Comments and Addenda

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Effective-potential approach to the quantization of the sine-Gordon theory in 1+1 dimensions

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We show that the effective-potential approach may be used to calculate the quantum-corrected mass of the one-soliton solution of the sine-Gordon theory in $1+1$ dimensions.

Nonlinear theories have been the subject of extensive investigations in the context of soliton and instanton solutions.[1,2] One of the most thoroughly investigated theories is the sine-Gordon theory in $1+1$ dimensions.[2] Within an analytical framework, N-soliton solutions for this model have been found, the equivalence with the Thirring model shown,[3] and the quantum-corrected mass of the soliton computed,[4] etc.

The quantization of the sine-Gordon theory in $1+1$ dimensions has been carried out in a qualified sense by Dashen, Hasslacher, and Neveu[4] (hereafter referred to as DHN). They have calculated the quantum corrections to the masses of the soliton states. In this note, we suggest an alternate method to calculate these quantum corrections and show that to order $\hbar$, the results we obtain are identical to those obtained by DHN.

The possibility of employing the effective-potential approach to calculate quantum corrections was suggested earlier.[5] We have used this approach to calculate quantum corrections to the soliton mass. The Lagrangian for the sine-Gordon theory is

$$\mathcal{L}(x)=\tfrac{1}{2}(\partial_\mu\phi)^2+\frac{m^4}{\lambda}\left[\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right)-1\right]. \tag{1}$$

For this Lagrangian, the vacuums are $\phi_v=0, n\pi$ ($n=1,2,\ldots$), the one-soliton solution is

$$\phi_s=\frac{m}{\sqrt{\lambda}}\tan^{-1}e^{mx}, \tag{2}$$

and the classical mass of the soliton is

$$E(\phi_s)-E(\phi_v)=\frac{8m^3}{\lambda}. \tag{3}$$

For calculating the quantum corrections to the mass of the soliton, we need the effective potential. For the case when ϕ is a constant, i.e., $\phi=\phi_v$, this is straightforward, and $V_{\rm eff}$, to one loop, turns out to be[5]

$$V_{\rm eff}=\frac{m^2\hbar}{8\pi}\left[\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right)-\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right)\ln\left(m^2\cos\frac{\sqrt{\lambda}}{m}\phi\right)\right.$$
$$\left.-m^2+m^2\ln m^2+m^2\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right)\ln\Lambda^2-m^2\ln\Lambda^2\right], \tag{4}$$

where Λ is a cutoff and we have used $c=1$. We shall use $\phi=\phi_v$ in calculating $E(\phi_v)$. We assume that the above expression for $V_{\rm eff}$ can also be used for ϕ_s, deferring for the moment to comment upon this. Before using this $V_{\rm eff}$, we need to renormalize the Lagrangian. We add a counterterm of the form $A[\cos(\sqrt{\lambda}/m)\phi-1]$ and demand $V''(\phi)|_{\phi=0}=m^2$ whence

$$\mathcal{L}^{\rm renorm}(x)=\tfrac{1}{2}(\partial_\mu\phi)^2+\frac{m^4}{\lambda}\left[\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right)-1\right]$$
$$+\frac{m^2\hbar}{8\pi}\left[\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right)\right.$$
$$\left.-\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right)\ln\cos\left(\frac{\sqrt{\lambda}}{m}\phi\right)-1\right]. \tag{5}$$

For $\sqrt{\lambda}/m$ small, it is now straightforward to see that quantum effects manifest themselves through the "scaling"

$$\frac{m^4}{\lambda} \to \frac{m^4}{\lambda} - \frac{m^2\hbar}{8\pi}. \tag{6}$$

We then obtain the quantum-corrected one-soliton mass (to order $\hbar$) using the above scaling, i.e., $8m^3/\lambda \to 8m^3/\lambda - m/\pi$ in agreement with the results of DHN.

We now comment on the assumption that the effective potential in Eq. (4) may also be used for $\phi_s(x)$. It seems that, in effect, it is equivalent to the assumption that the effective action is a local functional of $\phi(x)$, i.e., it depends on $\phi(x)$ but not its derivatives. This is apparent from the approach of Salam and Strathdee[5] for the calculation of the effective potential, which leads to the same differential equation for V_{eff} for the cases $\phi = \text{constant}$, and $\phi = \phi(x)$ with the restriction of locality on V_{eff}. We conclude by pointing out that while the method of DHN for the sine-Gordon theory in $1+1$ dimensions leads to results which may well be exact,[2] the above calculation of the mass of the soliton is true only to order $\hbar$. However, it seems that in theories where to order $\hbar$ the assumption that the effective action is a local functional of the fields is valid, the effective-potential approach provides a simpler alternative to other approaches.

[1] A. C. Scott, F. Y. F. Chu, and D. W. McLaughlin, Proc. IEEE 61, 1443 (1973).
[2] See, for example, R. Rajaraman, Phys. Rep. 21C, 5 (1975); R. Jackiw, Rev. Mod. Phys. 49, 681 (1977).
[3] S. Coleman, Phys. Rev. D 11, 2088 (1975).
[4] R. Dashen, B. Hasslacher, and A. Neveu, Phys. Rev. D 11, 3424 (1975).
[5] S. Coleman, in *Laws of Hadronic Matter*, proceedings of the 1973 International Summer School, "Ettore Majorana," Erice, Italy, edited by A. Zichichi (Academic, New York, 1975); Abdus Salam and J. Strathdee, Phys. Rev. D 9, 1129 (1974).