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Constructive evidence for flavor independence of the quark-antiquark potential

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The masses and leptonic widths of vector mesons may be used to construct quark-antiquark potentials by means of an inverse-scattering method. This technique has been applied to the charmonium system. A corresponding potential has been constructed from Υ and Υ' and found to be in substantial agreement with that constructed from ψ and ψ' if the Υ is composed of a charge $-1/3$ quark and the corresponding antiquark. This agreement may be interpreted as evidence for flavor independence of the quark-antiquark potential. Predictions for a hypothetical ζ family of higher-lying vector mesons also are noted.

I. INTRODUCTION

Bound states of heavy quarks can shed light on the strong interactions at distances within 1 fm and on the properties of the quarks themselves. The small decay widths of the ψ and Υ families have shown how feeble the strong interactions become at such short distances. The Υ family has supplied indirect but convincing evidence for a fifth quark b which is a color triplet with charge $-\frac{1}{3}$. Still, many questions remain. Do low-lying bound states of a very heavy quark and antiquark quantitatively resemble those of positronium, aside from overall mass and coupling constant scales, as predicted by perturbative quantum chromodynamics (QCD)? If heavier quarks exist, how can their properties (e.g., charge and color) be inferred from experiments on the hidden-flavor bosons that contain them?

A determination of the interquark potential which is free from theoretical biases can answer these questions. Such a description has been obtained using inverse-scattering techniques.[1] An interquark potential extracted from the ψ and ψ' masses and leptonic widths led to predictions of level spacings and leptonic widths of the Υ's in accord with experiment.[2]

The success of inverse-scattering predictions for the Υ system depended on several assumptions. The quark-antiquark potential was taken to be flavor-independent. This would not have been so, for example, if the Υ had been composed of color-sextet quarks; the potential then would have been stronger than for the color-triplet quarks in charmonium. The quarks in the Υ family were taken to have charge $\pm\frac{1}{3}$. The observed leptonic widths would not have been consistent with quarks of charge $\pm\frac{2}{3}$.[3] Under the same assumptions, many authors have succeeded in adjusting explicit potentials to fit both the ψ and Υ families.[4]

In this report we examine the assumptions of flavor independence and quark charge from another point of view. The Υ and Υ' themselves are used to construct a quark-antiquark potential, which may be compared with the potential constructed from the ψ and ψ'. The agreement between the two potentials, in the spatial region where such a comparison makes sense, provides constructive evidence for the flavor independence of the quark-antiquark potential.

The construction of a potential from Υ levels has further advantages.

(1) While the ψ and ψ' lie below charm threshold, all higher $^3S_1$ $c\bar{c}$ states can decay to pairs of charmed particles; their dynamics may undergo substantial modification as a result. We are not certain any nonrelativistic description, let alone a flavor-independent one, will survive this modification. Thus it is probably safe only to use states at or below flavor threshold to construct a quark-antiquark potential. The Υ family certainly has at least three $^3S_1$ levels below flavor threshold.[5,6] The fourth should be very close to threshold.[6] A potential based on three or four levels should contain more information than one based upon only two.[1]

(2) A potential constructed from Υ and Υ' should be more reliable at short distances than one based on ψ and ψ'; it may therefore provide the basis for a more trustworthy extrapolation to higher quark masses. We shall perform such an extrapolation to a hypothetical ζ family of vector mesons.

In the following section we review the inverse-scattering method for reflectionless potentials. In Sec. III we derive quarkonium potentials from the ψ and Υ families under several different assumptions. The results provide additional evidence in favor of the conventional charge and color assignments of the b quark, and support the idea of flavor independence of the strong interactions.

The brief concluding Sec. IV is devoted to predictions for the properties of more massive states and to summary comments.

II. REVIEW OF THE INVERSE-SCATTERING TECHNIQUE

Inverse-scattering techniques have been applied to the approximate reconstruction of confining potentials in Ref. 1, and issues of convergence have been further explored by Grosse and Martin.[7] Here we merely summarize the procedure. The reader is referred to Refs. 1 and 7 for detailed justification.

Consider the one-dimensional Schrödinger equation for a system with reduced mass μ,

$$\psi_i''(x) = [\kappa_i^2 + 2\mu V(x)]\psi_i(x) , \tag{1}$$

where primes denote derivatives with respect to the argument, with a symmetric potential

$$V(x) = V(-x) . \tag{2}$$

The odd-parity eigenfunctions of Eq. (1) are also solutions of the reduced radial equation for the s-wave problem in three dimensions.

Suppose the potential is known to have n bound states with binding energies $-\kappa_i^2/2\mu$ $(i = 1, \ldots, n)$. There is a unique symmetric reflectionless potential with just these binding energies.[1] It is

$$V(x) = -\frac{1}{\mu}\frac{d^2}{dx^2}\ln \det A , \tag{3}$$

where

$$A_{mp} = \delta_{mp} + \frac{\lambda_m \lambda_p}{\kappa_m + \kappa_p} , \tag{4}$$

$$\lambda_m = c_m e^{-\kappa_m x} , \tag{5}$$

and

$$\frac{c_m^2}{2\kappa_m} = \prod_{i \neq m} \left| \frac{\kappa_m + \kappa_i}{\kappa_m - \kappa_i} \right| . \tag{6}$$

The wave functions satisfy

$$\sum_{p=1}^{n} A_{mp}\, \psi_p(x) = \lambda_m(x) , \tag{7}$$

and are given explicitly by

$$\psi_m(x) = -\frac{1}{\lambda_m}\frac{\det A^{(m)}}{\det A} , \tag{8}$$

where

$$A_{ij}^{(m)} = \begin{cases} A_{ij} & (j \neq m) \\ -\lambda_i \lambda_j & (j = m) \end{cases} . \tag{9}$$

The potential given by (3) vanishes at $x = \pm\infty$. In order to approximate a confining potential, the absolute energies E_i must be related to binding energies by a free parameter E_0

$$E_i = E_0 - \kappa_i^2/2\mu . \tag{10}$$

For the n-level problem, the choice

$$E_0 = \tfrac{1}{2}(E_n + E_{n+1}) \tag{11}$$

gives excellent approximations to harmonic-oscillator, linear, and square-well potentials.[1]

These methods were applied to three-dimensional s-wave systems in the second of Refs. 1. The observed levels E_m $(m = 1, 2, \ldots, n)$ correspond to odd-parity levels in one-dimensional problem. Together with the parameter E_0 and the quark mass 2μ they fix the values of κ_2, $\kappa_4, \ldots,$ κ_{2n}. For the three-dimensional problem, the even-parity levels are unphysical. In the case of the spin-triplet quarkonium potential, equivalent information comes indirectly from the leptonic widths of the physical levels, which are related to the wave functions at the origin of the physical levels by[8]

$$\Gamma(\upsilon, {}^3S_1 \to e^+e^-) = \tfrac{16}{3}\pi \frac{N e_Q^2 \alpha^2 |\Psi(0)|^2}{M_v^2} . \tag{12}$$

Here N is the number of colors (3 for ordinary quarks), e_Q is the quark charge in units of the proton's charge, $\alpha \simeq \frac{1}{137}$ is the fine structure constant, M_v is the mass of the vector meson υ, and $|\Psi(0)|^2$ is the square of the (three-dimensional) wave function at the origin. The three-dimensional wave function Ψ is connected to the one-dimensional wave functions ψ which appear in Eqs. (7) and (8) by

$$|\Psi_m(0)|^2 = (\tfrac{1}{2}\pi)|\psi'_{2m}(0)|^2 . \tag{13}$$

In other words, the s-wave wave function at the origin in the three-dimensional problem is proportional to the slope of the reduced radial wave function. Knowledge of the positions and wave functions at the origin of the physical levels in the three-dimensional problem uniquely determines a potential which is symmetrical and reflectionless in one dimension. Given values of $|\Psi_m(0)|^2$ $(m = 1, 2, \ldots, n)$ may be reproduced by the choice of the n parameters κ_1, $\kappa_3, \ldots, \kappa_{2n-1}$ which govern the positions of the unphysical (even parity in one dimension) levels. This choice is unique;[1,7] it is accomplished in practice using an iterative numerical procedure.

III. QUARKONIUM POTENTIALS

The ψ and ψ' masses and leptonic widths shown in Table I were used in Ref. 1 to reconstruct the interquark potential. The quark mass $m_Q = 2\mu$ was varied between 1.1 and 1.5 GeV/c^2 for the charmonium system. Values of E_0 ranged between 3.75 GeV (just above the ψ') and 3.9 GeV. This last value is well above what one might expect

TABLE I. ψ and ψ' parameters used in reconstructing the interquark potential. (See Ref. 1.)

State	Mass (GeV/c^2)	Γ_{ee} (keV)	$\lvert\Psi(0)\rvert^2$ (GeV^3) Color 3, $e_Q=\frac{2}{3}$
ψ	3.095	4.8±0.6	0.0386
ψ'	3.684	2.1±0.3	0.0240

from Eq. (11), since $\psi(3684)$ corresponds to E_4 and $\psi(4028)$ corresponds to E_6.

Potentials resulting from these choices of E_0 and m_c were illustrated in Ref. 1.[9] The various potentials led to diverse predictions for such quantities as $M(\chi_c)$ (the spin-averaged position of the ^3P_J charmonium levels) and $M(\Upsilon')-M(\Upsilon)$.[1] The two monotonic potentials which describe these quantities correctly correspond to $E_0=3.8$ GeV and $m_c=1.1$ GeV/c^2 or $E_0=3.85$ GeV and $m_c=1.2$ GeV/c^2. The predictions they entail are summarized in Table II. The subsequently observed leptonic width of the Υ favors the first set of parameters, provided that $e_Q=-\frac{1}{3}$, $N=3$ for the quark in Υ. The corresponding potential is shown in Fig. 1.

The ψ, χ, and ψ' levels are shown on the left-hand side of Fig. 1, while those for the Υ system are shown on the right. The latter are obtained by solving the Schrödinger equation with a quark mass $m_b=4.554$ GeV/c^2, chosen to reproduce the mass of the Υ ground state. The right-hand scale is correspondingly $2(m_b-m_c)=6.908$ GeV higher than the left.

The Υ levels lie lower in the potential than the charmonium levels. If threshold occurs at a fixed interquark separation, as implied by the semi-

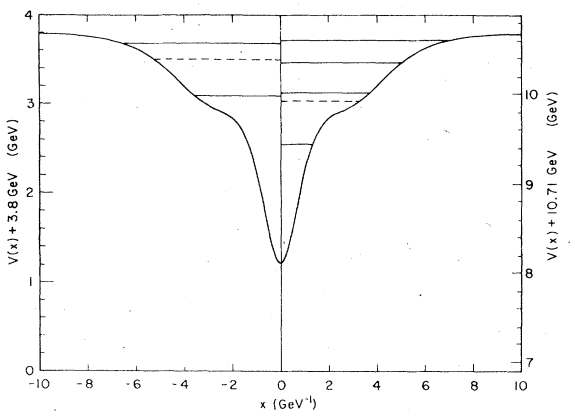

FIG. 1. Interquark potential constructed from the masses and leptonic widths of $\psi(3.095)$ and $\psi'(3.684)$, with $E_0=3.8$ GeV and $m_c=1.1$ GeV/c^2. The levels of charmonium are indicated on the left-hand side of the graph, while those of the Υ family are on the right-hand side. The solid lines denote $^3S_1$ levels; dashed lines indicate the 2^3P_J levels.

classical arguments of Ref. 5, the flavor threshold in the Υ system [above which dissociation into a $(\mathcal{B}\bar{\mathcal{B}})$ pair of mesons carrying the quantum number of the b quark is energetically allowed] should be at

$$2m_B \simeq 2m_D + 2(m_b - m_c) \simeq 10.64 \text{ GeV}. \qquad (14)$$

This is just about the expected position of the fourth Υ level. By contrast, flavor threshold lies just above the second level of the charmonium system.

A trend toward a more Coulomb-type interaction for heavier quarks may be seen in the Υ system

TABLE II. Extrapolations to Υ system on the basis of ψ and ψ' using inverse-scattering methods.

	$m_c=1.1$ GeV/c^2 $E_0=3.8$ GeV	$m_c=1.2$ GeV/c^2 $E_0=3.85$ GeV	Experiment
Υ M (GeV/c^2)	9.46 (Input)	9.46 (Input)	9.46 [a]
Γ_{ee} (keV) [b]	1.19	0.68	1.2 ± 0.2 [a]
Υ' M (GeV/c^2)	10.04	10.02	10.02 ± 0.01 [a]
Γ_{ee} (keV) [b]	0.31	0.26	0.33 ± 0.10 [a]
Υ'' M (GeV/c^2)	10.38	10.33	10.38 [c]
Γ_{ee} (keV) [b]	0.33	0.30	
Υ''' M (GeV/c^2)	10.63	10.60	
Γ_{ee} (keV) [b]	0.18	0.18	
$M_{\Upsilon'}-\langle M_{\chi_b}\rangle$ (GeV/c^2)	0.09	0.16	
m_b (GeV/c^2)	4.554	4.611	

[a] Based on a compilation (Ref. 10) of data in Ref. 2.

[b] $e_Q=-\frac{1}{3}$, $N=3$ is assumed. Other predictions may be obtained by suitable changes in Eq. (12).

[c] Reference 11.

TABLE III. Υ and Υ' parameters used in reconstructing the interquark potential.

State	Mass (GeV/c^2)	Γ_{ee} (keV)	$\lvert\Psi(0)\rvert^2$ (GeV^3) Color 3 $e_Q=-\frac{1}{3}$	Color 6 $e_Q=-\frac{1}{3}$	Color 3 $e_Q=\frac{2}{3}$
Υ	9.46	1.26	0.38	0.19	0.095
Υ'	10.02	0.36	0.12	0.06	0.03

as a predicted decrease in the $2S$-$2P$ splitting (91 MeV vs 162 MeV for charmonium). The decrease of the experimental ratio $|\Psi_{2S}(0)|^2/|\Psi_{1S}(0)|^2$ from charmonium ($\simeq 0.6$) to Υ ($\simeq 0.3$) also may indicate the onset of Coulomb behavior at short distances.[12]

We have tested the stability and self-consistency of the inverse method by constructing a potential directly from Υ and Υ' according to the color triplet $e_Q=-\frac{1}{3}$ assignment of Table III. The mass of $\Upsilon''(10.38$ GeV) is used to constrain E_0. Since Υ' (10.02) corresponds to E_4 and Υ'' to E_6, linear interpolation places

$$E_0 \simeq \tfrac{1}{4}(3E_4+E_6) \simeq 10.1 \text{ GeV}.$$

We have examined $E_0=10.05$, 10.1, 10.15, and 10.2 GeV and find that 10.1 GeV results in the smoothest potential. This value will be taken for illustration. The quark mass is taken as $m_b=4.5$ GeV/c^2, but variations from 4.4 to 4.9 GeV/c^2 have little effect. The result is shown as the solid line in Fig. 2, where it is compared with the charmonium potential of Fig. 1 (dashed curve).

The two potentials agree between 0.5 and 4 GeV^{-1}. Beyond the radius of Υ', the potential constructed from Υ and Υ' approaches E_0, because higher levels have been ignored. Very near the origin, the potential constructed from Υ and Υ' is a bit deeper than the charmonium potential. The heavier quarks in the Υ probe shorter distances and provide increased sensitivity to any (Coulomb) singularity that may be present.

The agreement between the charmonium and Υ potentials thus provides constructive evidence for flavor independence of the interquark potential, as long as the Υ is composed of a charge $-\frac{1}{3}$ color-triplet quark and the corresponding antiquark. This is a principal result of the present analysis.

The lines on Fig. 2 show the Υ and Υ' masses, which served as input, and the average $(2^3P_J)\chi_b$ mass, which was obtained by solving the Schrödinger equation. It is 95 MeV below the Υ' (to be compared with 91 MeV obtained above from the charmonium potential). The Υ'-$\langle\chi_b\rangle$ spacing is quite insensitive to the b-quark mass (in the range $4.4 \leq m_b \leq 4.9$ GeV/c^2) and varies from about 75 MeV for $E_0=10.2$ GeV to about 130 MeV for $E_0=10.05$ GeV.

The potential of Fig. 2 was constructed from the leptonic widths of Υ and Υ' under the assumption of color-triplet, charge $-\frac{1}{3}$ quarks. This assumption may be modified, and thereby tested. Table III contains the values of $|\Psi(0)|^2$ for other assignments: color sextets, $e_Q=-\frac{1}{3}$; and color triplets, $e_Q=\frac{2}{3}$. The corresponding potentials are shown in Fig. 3. The smaller values of $|\Psi(0)|^2$ have generated humps near $r=0$; the potentials look nothing like the acceptable charmonium potentials. A potential between sextets should be stronger at the origin than one between triplets; the potential of Fig. 3(a) is weaker near the origin. A color-sextet assignment for the b quark[13] is made implausible by the nonobservation[14] of stable charged particles with masses near 5 GeV/c^2. Evasions of this argument (which is reviewed in Ref. 12) may be imagined,[15] but these do not answer the objections raised here to the putative potential binding sextet quarks.

We now venture beyond existing data to investigate the stability of our results and to show how establishing the properties of another Υ level would make it possible to extend the present analysis. Three potentials constructed from Υ, Υ' and an assumed Υ'' (10.38) (see Table IV), with $E_0=10.45$ GeV, $m_b=4.5$ GeV/c^2, $e_Q=-\frac{1}{3}$, $N=3$, and

$$\Gamma(\Upsilon'' \to e^+e^-) = \begin{cases} \text{(a) } 0.19 \text{ keV}, \\ \text{(b) } 0.28 \text{ keV}, \\ \text{(c) } 0.36 \text{ keV}, \end{cases} \tag{15}$$

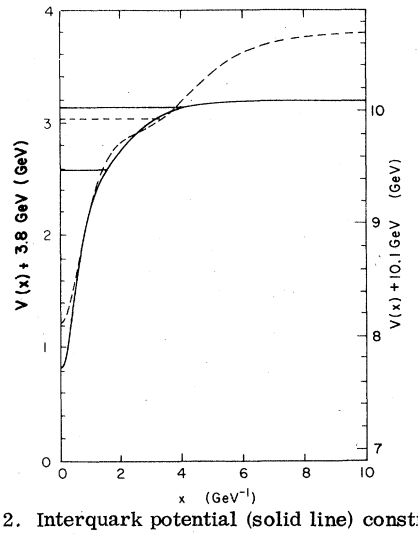

FIG. 2. Interquark potential (solid line) constructed from Υ and Υ' with $E_0=10.1$ GeV, $m_b=4.5$ GeV/c^2, and values of $|\Psi(0)|^2$ given in Table III. Here $e_Q=-\frac{1}{3}$, $N=3$. The charmonium potential of Fig. 1 is indicated by the dashed line for comparison. The $1S$ and $2S$ Υ levels are horizontal solid lines; the $2P$ $\langle\chi_b\rangle$ level is the horizontal dashed line. Relative scales of charmonium and Υ levels are shifted as in Fig. 1.

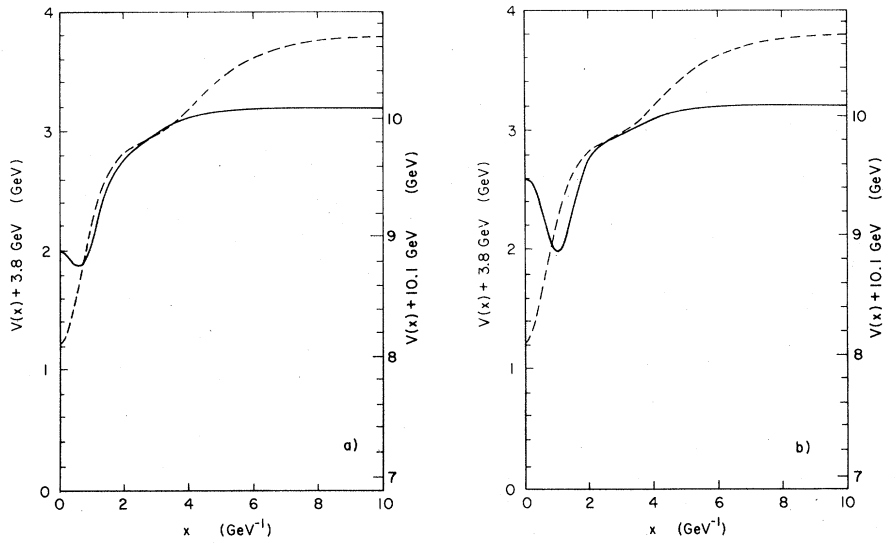

FIG. 3. Interquark potentials constructed from Υ and Υ' with $E_0 = 10.1$ GeV, $m_b = 4.5$ GeV/c^2, and (a) $e_Q = -\frac{1}{3}$, $N = 6$; (b) $e_Q = \frac{2}{3}$, $N = 3$. Dashed lines indicate the charmonium potential of Fig. 1.

are shown in Fig. 4. Each is compared with the charmonium potential for $E_0 = 3.8$ GeV, $m_c = 1.1$ GeV/c^2. If the potential is required to be monotonic, the extreme values (a) and (c) are excluded, while (b) is acceptable. The potential then is not modified appreciably at short distances by the addition of Υ'', while the agreement between the Υ and charmonium potential is extended to larger distances.

IV. OUTLOOK

The charmonium potential depicted in Fig. 1 made possible useful extrapolations to the Υ family. In a similar fashion, the Υ potential shown in Fig. 2 may be used to extrapolate to more massive quarkonium systems. On the left-hand side of Fig. 5 we show the positions of Υ levels (1^3S_1, 2^3P_J, 2^3S_1) in this potential. On the right-hand side of Fig. 5 are shown the energy levels obtained by solving the Schrödinger equation with a hypothetical quark mass $m_Q = 15$ GeV/c^2. We denote the hypothetical family ζ.

TABLE IV. Υ'' parameters used in reconstructing the interquark potentials of Fig. 4.

Γ_{ee} (keV)	$\lvert\Psi(0)\rvert^2$ (GeV^3) Color 3, $e_Q = -\frac{1}{3}$
(a) 0.19	0.07
(b) 0.28	0.10
(c) 0.36	0.13

The $2S$-$1S$ (ζ'-ζ) spacing is 686 MeV, somewhat larger than $M_{\psi'} - M_\psi$ or $M_{\Upsilon'} - M_\Upsilon$. Can this increase with m_Q be taken as evidence for a short-distance Coulomb singularity in the interquark potential? If so, the predicted $2S$-$2P$ spacing and the ratio $|\Psi_{2S}(0)|^2/|\Psi_{1S}(0)|^2$ should decrease in the passage from the Υ to the ζ family. Surprisingly, neither does. The $\zeta(2S)$-$\zeta(2P)$ spacing is 171 MeV (vs 95 MeV for Υ), and the ratio

$$|\Psi_{2S}(0)|^2/|\Psi_{1S}(0)|^2 = 0.42 \quad (\zeta \text{ family}) \tag{16}$$

is larger for the ζ family than for the Υ's:

$$|\Psi_{2S}(0)|^2/|\Psi_{1S}(0)|^2 = 0.32 \quad (\Upsilon \text{ family, input}). \tag{17}$$

Any potential constructed from Eqs. (3)–(6) must be finite at the origin. Nevertheless, the two-level reconstruction of a Coulomb potential[16] is much deeper than the Υ potential of Fig. 2. The reason may be traced to the modest leptonic widths and $|\Psi(0)|^2$ values of Υ and Υ'. A Coulomb plus linear potential which reproduces the ψ, ψ', Υ, and Υ' masses leads to leptonic widths which exceed measured values by about a factor of 2.[17] On the basis of this and other numerical experiments, we thus believe that if the leptonic widths of Υ and Υ' were so large as to imply a strong Coulomb-type singularity, the inverse method would yield a potential well much deeper than that in Fig. 2.

The present method makes use of values of $|\Psi(0)|^2$ obtained from leptonic widths via the nonrelativistic relation (12). It is possible that relativistic corrections allow a much larger value

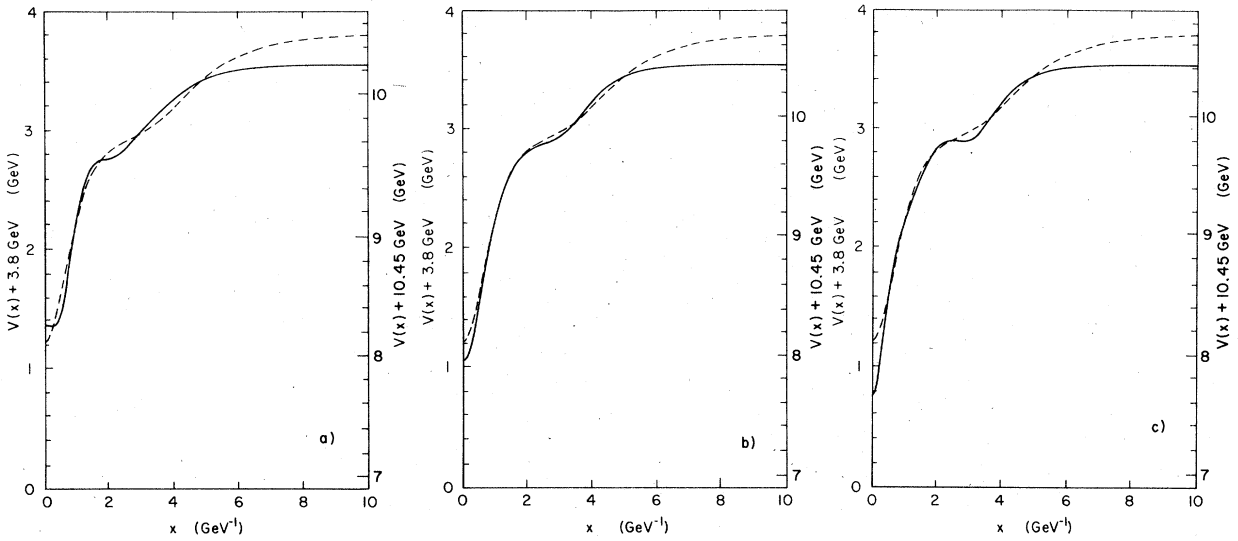

FIG. 4. Interquark potentials constructed from Υ, Υ', and a hypothetical $\Upsilon''(10.38)$ with $E_0 = 10.45$ GeV, $m_b = 4.5\,\mathrm{GeV}/c^2$, $e_Q = -\frac{1}{3}$, $N = 3$, and $\Gamma(\Upsilon'' \to e^+e^-) =$ (a) 0.19, (b) 0.28, (c) 0.36 keV. Dashed lines indicate the charmonium potential of Fig. 1.

of $|\Psi(0)|^2$ for a given leptonic width.[18] It is worth noting the sensitivity of our conclusions to such corrections.

One could, in principle, calculate the corrections to Eq. (12). However, the results presented in the first of Refs. 18 are valid only for a Coulomb potential, and the form for more general interactions is unknown. Alternatively, one could take the values of $|\Psi(0)|^2$ to be a universal multiple λ of those inferred from (12). This would entail flavor and level independence of the relativistic corrections. Such an approach is taken in Ref. 4, with $\lambda \simeq 2.9$. The depth of the potential is linear in this scale factor, as a consequence of the relation[19]

$$V(0) = -\frac{4\pi}{\mu} \sum_i \frac{|\Psi_i(0)|^2}{\kappa_{2i}} \,. \tag{18}$$

The scale factor then becomes an additional free parameter.

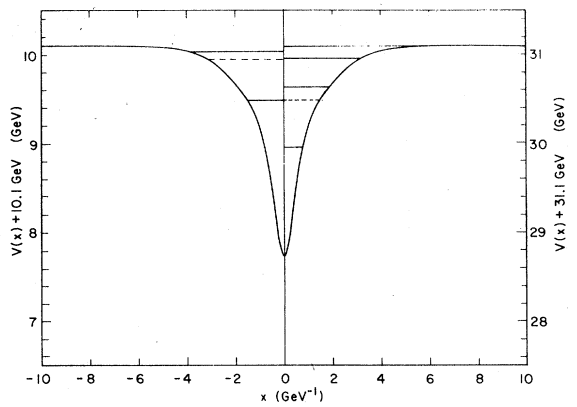

FIG. 5. Υ levels (left) and ζ levels (right) in the potential of Fig. 2. The ζ is a hypothetical family of $Q\bar{Q}$ vector mesons, with $m_Q = 15\,\mathrm{GeV}/c^2$. The right-hand scale is $2(m_Q - m_b) = 21$ GeV higher than the left. Solid lines are $^3S_1$ levels; dashed lines are 2^3P_J levels.

To test the ability of our methods to investigate flavor independence of the potential under such circumstances, we examined the spectrum and value of $|\Psi(0)|^2$ obtained from a recently published explicit potential[20] with Coulomb behavior (modified by asymptotic freedom) at the origin and linear behavior at large distances. In this potential, the values of $|\Psi(0)|^2$ are about a factor of 2 larger than one would infer from Eq. (12),[21] but the low-lying charmonium and Υ spectra are described correctly. We use two charmonium and three Υ levels. The potentials constructed from the ψ and Υ families then agree in the range 0.1–1 fm. The Υ potential is considerably deeper below 0.1 fm, in accord with the rather strong Coulomb singularity of the potential which is being approximated. As a result, the ζ levels in this potential behave in a thoroughly Coulomb-type manner: The $\zeta(2S)$-$\zeta(2P)$ spacing is less than for the Υ levels, and the ratio of $2S$ to $1S$ leptonic widths is smaller than for the Υ. The properties of the (yet to be discovered) ζ family will thus tell whether a nonrelativistic formula such as (12) can be used or whether strong relativistic corrections must be taken into account.

To conclude, an inverse-scattering method has given constructive evidence for flavor independence

of the quark-antiquark potential when a comparison is made of charmonium and Υ levels. Figure 2 indicates evidence for flavor independence over the range 0.1–0.8 fm. Tests at larger distances will require information on the leptonic width of Υ'' (~10.38). Our previous calculations[1] and the present exercises indicate that the Υ'' should have a leptonic width comparable to that of Υ', of order 0.3 keV, and thus should be detectable very soon.

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[1]H. B. Thacker, C. Quigg, and J. L. Rosner, Phys. Rev. D 18, 274 (1978); 18, 287 (1978).
[2]Ch. Berger *et al.*, Phys. Lett. 76B, 243 (1978); C. W. Darden *et al.*, *ibid.* 76B, 246 (1978); J. K. Bienlein *et al.*, *ibid.* 78B, 360 (1978); C. W. Darden *et al.*, *ibid.* 78B, 364 (1978).
[3]J. L. Rosner, C. Quigg, and H. B. Thacker, Phys. Lett. 74B, 350 (1978).
[4]Among the most recent and complete treatments is that by E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D 21, 203 (1980).
[5]E. Eichten and K. Gottfried, Phys. Lett. 66B, 286 (1977).
[6]C. Quigg and J. L. Rosner, Phys. Lett. 72B, 462 (1978).
[7]H. Grosse and A. Martin, Nucl. Phys. B148, 413 (1979).
[8]R. Van Royen and V. F. Weisskopf, Nuovo Cimento 50, 617 (1967); 51, 583 (1967).
[9]A plotting error resulted in the display of $E_0 + m_c V(r)$ rather than $E_0 + V(r)$ in Fig. 3 of the second of Refs. 1. The levels are correctly displayed, however, and no quantitative results are affected.
[10]G. Flügge, in *Proceedings of the XIX International Conference on High Energy Physics, Tokyo, 1978,* edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Tokyo, 1979), p. 793.
[11]K. Ueno *et al.*, Phys. Rev. Lett. 42, 486 (1979); L. M. Lederman (private communication).
[12]J. D. Jackson, C. Quigg, and J. L. Rosner, in *Proceedings of the XIX International Conference on High Energy Physics, Tokyo, 1978,* edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Phys. Soc. of Japan, Tokyo, 1979), p. 391.
[13]Y. J. Ng and S. H.-H. Tye, Phys. Rev. Lett. 41, 6 (1978); P. G. O. Freund and C. T. Hill, Phys. Rev. D 19, 2755 (1979); H. Fritzsch, Phys. Lett. 78B, 611 (1978).
[14]D. Cutts *et al.*, Phys. Rev. Lett. 41, 363 (1978); R. Vidal *et al.*, Phys. Lett. 77B, 344 (1978).
[15]H. Georgi and S. L. Glashow, Harvard Report No. HUTP-79/A027 (unpublished).
[16]See, for example, Fig. 1 of the second of Ref. 1.
[17]C. Quigg and J. L. Rosner, Phys. Lett. 71B, 153 (1977). See also Eichten *et al.*, Ref. 4.
[18]R. Barbieri, R. Gatto, R. Kögerler, and Z. Kunszt, Phys. Lett. 57B, 455 (1975); W. Celmaster, Phys. Rev. D 19, 1517 (1979); E. Poggio and H. J. Schnitzer, *ibid.* 20, 1175 (1979).
[19]Jonathan F. Schonfeld, Waikwok Kwong, Jonathan L. Rosner, C. Quigg, and H. B. Thacker, Fermilab Report No. Pub-79177-THY (unpublished).
[20]John L. Richardson, Phys. Lett 82B, 272 (1979).
[21]We are grateful to John L. Richardson for providing the values of $|\Psi(0)|^2$ corresponding to the potential in Ref. 20.