0}_{R\mu}) - 4\cos^2\alpha J^{\mathrm{em}}_\mu] + g\cos\beta(J^{W0}_{L\mu}-J^{W0}_{R\mu})\}Z_{2\mu} , \quad (32a)$$

where

$$J^{\mathrm{em}}_\mu = \tfrac{1}{4}(J^{W0}_{L\mu}+J^{W0}_{R\mu}) + \tfrac{1}{6}V^I_\mu . \quad (32b)$$

For leptons, the interaction Lagrangian in terms of physical fields is given by

$$\mathcal{L}_l^{W,\mathrm{em}}=\frac{ig}{2\sqrt{2}}[\bar{\nu}_l\gamma_\mu(1+\gamma_5)lW_{L\mu}^-+\mathrm{H.c.}]-ie\bar{l}\gamma_\mu lA_\mu$$
$$+\frac{i}{4\sqrt{2}}\{-(g^2+2g'^2)^{1/2}\cos\beta[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l-\bar{l}\gamma_\mu\gamma_5 l+(4\cos^2\alpha-1)\bar{l}\gamma_\mu l]$$
$$+g\sin\beta[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l-\bar{l}\gamma_\mu(1+\gamma_5)l]\}Z_{1\mu}$$
$$+\frac{i}{4\sqrt{2}}\{(g^2+2g'^2)^{1/2}\sin\beta[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l-\bar{l}\gamma_\mu\gamma_5 l+(4\cos^2\alpha-1)\bar{l}\gamma_\mu l]$$
$$+g\cos\beta[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l-\bar{l}\gamma_\mu(1+\gamma_5)l]\}Z_{2\mu}\,. \tag{33}$$

First we note from Eqs. (32) and (33) that for charged currents

$$H_W^{\mu e}=\frac{G_F}{\sqrt{2}}\{[\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\mu][\bar{e}\gamma_\mu(1+\gamma_5)\nu_e]\}+\mathrm{H.c.}\,, \tag{34a}$$

$$H_W^{lh}=\frac{iG_F}{\sqrt{2}}\{[\bar{l}\gamma_\mu(1+\gamma_5)\nu_l]J_{L\mu}^W+\mathrm{H.c.}\}\,, \tag{34b}$$

where

$$\frac{G_F}{\sqrt{2}}=\frac{g^2}{8m_{W_L}^{\ 2}}=\frac{1}{2\lambda^2}\,. \tag{34c}$$

From Eq. (32), the effective Hamiltonian for weak hadronic decays is given by

$$H_W^{hh}=\frac{G_F}{\sqrt{2}}\left(J_{L\mu}^W\bar{J}_{L\mu}^W+\frac{m_{W_L}^{\ 2}}{m_{W_R}^{\ 2}}J_{R\mu}^W\bar{J}_{R\mu}^W+\mathrm{H.c.}\right)\,. \tag{35}$$

This Hamiltonian is parity conserving if $m_{W_L}^{\ 2}=m_{W_R}^{\ 2}$. This cannot be true; on the other hand, to reproduce the experimental result, we must have

$$m_{W_R}^{\ 2}\gg m_{W_L}^{\ 2} \tag{36a}$$

or

$$(\lambda'^2+\nu^2)\gg\lambda^2\,. \tag{36b}$$

From Eqs. (34a) and (34b), we see that for charged weak currents we get exactly the same result as in the standard model. This is a consequence of the fact that we have taken the neutrino as two component left-handed objects. We have the result that, irrespective of whether the hadronic (quark) sector is parity conserving or not, the basic semileptonic weak Hamiltonian has $(V-A)$ structure for ordinary leptons.

We now discuss the neutral weak-current Hamiltonian. We consider two cases: (i) $\lambda'^2\gg\lambda^2$; i.e., $\eta\approx1$; (ii) $\lambda'^2\approx\lambda^2$, i.e., $\eta\ll1$. In the former case, we show that we can make

$$m_{Z_1}^{\ 2}\gg m_{Z_2}^{\ 2}\approx m_{W_L}^{\ 2}\,,$$

but $m_{Z_1}^{\ 2}$ may be smaller than $m_{W_R}^{\ 2}$. In this case we get for the currently available energies exactly the same results as in the standard SW model. For case (ii), $m_{Z_2}^{\ 2}\approx m_{Z_1}^{\ 2}$, and again we get the same results as in the SW model except for the y dependence of the asymmetry parameter $A=(\sigma_R-\sigma_L)/(\sigma_R+\sigma_L)$ for the scattering of polarized electrons on nucleon targets.

We now discuss case (i), where we set $\epsilon=\lambda^2/\lambda'^2\ll1$. Then to first order in ϵ, we see from Eqs. (30d) and (30b) that

$$m_{Z_2,Z_1}^{\ 2}=\frac{g^2\lambda'^2}{16\sin^2\alpha}(1+\epsilon)\times\left[(1+\sin^2\alpha)\mp(1+\sin^2\alpha)\left(1-8\frac{\sin^2\alpha}{(1+\sin^2\alpha)^2}\epsilon\right)\right]\,, \tag{37a}$$

so that

$$m_{Z_2}^{\ 2}\approx\frac{2}{1+\sin^2\alpha}m_{W_L}^{\ 2}\,, \tag{37b}$$

$$m_{Z_1}^{\ 2}\approx\frac{2g^2}{16\sin^2\alpha}\lambda'^2(1+\sin^2\alpha)\gg m_{Z_2}^{\ 2}\,, \tag{37c}$$

but smaller than $m_{W_R}^{\ 2}$. Note that the relation (37b) is the same as the Salam-Weinberg relation if we identify $\cos^2\alpha$ with $2\sin^2\theta_W$. Because of relation (37c), for "low" energies, in writing the effective Hamiltonians, we need to consider neutral weak interactions mediated by the Z_2 boson only, and these Hamiltonians, as derived from Eqs. (32a), (33), (30c), (37b), and (34c) in the limit $\epsilon\to0$, are

$$H_{\mathrm{eff}}^{N(\nu_l l)}\approx\frac{G_F}{2\sqrt{2}}[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l]\times[(2\cos^2\alpha-1)\bar{l}\gamma_\mu l-\bar{l}\gamma_\mu\gamma l]\,, \tag{38}$$

$$H_{\mathrm{eff}}^{N(\nu h)}\approx-i\frac{G_F}{\sqrt{2}}[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l]\times(\tfrac{1}{2}J_{L\mu}^{W0}-\cos^2\alpha J_\mu^{\mathrm{em}})\,, \tag{39}$$

$$H_{\mathrm{eff}}^{N(lh)}\approx-i\frac{G_F}{\sqrt{2}}[-\bar{l}\gamma_\mu\gamma_5 l+(2\cos^2\alpha-1)\bar{l}\gamma_\mu l]\times(\tfrac{1}{2}J_\mu^{W0}-\cos^2\alpha J_\mu^{\mathrm{em}})\,. \tag{40}$$

These are the same as in the SW models if we identify $\cos^2\alpha$ with $2\sin^2\theta_W$.

For case (ii), $\eta \ll 1$, so that $\rho \to 0$ and hence angle $\beta \to 0$, and we get from Eq. (30b)

$$\frac{m_{Z_1}{}^2}{m_{Z_2}{}^2} = \frac{1}{\sin^2\alpha}, \tag{41a}$$

with

$$\frac{1}{m_{Z_2}{}^2} = (1-\eta)\frac{1}{m_{W_L}{}^2}. \tag{41b}$$

In this case, we get "low energy" effective Hamiltonians for neutral weak interactions from Eqs. (32a), (33), (41), and (34c) in the limit $\beta \to 0$, as follows:

$$H_{\text{eff}}^{N(\nu_\mu e)} \approx \frac{G_F}{2\sqrt{2}}(1-\eta)[\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\nu_\mu][-\bar{e}\gamma_\mu\gamma_5 e + (2\cos^2\alpha - 1)e\gamma_\mu e], \tag{42}$$

$$H_{\text{eff}}^{N(\nu h)} \approx -i\frac{G_F}{\sqrt{2}}(1-\eta)[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l][\tfrac{1}{2}J_{L\mu}^{W^0} - \cos^2\alpha J_\mu^{\text{em}}], \tag{43}$$

$$H_{\text{eff}}^{N(eh)} \approx -i\frac{G_F}{\sqrt{2}}(1-\eta)\{[(2\cos^2\alpha - 1)\bar{e}\gamma_\mu e - \bar{e}\gamma_\mu\gamma_5 e](\tfrac{1}{2}J_{L\mu}^{W^0} - \cos^2\alpha J_\mu^{\text{em}}) + [(2\cos^2\alpha)\bar{e}\gamma_\mu e][\tfrac{1}{2}J_{R\mu}^{W^0} - \cos^2\alpha J_\mu^{\text{em}}]\}. \tag{44}$$

V. EXPERIMENTAL CONSEQUENCES OF THE SYMMETRY GROUP $G_{\text{wk}} = SU_L(2) \times SU_R(2) \times U(1)$

We now make the assumption that the matrix elements of bilinears containing s and c quarks between protons or neutrons are negligible as compared with matrix elements of bilinears containing u and d quarks. Thus in neutral weak and electromagnetic currents we retain terms containing u and d quarks only. With this approximation

$$J_{L,R\mu}^{W^0} = i[\bar{u}\gamma_\mu(1\pm\gamma_5)u - \bar{d}\gamma_\mu(1\pm\gamma_5)d], \tag{45a}$$

$$J_\mu^{\text{em}} = i(\tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d). \tag{45b}$$

If we write the effective Hamiltonian[1]

$$H_{\text{eff}}^{N(\nu h)} = \frac{G_F}{\sqrt{2}}[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l] \times [u_L\bar{u}\gamma_\mu(1+\gamma_5)u + d_L\bar{d}\gamma_\mu(1+\gamma_5)d + u_R\bar{u}\gamma_\mu(1-\gamma_5)u + d_R\bar{d}\gamma_\mu(1-\gamma_5)d], \tag{46a}$$

then several authors have determined, from the experimental data on deep-inelastic scattering of ν_μ on the deuteron target, u_L, u_R, d_L, and d_R. Their results are given below[1]:

$$\begin{aligned} u_L &= 0.35 \pm 0.07, \\ d_L &= -0.40 \pm 0.07, \\ u_R &= -0.19 \pm 0.06, \\ d_R &= 0.0 \pm 0.11. \end{aligned} \tag{46b}$$

For the L-R model with $\eta \approx 1$, we have from Eqs. (39), (45), and (46)

$$\begin{aligned} u_L &= \tfrac{1}{2}(1 - \tfrac{2}{3}\cos^2\alpha), \\ u_R &= (-\tfrac{1}{3}\cos^2\alpha), \\ d_L &= -\tfrac{1}{2}(1 - \tfrac{1}{3}\cos^2\alpha), \\ d_R &= \tfrac{1}{6}\cos^2\alpha. \end{aligned} \tag{47}$$

For the case $\eta \ll 1$, we have from Eqs. (43), (45), and (46)

$$\begin{aligned} u_L &= (1-\eta)\tfrac{1}{2}(1 - \tfrac{2}{3}\cos^2\alpha), \\ u_R &= (1-\eta)(-\tfrac{1}{3}\cos^2\alpha), \\ d_L &= -(1-\eta)\tfrac{1}{2}(1 - \tfrac{1}{3}\cos^2\alpha), \\ d_R &= (1-\eta)\tfrac{1}{6}\cos^2\alpha. \end{aligned} \tag{48}$$

The predictions for the asymmetry parameter A for the scattering of polarized electrons on deuteron targets for the two cases are as follows:

$$\text{(a)}\ \eta \approx 1:\ A \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{9G_F q^2}{20\sqrt{2}\,\pi\alpha}\left[(1 - \tfrac{10}{9}\cos^2\alpha) + (1 - 2\cos^2\alpha)\frac{1-(1-y)^2}{1+(1-y)^2}\right], \tag{49}$$

$$\text{(b)}\ \eta \ll 1:\ A = -\frac{9G_F(1-\eta)}{20\sqrt{2}\,\pi\alpha}\left\{(1 - \tfrac{10}{9}\cos^2\alpha) + [(1 - 2\cos^2\alpha) + 2\cos^2\alpha]\frac{1-(1-y)^2}{1+(1-y)^2}\right\}. \tag{50}$$

TABLE I. Comparison of predictions of the L-R model with $\eta \approx 1$ and $\eta \approx 0$ with the experimental data for deep-inelastic scattering of neutrinos and polarized electrons on deuterons. Note that the case $\eta \approx 1$ with $\cos^2\alpha = 2\sin^2\theta_W$ is identical with the SW model.

	Experimental values	$\cos^2\alpha = 0.4$ $\eta \approx 1$ ($\eta \approx 0$)	$\cos^2\alpha = 0.4$ ($\eta = 0.2$)	$\cos^2\alpha = 0.5$ $\eta \approx 1$ ($\eta \approx 0$)	$\cos^2\alpha = 0.5$ ($\eta = 0.2$)
u_L	0.35 ± 0.07	0.37	0.30	0.33	0.26
u_R	-0.19 ± 0.06	−0.13	−0.10	−0.17	−0.14
d_L	-0.04 ± 0.07	−0.43	−0.34	−0.42	−0.34
d_R	0.0 ± 0.11	0.06	0.05	0.08	0.06
A/q^2 (10^{-5} GeV^{-2}) at $y = 0.21$	-9.6 ± 1.6	−9.5		−7.0	
		(−12.4)	(−9.9)	(−10.6)	(−8.5)

The weak neutral currents can give rise to parity violation in atomic transition. The optical rotation ρ_0 for the two transitions in bismuth which have been measured is given by

$$\rho_0 \approx -4.4 \times 10^{-9} V_{\text{had}} g_A^e \text{ rad (for 8757 Å)},$$
$$\rho_0 \approx -6.0 \times 10^{-9} V_{\text{had}} g_A^e \text{ rad (for 4676 Å)} \tag{51}$$
$$(Z = 83 \text{ and } N = 126 \text{ for bismuth}),$$

where

$$V_{\text{had}} = g_u^V(2Z + N) + g_d^V(Z + 2N), \tag{52}$$

where Z and N are the number of protons and neutrons in an atom.

As can easily be seen from Eqs. (40) and (44), we have the following values for g_A^e, g_u^V, and g_d^V:

(a) $\eta \approx 1$: $g_e^A = -1$, $g_u^V = \frac{1}{2} - \frac{2}{3}\cos^2\alpha$,

$$g_d^V = -\tfrac{1}{2} + \tfrac{1}{3}\cos^2\alpha, \tag{53}$$

(b) $\eta \ll 1$: $g_e^A = -1$, $g_u^V = (1-\eta)(\frac{1}{2} - \frac{2}{3}\cos^2\alpha)$,

$$g_d^V = (1-\eta)(-\tfrac{1}{2} + \tfrac{1}{3}\cos^2\alpha). \tag{54}$$

It is clear that for the atomic parity-violation parameter ρ_0, both the spontaneous symmetry-breaking cases ($\eta \approx 1$ and $\eta \ll 1$) for the L-R model give results similar to the standard SW model.

To sum up, the predictions at "low" energies for the L-R model for both cases ($\eta \approx 1$ and $\eta \ll 1$) are exactly the same as those of the standard model except that for the case $\eta \ll 1$, the asymmetry parameter A for the deep-inelastic scattering of polarized electrons on the deuteron has a different y dependence. Even the experimental value of A at $y = 0.21$ is compatible with that predicted in this case, as can be seen from Table I. The y dependence of A for the case $\eta \ll 1$ is, how-

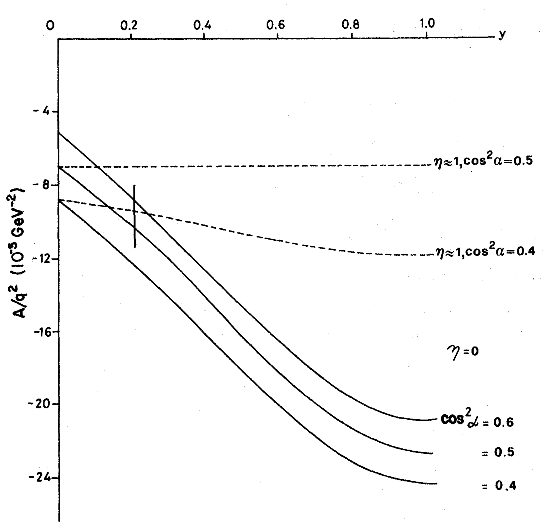

FIG. 1. The y dependence of the asymmetry parameter A.

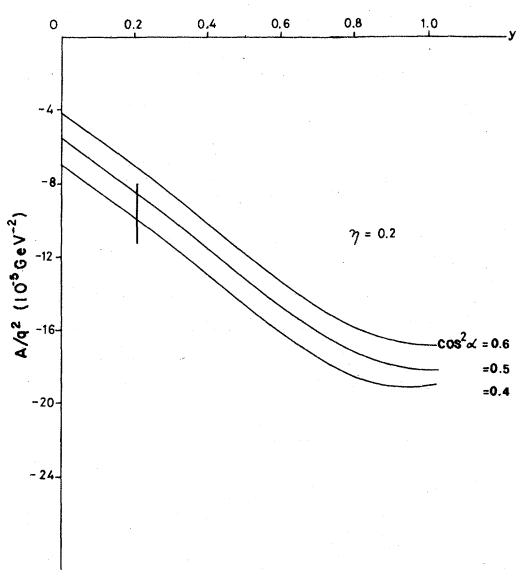

FIG. 2. The y dependence of the asymmetry parameter A for the case $\eta = 0.2$.

ever, very much different from that in the standard model or the L-R model with $\eta \approx 1$ as can be seen from Figs. 1 and 2.

Note added. After this work was submitted for publication, we were informed that the recent SLAC data seem to rule out the y dependence of the asymmetry parameter A predicted by the L-R model for the case $\eta \ll 1$.

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