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# Quantum field theory at finite temperature in a rotating system

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The finite-temperature Green's-function formalism is extended to the case of relativistic rotating systems. Free scalar, spinor, and electromagnetic Green's functions for a rotating system are obtained. As an application of the formalism, the neutrino parity-violating current is calculated. The result agrees with previous calculations where different methods have been used.

## I. INTRODUCTION

Methods of quantum field theory have proved to be very fruitful in statistical physics problems. Originally introduced in the framework of nonrelativistic many-body theory,[1] they were later extended to relativistic systems and to gauge theories.[2] However, these methods as they stand are not applicable to rotating equilibrium systems. The purpose of the present paper is to fill this gap and to extend the finite-temperature Green's-function formalism to relativistic rotating systems. A particular application that I have in mind is to the calculation of macroscopic parity-violating currents. It has been recently shown[3] that intrinsic parity nonconservation for neutrinos gives rise to equilibrium neutrino and antineutrino currents in a rotating thermal radiation. The direction of these currents is parallel to the angular velocity vector. It has been argued that currents of other particles can also occur as a result of parity-violating weak interactions. The main purpose of this paper is to develop an adequate formalism for studying such effects.

The equilibrium current of a spinor field described by a field operator $\Psi(\vec{x},t)$ is given by

$$\langle J^\mu(\vec{x})\rangle = \mathrm{Tr}\{\rho J^\mu(\vec{x},t)\}. \quad (1)$$

Here,

$$J^\mu = \tfrac{1}{2}[\bar{\Psi},\gamma^\mu\Psi] \quad (2)$$

is the current density operator,

$$\rho = C\exp\left[-\beta\left(H - \vec{M}\cdot\vec{\Omega} - \sum_i \mu_i N_i\right)\right] \quad (3)$$

is the statistical operator,

$$C^{-1} = \mathrm{Tr}\exp\left[-\beta\left(H - \vec{M}\cdot\vec{\Omega} - \sum_i \mu_i N_i\right)\right]$$

is the normalization factor, $\beta = T^{-1}$, $T$ is the temperature, $\vec{\Omega}$ is the angular velocity, $H$ and $\vec{M}$ are the Hamiltonian and the angular momentum of the system, respectively, and $\mu_i$ and $N_i$ are, respectively, the chemical potential and the number of particles of the $i$th particle species. The expression (3) for the statistical operator in a rotating system has been derived by Landau and Lifshitz.[4] However, the argument leading to Eq. (3) is scattered over the book and is partly based on a nonrelativistic expression for the energy. For these reasons, I feel that a brief derivation is in order here. It is given in Appendix A. In the following, I assume for simplicity that the chemical potentials of all particles are equal to zero.

In equilibrium the right-hand side of Eq. (1) is independent of time and we can set $t=0$. Introducing the Matsubara field operator

$$\psi(\vec{x},\tau) = \exp[\tau(H-\vec{M}\cdot\vec{\Omega})]\Psi(\vec{x},0)\exp[-\tau(H-\vec{M}\cdot\vec{\Omega})] \quad (4)$$

and the finite-temperature Green's function

$$\mathcal{S}_{\alpha\beta}(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2) = \mathrm{Tr}\{\rho T_\tau \psi_\alpha(\vec{x}_1,\tau_1)\bar{\psi}_\beta(\vec{x}_2,\tau_2)\}, \quad (5)$$

we can rewrite Eq. (1) as

$$\langle J^\mu(\vec{x})\rangle = -\mathrm{Tr}\{\gamma^\mu \mathcal{S}(\vec{x},\tau;\vec{x},\tau+\epsilon)\}_{\epsilon\to 0}. \quad (6)$$

Here, $T_\tau$ is the usual ordering operator,

$$T_\tau \psi_\alpha(\vec{x}_1,\tau_1)\bar{\psi}_\beta(\vec{x}_2,\tau_2) = \psi_\alpha(\vec{x}_1,\tau_1)\bar{\psi}_\beta(\vec{x}_2,\tau_2) \quad \text{for } \tau_1 > \tau_2$$

and

$$T_\tau \psi_\alpha(\vec{x}_1,\tau_1)\bar{\psi}_\beta(\vec{x}_2,\tau_2) = -\bar{\psi}_\beta(\vec{x}_2,\tau_2)\psi_\alpha(\vec{x}_1,\tau_1) \quad \text{for } \tau_1 < \tau_2,$$

the trace in Eq. (6) is taken with respect to spinor indices, and the limit $\epsilon\to 0$ is taken symmetrically for $\epsilon\to +0$ and $\epsilon\to -0$.

The diagram technique for the calculation of the Green's function $\mathcal{S}$ can be derived in the usual way.[1,2] Using the standard argument, Eq. (5) can be transformed to

$$\begin{aligned}\mathcal{S}_{\alpha\beta}(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2) &= \langle T_\tau \psi_\alpha(\vec{x}_1,\tau_1)\bar{\psi}_\beta(\vec{x}_2,\tau_2)U(\beta)\rangle_0 \langle U(\beta)\rangle_0^{-1} \\ &= \langle T_\tau \psi_\alpha(\vec{x}_1,\tau_1)\bar{\psi}_\beta(\vec{x}_2,\tau_2)U(\beta)\rangle_{0,\,\mathrm{conn}}, \quad (7)\end{aligned}$$

where

$$U(\beta)=1+\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\int_0^\beta\cdots\int_0^\beta d\tau_1\cdots d\tau_n T_\tau \times\{H_i(\tau_1)\cdots H_i(\tau_n)\}, \quad (8)$$

$$\langle\cdots\rangle_0\equiv\mathrm{Tr}\{\rho_0\cdots\}, \quad (9)$$

$$\rho_0=\exp[-\beta(H_0-\vec{M}_0\cdot\vec{\Omega})]/\mathrm{Tr}\exp[-\beta(H_0-\vec{M}_0\cdot\vec{\Omega})], \quad (10)$$

the subscript "conn" means the contribution of connected diagrams only, $H_0$ and $\vec{M}_0$ are the free-field Hamiltonian and angular momentum, respectively,

$$H_i=H-H_0-\vec{\Omega}\cdot(\vec{M}-\vec{M}_0) \quad (11)$$

and

$$H_i(\tau)=e^{\tau(H_0-\vec{M}_0\cdot\vec{\Omega})}H_i e^{-\tau(H_0-\vec{M}_0\cdot\vec{\Omega})}. \quad (12)$$

It should be noted that in some important cases (e.g., spinor electrodynamics) the angular momentum operator $\vec{M}$ coincides with its free-field form $\vec{M}_0$ and thus $H_i=H-H_0$.

The rules of diagram technique in coordinate space are the same as those for a nonrotating system, the only difference being that in the free-particle Green's functions

$$S_{\alpha\beta}(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2)=\langle T_\tau\psi_\beta(\vec{x}_1,\tau_1)\bar{\psi}_\beta(\vec{x}_2,\tau_2)\rangle_0, \quad (13)$$

etc., the averaging is taken using the statistical operator $\rho_0$ given by Eq. (10), rather than $e^{-\beta H_0}/\mathrm{Tr}e^{-\beta H_0}$.

In this paper, I shall calculate the Green's functions for free scalar, spinor, and electromagnetic fields in coordinate and momentum representations. As an application of the formalism, the neutrino parity-violating current will be calculated. The result agrees with Ref. 3 where the neutrino current has been found using different methods. The equilibrium currents of interacting fields will be discussed elsewhere.

## II. FINITE-TEMPERATURE GREEN'S FUNCTIONS IN A ROTATING SYSTEM

### A. Scalar field

The field operator of a scalar field of mass $\mu$ satisfies the Klein-Gordon equation

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}-\frac{\partial^2}{\partial z^2}+\mu^2\right)\Psi(\vec{x},t)=0, \quad (14)$$

where $(r,\phi,z)$ are the cylindrical coordinates and I use the system of units in which $\hbar=c=k=1$ ($k$ is the Boltzmann constant). The scalar wave functions with energy $\omega$, $z$ component of momentum $p$, and $z$ component of angular momentum $m$ are given by

$$\psi_{\omega pm}(\vec{x})e^{-i\omega t}=2^{-3/2}\pi^{-1}J_m(\alpha r)\exp(ipz+im\phi-i\omega t), \quad (15)$$

where

$$\alpha=(\omega^2-p^2-\mu^2)^{1/2}\equiv(p_0^2-p^2)^{1/2}. \quad (16)$$

The functions $\psi_{\omega pm}(\vec{x})$ are normalized according to

$$\int\psi_{\omega pm}(\vec{x})\psi^*_{\omega' p' m'}(\vec{x})\,d^3x=(2\omega)^{-1}\delta_{mm'}\delta(p-p')\delta(\omega-\omega'), \quad (17)$$

where $d^3x=r\,dr\,d\phi\,dz$.

The field operator $\Psi(\vec{x},t)$ can be written as

$$\Psi(\vec{x},t)=\sum_m\int_\mu^\infty d\omega\int_{-p_0}^{p_0}dp[a_{\omega pm}\psi_{\omega pm}(\vec{x})e^{-i\omega t}+a^\dagger_{\omega pm}\psi^*_{\omega pm}(\vec{x})e^{i\omega t}], \quad (18)$$

where $a^\dagger_{\omega pm}$ and $a_{\omega pm}$ are creation and annihilation operators,

$$[a_{\omega pm},a^\dagger_{\omega' p' m'}]=\delta_{mm'}\delta(p-p')\delta(\omega-\omega'). \quad (19)$$

Let us choose our $z$ axis in the direction of $\vec{\Omega}$. Then using the relations

$$\begin{aligned}e^{\tau(H_0-\vec{M}\cdot\vec{\Omega})}a_{\omega pm}e^{-\tau(H_0-\vec{M}\cdot\vec{\Omega})}&=a_{\omega pm}e^{-\tau(\omega-m\Omega)},\\ e^{\tau(H_0-\vec{M}\cdot\vec{\Omega})}a^\dagger_{\omega pm}e^{-\tau(H_0-\vec{M}\cdot\vec{\Omega})}&=a^\dagger_{\omega pm}e^{\tau(\omega-m\Omega)},\end{aligned} \quad (20)$$

we can write the Matsubara operator (4) as

$$\psi(\vec{x},\tau)=\sum_m\int_\mu^\infty d\omega\int_{-p_0}^{p_0}dp\,[a_{\omega pm}e^{-\tau(\omega-m\Omega)}\psi_{\omega pm}(\vec{x})+a^\dagger_{\omega pm}e^{\tau(\omega-m\Omega)}\psi^*_{\omega pm}(\vec{x})]. \quad (21)$$

The finite-temperature Green's function is given by

$$D(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2)=\langle T_\tau\psi(\vec{x}_1,\tau_1)\psi(\vec{x}_2,\tau_2)\rangle_0$$

$$=\sum_m\int_\mu^\infty d\omega\int_{-p_0}^{p_0}dp\left[\begin{pmatrix}n_{\omega m}+1\\ n_{\omega m}\end{pmatrix}e^{-\tau(\omega-m\Omega)}\psi_{\omega pm}(\vec{x}_1)\psi^*_{\omega pm}(\vec{x}_2)+\begin{pmatrix}n_{\omega m}\\ n_{\omega m}+1\end{pmatrix}e^{\tau(\omega-m\Omega)}\psi^*_{\omega pm}(\vec{x}_1)\psi_{\omega pm}(\vec{x}_2)\right]. \quad (22)$$

Here,

$$n_{\omega m}=(e^{\beta(\omega-m\Omega)}-1)^{-1} \tag{23}$$

is the Bose-Einstein distribution for a rotating system,[5] $\tau=\tau_1-\tau_2$, the upper and lower lines in parentheses correspond to $\tau_1>\tau_2$ and $\tau_1<\tau_2$, respectively, and I have used the fact that

$$\langle a^\dagger_{\omega pm}a_{\omega' p' m'}\rangle_0=n_{\omega m}\delta_{mm'}\delta(p-p')\delta(\omega-\omega'),$$
$$\langle a_{\omega pm}a^\dagger_{\omega' p' m'}\rangle_0=(n_{\omega m}+1)\delta_{mm'}\delta(p-p')\delta(\omega-\omega'),$$
$$\langle a_{\omega pm}a_{\omega' p' m'}\rangle_0=\langle a^\dagger_{\omega pm}a^\dagger_{\omega' p' m'}\rangle_0=0.$$

It is easily seen from Eq. (22) that[8] for $-\beta<\tau<0$,

$$D(\tau+\beta)=D(\tau) \tag{24}$$

and thus the function $D$ in the interval $-\beta<\tau<\beta$ can be expanded in a Fourier series as

$$D(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2)=\beta^{-1}\sum_n e^{-\nu_n\tau}D(\vec{x}_1,\vec{x}_2,\nu_n), \tag{25}$$

where $\nu_n=2\pi in\beta^{-1}$. The function $D(\vec{x}_1,\vec{x}_2,\nu_n)$ can be found from

$$D(\vec{x}_1,\vec{x}_2,\nu_n)=\tfrac{1}{2}\int_{-\beta}^{\beta}e^{\nu_n\tau}D(\vec{x}_1,\tau;\vec{x}_2,0)\,d\tau$$
$$=\int_0^{\beta}e^{\nu_n\tau}D(\vec{x}_1,\tau;\vec{x}_2,0)d\tau. \tag{26}$$

Substituting Eq. (22) in Eq. (26) and integrating over $\tau$, we obtain

$$D(\vec{x}_1,\vec{x}_2,\nu_n)$$
$$=-\sum_m\int_\mu^\infty d\omega\int_{-p_0}^{p_0}dp\left(\frac{\psi_{\omega pm}(\vec{x}_1)\psi^*_{\omega pm}(\vec{x}_2)}{\nu_n-\omega+m\Omega}-\frac{\psi^*_{\omega pm}(\vec{x}_1)\psi_{\omega pm}(\vec{x}_2)}{\nu_n+\omega-m\Omega}\right). \tag{27}$$

In most physical situations, $\Omega\ll T$ and one can be interested in calculating the first few terms in the expansions of physical quantities in powers of $\beta\Omega=(\hbar\Omega/kT)$. Expanding $(\nu_n-\omega+m\Omega)^{-1}$ and $(\nu_n+\omega-m\Omega)^{-1}$ in powers of $\Omega$, we obtain

$$D(\vec{x}_1,\vec{x}_2,\nu_n)=-\int_\mu^\infty\frac{d\omega}{\nu_n-\omega}\sum_{k=0}^{\infty}\left(\frac{i\Omega}{\nu_n-\omega}\frac{\partial}{\partial\phi_1}\right)^kA+\int_\mu^\infty\frac{d\omega}{\nu_n+\omega}\sum_{k=0}^{\infty}\left(\frac{i\Omega}{\nu_n+\omega}\frac{\partial}{\partial\phi_1}\right)^kA^*, \tag{28}$$

where

$$A=\int_{-p_0}^{p_0}dp\sum_m\psi_{\omega pm}(\vec{x}_1)\psi^*_{\omega pm}(\vec{x}_2). \tag{29}$$

The quantity $A$ can be calculated directly from Eq. (29) using the cylindrical wave functions (17). This is done in Appendix B. The result is

$$A=(4\pi^2R)^{-1}\sin p_0R, \tag{30}$$

where

$$R=|\vec{x}_1-\vec{x}_2|=[r_1^2+r_2^2-2r_1r_2\cos(\phi_1-\phi_2)+(z_1-z_2)^2]^{1/2}.$$

Another representation for the Green's function $D$ can be obtained if we note that Eq. (28) can be rewritten in the form

$$D(\vec{x}_1,\vec{x}_2,\nu_n)=\sum_{k=0}^{\infty}(k!)^{-1}\left(-i\Omega\frac{\partial}{\partial\nu_n}\frac{\partial}{\partial\phi_1}\right)^kD_0(\vec{x}_1,\vec{x}_2,\nu_n)$$
$$=\exp\left(-i\Omega\frac{\partial}{\partial\nu_n}\frac{\partial}{\partial\phi_1}\right)D_0(\vec{x}_1,\vec{x}_2,\nu_n). \tag{31}$$

Here, $D_0$ is the scalar Green's function for a nonrotating system which is given by the well-known expression[2]

$$D_0(\vec{x}_1,\vec{x}_2,\nu_n)=-(2\pi)^{-3}\int d^3p\,e^{i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)}\times(\nu_n^2-p^2-\mu^2)^{-1}. \tag{32}$$

Noticing that

$$\Omega\partial/\partial\phi_1=\vec{\Omega}\cdot(\vec{x}_1\times\vec{\nabla}_1), \tag{33}$$

we obtain finally

$$D(\vec{x}_1,\vec{x}_2,\nu_n)=\exp\left[-i\vec{\Omega}\cdot(\vec{x}_1\times\vec{\nabla}_1)\frac{\partial}{\partial\nu_n}\right]\times D_0(\vec{x}_1,\vec{x}_2,\nu_n). \tag{34}$$

### B. Spinor field

The spinor field equation in cylindrical coordinates is[3]

$$\left(\gamma^0\frac{\partial}{\partial t}-\vec{\gamma}\vec{\nabla}'-\gamma^2\Gamma_2-\mu\right)\Psi'(\vec{x},t)=0, \tag{35}$$

where[9,10]

$$\vec{\nabla}'=\left(\frac{\partial}{\partial r},\frac{1}{r}\frac{\partial}{\partial\phi},\frac{\partial}{\partial z}\right) \tag{36}$$

and

$$\Gamma_2=\frac{i}{2}\begin{bmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{bmatrix}. \tag{37}$$

For neutrinos $\mu=0$, and Eq. (35) is supplemented by the condition[11]

$$(1+\gamma^5)\Psi'(\vec{x},t)=0. \tag{38}$$

The primes in Eqs. (35), (36), and (38) indicate that the corresponding quantities are taken in cylindrical coordinates. Unprimed quantities correspond to Cartesian coordinates.

The spinor field operator can be written as

$$\Psi'(\vec{x},t)=\sum_{m,h}\int_{\mu}^{\infty}d\omega\int_{-p_0}^{p_0}dp[a_{\omega pmh}\psi_{\omega pmh}(\vec{x})e^{-i\omega t}+b^{\dagger}_{\omega pmh}\chi_{\omega pmh}(\vec{x})e^{i\omega t}], \tag{39}$$

where $h$ stands for helicity, $a_{\omega pmh}$ and $b_{\omega pmh}$ satisfy the usual anticommutation relations, $\psi_{\omega pmh}(\vec{x})$ and $\chi_{\omega pmh}(\vec{x})$ are the particle and antiparticle wave functions, respectively,

$$\chi_{\omega pmh}=\gamma^2\psi^*_{\omega pmh}. \tag{40}$$

The specific form of $\psi_{\omega pmh}$ is unimportant for the following discussion. We shall only use the fact that the $\phi$ dependence of $\psi_{\omega pmh}$ is given by the factor $\exp(im\phi)$.

Following the lines of the previous subsection, we obtain

$$S'(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2)=\beta^{-1}\sum_l\exp(-\zeta_l\tau)S'(\vec{x}_1,\vec{x}_2,\zeta_l), \tag{41}$$

where $\zeta_l=i\pi(2l+1)\beta^{-1}$,

$$S'(\vec{x}_1,\vec{x}_2,\zeta_l)=-\int_{\mu}^{\infty}\frac{d\omega}{\zeta_l-\omega}\sum_{k=0}^{\infty}\left(\frac{i\Omega}{\zeta_l-\omega}\frac{\partial}{\partial\phi_1}\right)^kA-\int_{\mu}^{\infty}\frac{d\omega}{\zeta_l+\omega}\sum_{k=0}^{\infty}\left(\frac{i\Omega}{\zeta_l+\omega}\frac{\partial}{\partial\phi_1}\right)^kB, \tag{42}$$

$$A_{\alpha\beta}=\sum_{m,h}\int_{-p_0}^{p_0}dp\,\psi^{\alpha}_{\omega pmh}(\vec{x}_1)\bar{\psi}^{\beta}_{\omega pmh}(\vec{x}_2),$$
$$B_{\alpha\beta}=\sum_{m,h}\int_{-p_0}^{p_0}dp\,\chi^{\alpha}_{\omega pmh}(\vec{x}_1)\bar{\chi}^{\beta}_{\omega pmh}(\vec{x}_2). \tag{43}$$

Equation (42) can be rewritten in the form

$$S'(\vec{x}_1,\vec{x}_2,\zeta_l)=\exp\left(-i\Omega\frac{\partial}{\partial\zeta_l}\frac{\partial}{\partial\phi_1}\right)S_0'(\vec{x}_1,\vec{x}_2,\zeta_l), \tag{44}$$

where $S_0'$ is the free spinor Green's function for a nonrotating system in cylindrical coordinates, which can be obtained from the Cartesian Green's function

$$S_0(\vec{x}_1,\vec{x}_2,\zeta_l)=-\frac{1}{(2\pi)^3}\int d^3p\,e^{i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)}\frac{\gamma^0\zeta_l-\vec{\gamma}\cdot\vec{p}+\mu}{\zeta_l^2-p^2-\mu^2} \tag{45}$$

by means of a coordinate transformation.

If the local coordinate system at some point is rotated by angle $\phi$ around the $z$ axis, then all the vector quantities at that point transform according to

$$V'=\hat{\alpha}V, \tag{46}$$

where the transformation matrix $\hat{\alpha}$ is given by

$$\hat{\alpha}=\begin{pmatrix}1&0&0&0\\0&\cos\phi&\sin\phi&0\\0&-\sin\phi&\cos\phi&0\\0&0&0&1\end{pmatrix}. \tag{47}$$

The corresponding spinor transformation is

$$\Psi'=U\Psi,\quad \bar{\Psi}'=\bar{\Psi}U^{\dagger}, \tag{48}$$

where

$$U(\phi)=\exp(\tfrac{1}{2}i\phi\Sigma_3) \tag{49}$$

and

$$\vec{\Sigma}=\begin{pmatrix}\vec{\sigma}&0\\0&\vec{\sigma}\end{pmatrix}. \tag{50}$$

For the $\gamma$ matrices,

$$\gamma'=\hat{\alpha}U\gamma U^{\dagger}=\gamma. \tag{51}$$

In cylindrical coordinates, the local coordinate axes at point $(r,\phi,z)$ are rotated by angle $\phi$ with respect to Cartesian ones, and thus the cylindrical and Cartesian spinor Green's functions are related by

$$S_0'(\vec{x}_1,\vec{x}_2,\zeta_l)=U(\phi_1)S_0(\vec{x}_1,\vec{x}_2,\zeta_l)U^{\dagger}(\phi_2). \tag{52}$$

From Eqs. (44) and (52) we find that the spinor Green function for a rotating system in Cartesian coordinates is given by

$$S(\vec{x}_1,\vec{x}_2,\zeta_l)=U^{\dagger}(\phi_1)\times\exp\left(-i\Omega\frac{\partial}{\partial\zeta_l}\frac{\partial}{\partial\phi_1}\right)U(\phi_1)S_0(\vec{x}_1,\vec{x}_2,\zeta_l). \tag{53}$$

Using the well-known theorem that if the commutator of operators $A$ and $B$ commutes with both $A$ and $B$, then

$$e^Ae^B=e^Be^Ae^{[A,B]}$$

and we obtain

$$S(\vec{x}_1,\vec{x}_2,\zeta_l)=\exp\left[-i\vec{\Omega}\cdot(\vec{x}_1\times\vec{\nabla}_1)\frac{\partial}{\partial\zeta_l}+\tfrac{1}{2}\vec{\Omega}\cdot\vec{\Sigma}\frac{\partial}{\partial\zeta_l}\right]\times S_0(\vec{x}_1,\vec{x}_2,\zeta_l). \tag{54}$$

Replacing $\partial/\partial\phi_1$ by $-\partial/\partial\phi_2$ in Eq. (44), we find another representation for the spinor Green's function:

$$S(\vec{x}_1,\vec{x}_2,\zeta_l)=\exp\left[i\vec{\Omega}\cdot(\vec{x}_2\times\vec{\nabla}_2)\frac{\partial}{\partial\zeta_l}\right]\times S_0(\vec{x}_1,\vec{x}_2,\zeta_l)\exp\left(\tfrac{1}{2}\vec{\Omega}\cdot\vec{\Sigma}\frac{\overleftarrow{\partial}}{\partial\zeta_l}\right). \tag{55}$$

Here the arrow over $\partial/\partial\zeta_l$ indicates that this differential operator acts to the left. In the case of neutrinos, one has to set $\mu=0$ and multiply Eqs. (54) and (55) by $\frac{1}{2}(1+\gamma^5)$ on the right.

### C. Electromagnetic field

As we know, the quantization of the electromagnetic field is complicated by the presence of unphysical states in some gauges. It is not obvious that we can do statistical mechanics in any gauge, since that would result in nonzero equilibrium numbers of particles in unphysical states. For this reason, in deriving the expression for the photon Green's function, I shall assume that the Coulomb gauge is used in which only physical states are present. Then it will be shown (Appendix C) that the theory is invariant with respect to gauge transformations.

Quite similarly to the cases of scalar and spinor fields, it can be shown that

$$D'_{\mu\nu}(\vec{x}_1,\vec{x}_2,\nu_n)=\exp\left(-i\Omega\frac{\partial}{\partial\nu_n}\frac{\partial}{\partial\phi_1}\right)D'_{0\mu\nu}(\vec{x}_1,\vec{x}_2,\nu_n)\,, \tag{56}$$

where $\nu_n=2\pi in\beta^{-1}$, $D'_{\mu\nu}$ is the photon Green's function in cylindrical coordinates, and $D'_{0\mu\nu}$ is the corresponding function for a nonrotating system. The cylindrical and Cartesian photon Green's functions are related by

$$D'_{\mu\nu}=\alpha_{\mu\lambda}(\phi_1)D_{\lambda\sigma}\alpha^\dagger_{\sigma\nu}(\phi_2)\,, \tag{57}$$

where $\alpha_{\mu\nu}(\phi)$ is given by Eq. (47). The matrix $\hat{\alpha}(\phi)$ can be represented as[12]

$$\hat{\alpha}(\phi)=\exp(\hat{M}\phi)\,, \tag{58}$$

where

$$\hat{M}=\begin{bmatrix}0&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&0\end{bmatrix} \tag{59}$$

or $M_{\mu\nu}=\epsilon_{0\mu\nu3}$. From Eqs. (56)–(59) we get

$$\hat{D}(\vec{x}_1,\vec{x}_2,\nu_n)=\exp\left[-i\Omega(\vec{x}_1\times\vec{\nabla}_1)\frac{\partial}{\partial\nu_n}-i\Omega\hat{M}\frac{\partial}{\partial\nu_n}\right]\times\hat{D}_0(\vec{x}_1,\vec{x}_2,\nu_n)\,. \tag{60}$$

So far we assumed that in Eqs. (56) and (60), $\hat{D}_0$ is taken in the Coulomb gauge. It is shown in Appendix C that the physical results of the theory do not change if $D_{0\mu\nu}$ is replaced by

$$\tilde{D}_{0\mu\nu}=D_{0\mu\nu}+\nabla_{(1)\mu}\Lambda_{(1)0\nu}+\nabla_{(2)\nu}\Lambda_{(2)0\mu}\,, \tag{61}$$

where

$$\nabla_{(1)0}=-\nabla_{(2)0}=-i\nu_n,\quad \nabla_{(1)i}=\partial/\partial x_{1i}\,,\quad \nabla_{(2)i}=\partial/\partial x_{2i}$$

and $\Lambda_{(1)0\mu}(\vec{x}_1,\vec{x}_2,\nu_n)$ and $\Lambda_{(2)0\mu}(\vec{x}_1,\vec{x}_2,\nu_n)$ are arbitrary functions of $\vec{x}_1$, $\vec{x}_2$, and $\nu_n$. In other words, the theory is gauge invariant and one can use an arbitrary gauge for $D_{0\mu\nu}$. In particular, one can choose the Feynman gauge in which

$$D_{0\mu\nu}(\vec{x}_1,\vec{x}_2,\nu_n)=-g_{\mu\nu}D_0(\vec{x}_1,\vec{x}_2,\nu_n)\,, \tag{62}$$

where $D_0(\vec{x}_1,\vec{x}_2,\nu_n)$ is given by Eq. (32) and $g_{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$.

## III. GREEN'S FUNCTIONS IN MOMENTUM SPACE

For a nonrotating system, the Green's functions depend only on the difference $\vec{x}_1-\vec{x}_2$. For example [see Eq. (32)],

$$D_0(\vec{x}_1,\vec{x}_2,\nu_n)=D_0(\vec{x}_1-\vec{x}_2,\nu_n)\,. \tag{63}$$

Rotating systems are not translationally invariant and, as a result, $D(\vec{x}_1,\vec{x}_2,\nu_n)$ depends on both $\vec{x}_1$ and $\vec{x}_2$. The scalar Green's function in momentum space $D(\vec{p}_1,\vec{p}_2,\nu_n)$ can be defined as

$$D(\vec{p}_1,\vec{p}_2,\nu_n)=(2\pi)^{-3}\int d^3x_1\int d^3x_2 e^{-i\vec{p}_1\cdot\vec{x}_1}e^{i\vec{p}_2\cdot\vec{x}_2}D(\vec{x}_1,\vec{x}_2,\nu_n)\,. \tag{64}$$

The inverse transformation is

$$D(\vec{x}_1,\vec{x}_2,\nu_n)=(2\pi)^{-3}\int d^3p_1\int d^3p_2 e^{i\vec{p}_1\cdot\vec{x}_1}e^{-i\vec{p}_2\cdot\vec{x}_2}D(\vec{p}_1,\vec{p}_2,\nu_n)\,. \tag{65}$$

Substituting Eqs. (34) and (32) in Eq. (64), we obtain

$$\begin{aligned}D(\vec{p}_1,\vec{p}_2,\nu_n)&=(2\pi)^{-6}\int d^3x_1\int d^3x_2\int d^3p\exp(-i\vec{p}_1\cdot\vec{x}_1+i\vec{p}_2\cdot\vec{x}_2+i\vec{p}\cdot\vec{x}_1-i\vec{p}\cdot\vec{x}_2)\exp\left[\vec{\Omega}\cdot(\vec{x}_1\times\vec{p})\frac{\partial}{\partial\nu_n}\right]D_0(\vec{p},\nu_n)\\&=(2\pi)^{-3}\int d^3x_1\exp[i\vec{x}_1\cdot(\vec{p}_2-\vec{p}_1)]\exp\left[\vec{x}_1\cdot(\vec{p}_2\times\vec{\Omega})\frac{\partial}{\partial\nu_n}\right]D_0(\vec{p}_2,\nu_n)\\&=(2\pi)^{-3}\exp\left[i\nabla_{p1}\cdot(\vec{p}_2\times\vec{\Omega})\frac{\partial}{\partial\nu_n}\right]\int d^3x_1\exp[i\vec{x}_1\cdot(\vec{p}_2-\vec{p}_1)]D_0(\vec{p}_2,\nu_n)\\&=\exp\left[i\vec{\Omega}\cdot(\nabla_{p1}\times\vec{p}_2)\frac{\partial}{\partial\nu_n}\right]\delta(\vec{p}_1-\vec{p}_2)D_0(\vec{p}_2,\nu_n)\,.\end{aligned} \tag{66}$$

Here, $\vec{\nabla}_{p1} = \partial/\partial \vec{p}_1$ and

$$D_0(\vec{p}, \nu_n) = -(\nu_n^2 - p^2 - \mu^2)^{-1}. \tag{67}$$

Noticing that

$$(\vec{\nabla}_{p1} \times \vec{\nabla}_{p2})\delta(\vec{p}_1 - \vec{p}_2) = 0$$

and

$$\vec{p}\,\frac{\partial}{\partial \nu_n} D_0(\vec{p}, \nu_n) = -\nu_n \vec{\nabla}_p D_0(\vec{p}, \nu_n),$$

we can write Eq. (66) as

$$D(\vec{p}_1, \vec{p}_2, \nu_n) = \exp[-i\nu_n \vec{\Omega}\cdot(\vec{\nabla}_{p1} \times \vec{\nabla}_{p2})] \times \delta(\vec{p}_1 - \vec{p}_2) D_0(\vec{p}_2, \nu_n). \tag{68}$$

Quite similarly, we obtain the momentum-space representations for the spinor and electromagnetic Green's functions:

$$S(\vec{p}_1, \vec{p}_2, \zeta_l) = \exp\left[i\vec{\Omega}\cdot(\vec{\nabla}_{p1} \times \vec{p}_2)\frac{\partial}{\partial \zeta_l} + \tfrac{1}{2}\vec{\Omega}\cdot\vec{\Sigma}\frac{\partial}{\partial \zeta_l}\right] \times \delta(\vec{p}_1 - \vec{p}_2) S_0(\vec{p}_2, \zeta_l), \tag{69}$$

$$\hat{D}(\vec{p}_1, \vec{p}_2, \nu_n) = \exp\left[i\vec{\Omega}\cdot(\vec{\nabla}_{p1} \times \vec{p}_2)\frac{\partial}{\partial \nu_n} - i\Omega \hat{M}\frac{\partial}{\partial \nu_n}\right] \times \delta(\vec{p}_1 - \vec{p}_2) D_0(\vec{p}_2, \nu_n). \tag{70}$$

Another representation for the spinor Green's function can be found from Eq. (55):

$$S(\vec{p}_1, \vec{p}_2, \zeta_l) = \exp\left[-i\vec{\Omega}\cdot(\vec{\nabla}_{p2} \times \vec{p}_1)\frac{\partial}{\partial \zeta_l}\right] \times \delta(\vec{p}_1 - \vec{p}_2) S_0(\vec{p}_1, \zeta_l) \exp\left(\tfrac{1}{2}\vec{\Omega}\cdot\vec{\Sigma}\frac{\overleftarrow{\partial}}{\partial \zeta_l}\right). \tag{71}$$

Here,

$$S_0(\vec{p}, \zeta_l) = -(\gamma^0 \zeta_l - \vec{\gamma}\cdot\vec{p} + \mu)(\zeta_l^2 - p^2 - \mu^2)^{-1}, \tag{72}$$

$$D_{0\mu\nu}(\vec{p}, \nu_n) = g_{\mu\nu}(\nu_n^2 - p^2)^{-1}, \tag{73}$$

and I use the Feynman gauge for $D_{0\mu\nu}$.

## IV. NEUTRINO PARITY-VIOLATING CURRENT

The formalism developed in the previous sections can now be applied to calculate the equilibrium neutrino current in a rotating thermal radiation. The physical mechanism of this effect is easily understood if we recall that neutrinos have negative helicity, i.e., their spin is always antiparallel to the direction of their motion. In a rotating system neutrinos are partially polarized in the direction of the angular velocity $\vec{\Omega}$, and thus their average velocity is antiparallel to $\vec{\Omega}$. Antineutrinos have positive helicity and their current is parallel to the angular velocity vector.

According to Eq. (6),

$$\langle \vec{J}(\vec{x}) \rangle = -\mathrm{Tr}\{\vec{\gamma} S^{(\nu)}(\vec{x}, 0; \vec{x}, \epsilon)\}_{\epsilon\to 0}, \tag{74}$$

where $S^{(\nu)}(\vec{x}_1, \tau_1; \vec{x}_2, \tau_2)$ is the neutrino Green's function. For simplicity, I shall calculate the current on the rotation axis ($\vec{x} = 0$). Since

$$S^{(\nu)}(\vec{x}_1, \tau_1; \vec{x}_2, \tau_2) = \beta^{-1}(2\pi)^{-3}\sum_l \exp[-\zeta_l(\tau_1 - \tau_2)] \int d^3p_1 \int d^3p_2 \exp(i\vec{p}_1\cdot\vec{x}_1 - i\vec{p}_2\cdot\vec{x}_2) S^{(\nu)}(\vec{p}_1, \vec{p}_2, \zeta_l), \tag{75}$$

we can rewrite Eq. (74) as

$$\langle \vec{J}(0) \rangle = -\beta^{-1}(2\pi)^{-3}\sum_l e^{\epsilon\zeta_l} \int d^3p_1 \int d^3p_2 \mathrm{Tr}\{\vec{\gamma} S^{(\nu)}(\vec{p}_1, \vec{p}_2, \zeta_l)\}_{\epsilon\to 0}. \tag{76}$$

Substituting $S(\vec{p}_1, \vec{p}_2, \zeta_l)$ from Eq. (70), it is easily seen that terms with $(\vec{\nabla}_{p1} \times \vec{p}_2)$ do not contribute to $\langle \vec{J}(0) \rangle$ and thus

$$\langle \vec{J}(0) \rangle = -\beta^{-1}(2\pi)^{-3}\sum_l \exp(\epsilon\zeta_l)\,\mathrm{Tr}\left\{\vec{\gamma}\exp\left(\tfrac{1}{2}\vec{\Omega}\cdot\vec{\Sigma}\frac{\partial}{\partial \zeta_l}\right)\int d^3p\, S_0^{(\nu)}(\vec{p}, \zeta_l)\right\}\Bigg|_{\epsilon\to 0}, \tag{77}$$

where

$$S_0^{(\nu)}(\vec{p}, \zeta_l) = -\tfrac{1}{2}(\gamma^0\zeta_l - \vec{\gamma}\cdot\vec{p})(\zeta_l^2 - p^2)^{-1}(1 + \gamma^5). \tag{78}$$

Expanding $\exp(\frac{1}{2}\vec{\Omega}\cdot\vec{\Sigma}\,\partial/\partial\zeta_l)$ in powers of $\Omega$ and keeping only the linear term, we obtain, after calculating the trace,

$$\langle \vec{J}(0) \rangle = \vec{\Omega}\beta^{-1}(2\pi)^{-3}\sum_l e^{\epsilon\zeta_l}\int d^3p(\zeta_l^2 + p^2)(\zeta_l^2 - p^2)^{-2}\Big|_{\epsilon\to 0}. \tag{79}$$

Using a standard device[1,2] the sum in Eq. (79) can be transformed into an integral

$$\langle \vec{J}(0)\rangle = i\vec{\Omega}(2\pi)^{-4}\int_C d\omega[f_0(\omega)-f_0(-\omega)] \times \int d^3p(\omega^2+p^2)(\omega^2-p^2)^{-2}, \quad (80)$$

where

$$f_0(\omega)=(e^{\beta\omega}+1)^{-1} \quad (81)$$

and the contour $C$ runs around the imaginary axis in the counterclockwise direction. The integral over $\omega$ is easily evaluated and we get

$$\langle \vec{J}(0)\rangle = \vec{\Omega}(2\pi)^{-3}\int d^3p f_0'(p) = -\vec{\Omega}\pi^{-2}\int_0^\infty f_0(p)p\,dp = -\tfrac{1}{12}\vec{\Omega}T^2. \quad (82)$$

This is in agreement with Ref. 3 where the neutrino current has been calculated using different methods.

Using Eq. (77) one can calculate contributions to $\langle\vec{J}(0)\rangle$ proportional to the higher powers of $\Omega$. It is shown in Appendix D that the resulting series can be summed to give a simple closed expression. The result is

$$\langle \vec{J}(0)\rangle = -\vec{\Omega}(T^2/12+\Omega^2/48\pi^2), \quad (83)$$

again in agreement with Ref. 3.

A word of caution should be said concerning the interpretation of Eq. (83). As was mentioned earlier,[5] a rotating system cannot be infinite, its maximum size in the plane of rotation being $R_{\max}=\Omega^{-1}$. The energy spectrum of particles in a finite system is different from that in infinite space. A considerable deviation occurs for the low-lying energy states with energies $\epsilon \lesssim R^{-1}$. From Eq. (82) we see that the contribution of this part of the spectrum to $\langle\vec{J}(0)\rangle$ is of order $\vec{\Omega}R^{-2}$. If $T\gg R^{-1}$, this contribution is much smaller than the first term in Eq. (83), but it is always greater than or comparable to the second term.

The conclusion is that for $T\gg R^{-1}$, Eq. (82) gives a good approximation while for $T\lesssim R^{-1}$ the boundary effects are important and Eq. (83) is inaccurate. In particular, one cannot argue from Eq. (83) that[13,14] $\langle\vec{J}(0)\rangle\neq 0$ at $T=0$.

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## APPENDIX A

In this appendix, I shall derive Eq. (3) for the statistical operator in a rotating system. Let us consider an equilibrium system divided into several weakly interacting subsystems. The interaction between subsystems is important in establishing thermal equilibrium but can be neglected otherwise. In this case, the subsystems are statistically independent and thus the statistical operator for two subsystems is equal to the product of statistical operators for the individual subsystems[15]: $\rho_{12}=\rho_1\rho_2$ or

$$\ln\rho_{12}=\ln\rho_1+\ln\rho_2. \quad (A1)$$

From the equation for the statistical operator

$$\partial\rho/\partial t = i[H,\rho], \quad (A2)$$

it follows that in equilibrium

$$[H,\rho]=0. \quad (A3)$$

Equations (A1) and (A3) imply that $\ln\rho$ is an additive integral of motion. The only additive integrals of motion for a mechanical system are the energy $H$, momentum $\vec{P}$, and angular momentum $\vec{M}$. To these we have to add the particle numbers $N_i$ (or the conserved charges $Q_i$ if interactions with transformations between different particles are allowed). Therefore, $\ln\rho$ has to be equal to a linear combination of these quantities:

$$\ln\rho=\alpha+\beta H+\vec{\gamma}\cdot\vec{M}+\vec{\delta}\cdot\vec{P}+\sum_i\lambda_i N_i, \quad (A4)$$

where the constants $\alpha$, $\beta$, $\vec{\gamma}$, $\vec{\delta}$, and $\lambda_i$ have the same values for all subsystems.

The entropy of a subsystem is given by

$$S=-\langle\ln\rho\rangle=-\mathrm{Tr}(\rho\ln\rho) \quad (A5)$$

and thus

$$dS=-\beta dE-\vec{\gamma}\cdot d\vec{M}-\vec{\delta}\cdot d\vec{P}-\sum_i\lambda_i dN_i, \quad (A6)$$

where $E=\langle H\rangle$ and $\vec{M}$, $\vec{P}$, and $N_i$ stand for the statistical averages of the corresponding operators. Comparing this equation with

$$dE=TdS+\vec{V}\cdot d\vec{P}+\vec{\Omega}\cdot d\vec{M}+\sum_i\mu_i dN_i, \quad (A7)$$

where $\vec{V}$ is the center-of-mass velocity, $\vec{\Omega}$ is the angular velocity, and $\mu_i$ are the chemical potentials, we find

$$\beta=-T^{-1},\quad \vec{\gamma}=\vec{\Omega}/T,\quad \vec{\delta}=\vec{V}/T,\quad \lambda_i=\mu_i/T. \quad (A8)$$

In the rest frame of the system $\vec{V}=0$ and we come to Eq. (3). The normalization constant $C$ is determined from the condition $\mathrm{Tr}\rho=1$.

## APPENDIX B

In this appendix we shall calculate

$$A=\int_{-p_0}^{p_0}dp\sum_m\psi_{\omega pm}(\vec{x}_1)\psi^*_{\omega pm}(\vec{x}_2). \quad (B1)$$

Substituting the wave functions from Eq. (17), we obtain

$$A=(8\pi^2)^{-1}\int_{-p_0}^{p_0}dp\,e^{ipz}\sum_{m=-\infty}^{\infty}J_m(\alpha r_1)J_m(\alpha r_2)e^{im\phi}\,, \quad \text{(B2)}$$

where $z=z_1-z_2$, $\phi=\phi_1-\phi_2$. The sum in Eq. (B2) equals[10] $J_0(\alpha\rho)$, where $\rho=(r_1{}^2+r_2{}^2-2r_1r_2\cos\phi)^{1/2}$. Now Eq. (B2) can be written as

$$A=(4\pi^2)^{-1}\int_0^{p_0}dp\,J_0(\alpha p)\cos(pz)\,. \quad \text{(B3)}$$

Defining a new variable by $p/p_0=\cos\theta$, we obtain

$$A=p_0(4\pi^2)^{-1}\int_0^{\pi/2}d\theta\,\sin\theta J_0(p_0\rho\sin\theta)\cos(p_0z\cos\theta)\,. \quad \text{(B4)}$$

The integral over $\theta$ equals[10]

$$(\pi/2p_0R)^{1/2}J_{1/2}(p_0R)\,, \quad \text{(B5)}$$

where $R=(\rho^2+z^2)^{1/2}$. Thus,

$$A=(4\pi^2R)^{-1}\sin p_0R\,. \quad \text{(B6)}$$

## APPENDIX C

In this appendix we shall discuss the gauge transformations of the electromagnetic Green's function. The gauge invariance is closely related to the charge conservation law which, in cylindrical coordinates, can be written as

$$\partial J^{0\prime}/\partial t+\vec{\nabla}'\cdot\vec{J}'=0\,. \quad \text{(C1)}$$

Here, as before, primed quantities correspond to cylindrical coordinates and $\vec{\nabla}'=(\partial/\partial r, r^{-1}\partial/\partial\phi, \partial/\partial z)$. Since $\partial J^{0\prime}/\partial t=i[H,J^{0\prime}]$, we can rewrite (C1) as

$$i[H,J^{0\prime}]+\vec{\nabla}'\cdot\vec{J}'=0\,. \quad \text{(C2)}$$

The Matsubara current operator is

$$j^{\mu\prime}(\vec{x},\tau)=e^{\tau(H-M_3\Omega)}J^{\mu\prime}(\vec{x},0)e^{-\tau(H-M_3\Omega)}\,. \quad \text{(C3)}$$

Differentiating it with respect to $\tau$ and using $[H,M_3]=0$, we find

$$\begin{aligned}\partial j^{0\prime}/\partial\tau&=[H,j^{0\prime}]-\Omega[M_3,j^{0\prime}]\\&=[H,j^{0\prime}]-i\Omega\partial j^{0\prime}/\partial\phi\,.\end{aligned} \quad \text{(C4)}$$

The continuity equation for $j^{\mu\prime}$ can now be obtained from Eqs. (C2)–(C4):

$$(i\partial/\partial\tau-\Omega\partial/\partial\phi)j^{0\prime}+\vec{\nabla}'\cdot\vec{j}'=0 \quad \text{(C5)}$$

or

$$\tilde{\nabla}'_\mu j^{\mu\prime}=0\,, \quad \text{(C6)}$$

where

$$\tilde{\nabla}'_0=i\partial/\partial\tau-\Omega\partial/\partial\phi\,,\quad \vec{\tilde{\nabla}}'=\vec{\nabla}'\,. \quad \text{(C7)}$$

Noticing that in all diagrams $D'_{\mu\nu}(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2)$ is multiplied on both sides by $j^{\mu\prime}$ and $j^{\nu\prime}$ and integrated over $d^3x_1d\tau$, $d^3x_2d\tau_2$ and that in all $\tau$ and $\phi$ integrations the integrands are periodic functions of $\tau$ and $\phi$, we conclude that the results of all calculations will not change if we replace $D'_{\mu\nu}$ by

$$\begin{aligned}\tilde{D}'_{\mu\nu}(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2)=&D'_{\mu\nu}(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2)\\&+\tilde{\nabla}'_{(1)\mu}\Lambda'_{(1)\nu}(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2)\\&+\tilde{\nabla}'_{(2)\nu}\Lambda'_{(2)\mu}(\vec{x}_1,\tau_1;\vec{x}_2,\tau_2)\,,\end{aligned} \quad \text{(C8)}$$

where $\Lambda'_{(1)\nu}$ and $\Lambda'_{(2)\mu}$ are arbitrary (periodic in $\phi$ and $\tau$) functions. A similar transformation can be written for $D'_{\mu\nu}(x_1,x_2,\nu_n)$:

$$\begin{aligned}\tilde{D}'_{\mu\nu}(\vec{x}_1,\vec{x}_2,\nu_n)=&D'_{\mu\nu}(\vec{x}_1,\vec{x}_2,\nu_n)\\&+\tilde{\nabla}'_{(1)\mu}\Lambda'_{(1)\nu}(\vec{x}_1,\vec{x}_2,\nu_n)\\&+\tilde{\nabla}'_{(2)\nu}\Lambda'_{(2)\mu}(\vec{x}_1,\vec{x}_2,\nu_n)\,,\end{aligned} \quad \text{(C9)}$$

where now

$$\begin{aligned}&\tilde{\nabla}'_{(1)0}=-i\nu_n-\Omega\partial/\partial\phi_1\,,\quad \vec{\tilde{\nabla}}'_{(1)}=\vec{\nabla}'_{(1)}\,,\\&\tilde{\nabla}'_{(2)0}=i\nu_n-\Omega\partial/\partial\phi_2\,,\quad \vec{\tilde{\nabla}}'_{(2)}=\vec{\nabla}'_{(2)}\,.\end{aligned} \quad \text{(C10)}$$

Noticing that

$$\exp\left(i\Omega\frac{\partial}{\partial\nu_n}\frac{\partial}{\partial\phi_i}\right)\left(i\nu_n+\Omega\frac{\partial}{\partial\phi_1}\right)=i\nu_n\exp\left(i\Omega\frac{\partial}{\partial\nu_n}\frac{\partial}{\partial\phi_1}\right), \quad \text{(C11)}$$

we see that the corresponding transformation for $D'_{0\mu\nu}$ is

$$\tilde{D}'_{0\mu\nu}=D'_{0\mu\nu}+\nabla'_{(1)\mu}\Lambda'_{(1)0\nu}+\nabla'_{(2)\nu}\Lambda'_{(2)0\mu}\,, \quad \text{(C12)}$$

with

$$\nabla'_{(1)0}=-i\nu_n,\quad \nabla'_{(2)0}=i\nu_n \quad \text{(C13)}$$

and

$$\Lambda'_{0\mu}(\vec{x}_1,\vec{x}_2,\nu_n)=\exp\left(i\Omega\frac{\partial}{\partial\nu_n}\frac{\partial}{\partial\phi_1}\right)\Lambda'_\mu(\vec{x}_1,\vec{x}_2,\nu_n)\,. \quad \text{(C14)}$$

In Cartesian coordinates,

$$\tilde{D}_{0\mu\nu}=D_{0\mu\nu}+\nabla_{(1)\mu}\Lambda_{(1)0\nu}+\nabla_{(2)\nu}\Lambda_{(2)0\mu}\,, \quad \text{(C15)}$$

where

$$\nabla_{(1)0}=-\nabla_{(2)0}=-i\nu_n,\quad \nabla_{(1)i}=\partial/\partial x_{1i},\quad \nabla_{(2)i}=\partial/\partial x_{2i}\,.$$

## APPENDIX D

In this appendix I shall find an exact expression for the neutrino parity-violating current on the rotation axis. It is clear from the symmetry of the problem that $\langle\vec{J}(0)\rangle$ is parallel (or antiparallel) to $\vec{\Omega}$, and thus it is sufficient to calculate $\vec{\Omega}\cdot\langle\vec{J}(0)\rangle$. Furthermore, it is easily seen that the term proportional to $\vec{\gamma}\cdot\vec{p}$ in $S_0^{(\nu)}(\vec{p},\zeta_l)$ vanishes upon integration over $\vec{p}$ and that terms independent of $\gamma^5$ drop out after taking the trace. Omitting all these terms we get

$$\vec{\Omega}\cdot\langle\vec{J}(0)\rangle = -(2\beta^{-1}(2\pi)^{-3}\sum_l \exp(\epsilon\zeta_l)\int d^3p\,\mathrm{Tr}\{\vec{\Omega}\cdot\vec{\Sigma}\exp(\tfrac{1}{2}\vec{\Omega}\cdot\vec{\Sigma}\partial/\partial\zeta_l)\}\zeta_l(\zeta_l^2-p^2)^{-1}, \tag{D1}$$

where I have used that $\vec{\Sigma}=\gamma^0\vec{\gamma}\gamma^5$.

Since $\vec{\Omega}\cdot\vec{\Sigma}=\Omega\Sigma_3$ and $\Sigma_3=\mathrm{diag}(1,-1,1,-1)$, we easily find

$$\mathrm{Tr}\{\vec{\Omega}\cdot\vec{\Sigma}\exp(\tfrac{1}{2}\vec{\Omega}\cdot\vec{\Sigma}\partial/\partial\zeta_l)\} = 2\Omega[\exp(\tfrac{1}{2}\Omega\partial/\partial\zeta_l)-\exp(-\tfrac{1}{2}\Omega\partial/\partial\zeta_l)]. \tag{D2}$$

Using the relation

$$\exp(\alpha d/dx)f(x)=f(x+\alpha), \tag{D3}$$

we can now rewrite Eq. (D1) as

$$\vec{\Omega}\cdot\langle\vec{J}(0)\rangle = -\Omega\beta^{-1}(2\pi)^{-3}\sum_l \exp(\epsilon\zeta_l)\int d^3p\{(\zeta_l+\Omega/2)[(\zeta_l+\Omega/2)^2-p^2]^{-1}-(\zeta_l-\Omega/2)[(\zeta_l-\Omega/2)^2-p^2]^{-1}\}. \tag{D4}$$

Using the same device as in Sec. IV to replace the sum over $l$ by an integral, we find

$$\begin{aligned}\vec{\Omega}\cdot\langle\vec{J}(0)\rangle &= \Omega(2\beta\pi^2)^{-1}\int_0^\infty dp\,p^2[f_0(p+\Omega/2)-f_0(p-\Omega/2)]\\ &= -(2\pi^2\beta^3)^{-1}\sinh\xi\int_0^\infty(\cosh x+\cosh\xi)^{-1}x^2dx,\end{aligned} \tag{D5}$$

where $\xi=\Omega\beta/2$. The integral in Eq. (D5) can be evaluated,[7]

$$\int_0^\infty(\cosh x+\cosh\xi)^{-1}x^2dx=\xi(\pi^2+\xi^2)/3\sinh\xi \tag{D6}$$

and we obtain

$$\langle\vec{J}(0)\rangle = -\vec{\Omega}(T^2/12+\Omega^2/48\pi^2), \tag{D7}$$

in agreement with Ref. 3.

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[1]For a review see, e.g., A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics*, translated and edited by R. Silverman (Prentice-Hall, Englewood Cliffs, N. Y., 1963).

[2]C. W. Bernard, Phys. Rev. D 9, 3312 (1974); L. Dolan and R. Jackiw, *ibid.* 9, 3320 (1974); S. Weinberg, *ibid.* 9, 3357 (1974). Other references can be found in R. Hakim, Riv. Nuovo Cimento 1, No. 6 (1978).

[3]A. Vilenkin, Phys. Rev. D 20, 1807 (1979); Phys. Lett. 80B, 150 (1978).

[4]L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, London, 1969).

[5]Note that $n_{\omega m}$ has a singularity at $\omega=m\Omega$. This singularity, however, is unphysical. A rotating system cannot have size greater than $\Omega^{-1}$ (otherwise the velocity at the boundary would exceed the velocity of light), and in a finite system the energy is quantized in such a way that $\omega$ is always greater than $m\Omega$. (There are some exceptions in which the field has exponentially growing modes. See Ref. 6.) As an example, consider an infinite cylinder of radius $R$ rotating around its axis. Requiring that $\Psi$ vanishes at the boundary, we find the energy levels $\omega_{nmp}=(p^2+\mu^2+\xi_{mn}{}^2R^{-2})^{1/2}$, where $\xi_{mn}$ is the $n$th root of $J_m(x)$. It can be shown (Ref. 7) that $\xi_{mn}>m$. Thus, $\omega_{nmp}>\xi_{mn}R^{-1}>m\Omega$. In the present paper we shall assume that the lowest energy modes are unimportant and thus the infinite-space solutions (17) can be used.

[6]Ya. B. Zeldovich, Pis'ma Zh. Eksp. Teor. Fiz. 14, 270 (1971) [JETP Lett. 14, 180 (1971)]; W. M. Press and S. A. Teukolsky, Nature 238, 211 (1972); A. Vilenkin, Phys. Lett. 78B, 301 (1978).

[7]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1965).

[8]In fact, Eq. (24) follows directly from the definition of the temperature Green's function (see Ref. 2) and holds for interacting as well as for free fields.

[9]In this paper I use *physical*, not coordinate components of vectors, so that $A_k=\vec{A}\cdot\vec{e}_k$, where $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ are unit vectors parallel to the local coordinate axes.

[10]The matrices $\gamma^\mu$ and $\gamma^5$ are taken in the representation of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[11]In Ref. 3, the matrix $\gamma^5$ and the angular quantum number $m$ were taken with a wrong sign. The correct equations are obtained by changing $L\to -L$ and $m\to -m$ in Eqs. (15), (19), and (20) of Ref. 3. All the following equations, including the final results, do not change.

[12]See, e.g., S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson, Evanston, Illinois, 1961).

[13]A nonvanishing zero-temperature value of $\langle\vec{J}(0)\rangle$ in Eq. (83) is due to the fact that the Fermi distribution function

$$f_{\omega m}=\{\exp[\beta(\omega-m\Omega)]+1\}^{-1}$$

does not vanish for $\omega<m\Omega$, even at $T=0$. It seems reasonable to assume that the finite size of the system modifies the particle spectrum in such a way that $\omega$ is

always greater than $m\Omega$. I have proved this statement for a scalar field (Ref. 5). Quite similarly, it can be proved for electromagnetic and nonrelativistic fermion fields. The case of relativistic fermions and neutrinos requires a special treatment because of the well-known difficulties with confinement of these particles to a fixed volume.

[14]This point has been missed in Ref. 3.

[15]$[\rho_1, \rho_2] = 0$, since the interaction of the subsystems is neglected.