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Cosmological limits on photon splitting

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General conservation laws and symmetries permit one photon to split into three all traveling in the original direction. But QED predicts that this process has vanishing probability. Observations of the 3-K microwave background and optical line spectra of quasars can be used to test this over cosmic length scales. We discuss photon splitting within the framework of a Lorentz-covariant, phenomenological theory. This theory predicts (a) spectral broadening, (b) frequency-dependent red-shift, and (c) multiple red-shifts for each source. Because these effects do not show up, we can set observational limits on the parameters that govern the splitting. In particular, the 3-K background implies that a photon of arbitrary wavelength λ propagating in flat space (i.e., without cosmological reddening) must have a decay lifetime larger than (6 cm/λ)t, where t is the present age of the universe.

I. INTRODUCTION

Physicists commonly assume that massless particles such as photons and neutrinos are stable. But Havas has emphasized[1] that the known conservation laws and symmetries do not guarantee this. Decays of the form $A\rightarrow B+C+\cdots+D$, with all particles, $A,\ldots,D$ massless, will conserve energy and momentum provided the particles all travel in the same direction and

$$\omega_A=\omega_B+\omega_C+\cdots+\omega_D\,.$$

In addition, there are some other constraints: Conservation of lepton number requires that μ- and e-type neutrinos be separately produced in particle-antiparticle pairs. The Pauli exclusion principle implies that no two decay neutrinos (or antineutrinos) can have the same energy,[2] and invariance under charge conjugation for processes with only photons incoming or outgoing implies that a single photon must always split up to an odd number[3] $\gamma\rightarrow k\gamma$, where $k=3,5,7,\ldots$.

Two conceivable decay modes of a photon, $\gamma\rightarrow 3\gamma$ and $\gamma\rightarrow\gamma+\nu+\bar{\nu}$, are illustrated in Fig. 1. The pure photon process is mediated by virtual electrons and positrons while the decay with neutrinos also involves a virtual W meson, hypothetical mediator of the weak force. We shall focus in this paper on photon triple splitting, $\gamma\rightarrow 3\gamma$, because it appears to be the dominant decay mode. The neutrino process should be negligible by comparison because the weak coupling constant is very small compared to the electromagnetic.[4] Likewise we neglect higher-order processes such as $\gamma\rightarrow 5\gamma$.

A detailed calculation[5] shows that photon triple breakup has zero probability in quantum electrodynamics. This had long been suspected because the phase-space volume of the collinear decay photons vanishes. (For further discussion of nonlinear effects in QED arising from vacuum polarization, such as photon-photon scattering, see the paper of Yueh, Ref. 6, and references therein.)

Our aim in this work is to report an observational check on this prediction. Granted that the numerical accuracy of QED on the laboratory scale is outstanding, a check on the cosmic length scale is nevertheless desirable. We here use optical observations of quasar spectral lines and radio measurements of the 3-K microwave background to set a stringent upper limit on the decay process $\gamma\rightarrow 3\gamma$.

But before we can assess this data, we need a theoretical framework and language for photon splitting. We therefore develop in this paper a phenomenological, Lorentz-covariant theory of the effect. We give a linear integro-differential equation governing the frequency distribution of

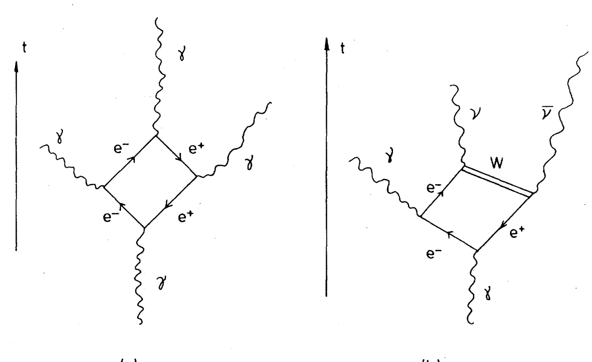

FIG. 1. Feynman diagram for splitting of a photon. (a) Shows how $\gamma\rightarrow 3\gamma$ could be mediated by virtual electrons and positrons. (b) Represents $\gamma\rightarrow\gamma+\nu+\bar{\nu}$. The neutrino lines are connected by a virtual W boson. Because the weak coupling is much smaller than the electromagnetic, process (a) should dominate (b).

decay photons. This determines how the spectrum changes with time. The equation contains two unknown quantities which must be determined either from quantum theory or from experiment. These are a constant A proportional to the lifetime of a photon and a function $f(x)$, $0 \leq x \leq 1$, determining the distribution of decay photons smaller in energy by a factor x than the original one. We aim to determine *observational limits* on A and $f(x)$ using photons that have traversed cosmic distances.

Our theoretical framework is general enough to embrace a wide class of microscopic models. From quantum theory we have taken only (1) the motivation (Fig. 1) for considering splitting, (2) the idea that the splitting is probabilistic, and (3) $E = pc = \hbar\nu$, the quantal formulas for momentum and energy.

We can classify the breakup into types I, II, or III accordingly as 1, 2, or all 3 fragments get most of the original energy (and share it more or less equally). It seems likely that only a type-I splitting would display sharp peaks. If so, a photon of frequency ω would produce a "high-energy" fragment of frequency peaked about $(1-\epsilon)\omega$, where $0 < \epsilon \ll 1$. This is interesting because, if sharp enough, the decay can preserve spectral lines; otherwise the lines tend to get smeared out and lost in the background noise.

Photon splitting leads to three new effects:

(a) broadening, i.e., increase of the relative width in frequency $\Delta\omega/\omega$,
(b) frequency-dependent red-shift, and
(c) multiple red-shifts for a single source.

Effects (b) and (c) apply only to a sharply peaked decay law, i.e., one favoring certain precise fragment frequencies. Note that the ratio $\Delta\omega/\omega$ in (a) is unaffected by Doppler, gravitational, or cosmological red-shifts, which change numerator and denominator by the same factor. Failure to detect any of these effects (a), (b), or (c) leads to limits on the decay parameters. We find that a photon of wavelength less than 6 cm propagating in empty space has, discounting reddening from cosmic expansion, a decay lifetime exceeding the present age of the universe.

Our basic assumptions are as follows:

(1) The splitting is irreversible. The number-density four-vector of photons obeys a simple first-order linear decay law modeled on the exponential decay of radioactive nuclei. A photon of frequency ω has a mean lifetime $\tau = A\omega$, where A is a universal constant.

(2) We represent the universe by the standard Robertson-Walker models with vanishing cosmological constant $\Lambda = 0$.

(3) Quasars are at the large distances given by assuming that their red-shifts arise mostly from cosmological expansion.

(4) The 3-K microwaves were decoupled from matter at an early epoch with a perfectly Planckian spectrum.

(5) Scattering of photons by real particles or fields (as opposed to virtual ones—see Fig. 1) is neglected. We also neglect decay channels other than three-photon breakup.

It was suggested by several authors[7] that type-I splitting might give an alternative explanation of the cosmological red-shift. Assume that only the high-energy fragment is observed. Then in a time $nA\omega_0$ a photon of frequency ω_0 undergoes about n splittings. Its final frequency

$$(1-\epsilon)^n \omega_0 \simeq (1-n\epsilon)\omega_0 \quad (\text{for } n \ll 1/\epsilon)$$

decreases in proportion to the distance traveled, the same as for the conventional Hubble red-shift. One can therefore object that assumption (3) is too strong: It disregards the possibility that most of the observed red-shift comes from splitting instead of cosmic expansion. We will see, however, that this is impossible because, contrary to observation, a red-shift due to splitting alone depends markedly on the initial frequency. We do not consider alternative, noncosmological possibilities for quasar red-shifts.

The 3-K microwaves provide much more stringent limits on splitting than the visible lines of quasars. The reason is that the microwaves, with typical wavelengths from about 1 mm to 10 cm have a frequency $\sim 10^3$–10^5 times smaller than that of visible light. Hence, for comparable travel times, the microwave photons undergo many more splittings than the optical ones.

Section II gives the basic theoretical foundations, namely, collinearity of the decay, and Havas's reasoning[1] concerning the lifetime of unstable, massless particles.

Section III develops the Lorentz-covariant decay law—first for flat spacetime in III A, then we generalize this to the standard homogeneous and isotropic cosmological model in III B. The specializations of the decay law to the microwaves and to the light from quasars follow different paths but end in two equations which look the same. In III C, we use this common equation to find the amount of reddening and broadening of any given spectrum.

Section IV compares the theory with observations of optical quasar lines. Here type-I decays are the only interesting candidates. We use the observed red-shift and width of absorption lines to put upper limits on the fractional energy loss per decay ϵ as a function of A, or, more intuitively, of that wavelength λ_0 which decays in, say, 2×10^{10} years in flat space. (There is an *apparent* loss of energy, although splitting conserves energy, because in type-I decays two of the three fragments normally escape detection.) We also explain why multiple red-shifts observed in absorption lines do not come from splitting.

We find in Sec. V that the 3-K microwave spectrum is narrower than a Planckian spectrum, not broader—contrary to the prediction from photon splitting. But taking inaccuracy of measurement into account, some small effect of splitting may be present. We find at the 99.9% confidence level (about 3 standard deviations) the wavelength of photons that can decay in the present age of the universe (which age we leave open) is larger than 6 cm. Section VI sums up our findings.

An appendix describes how we made a conservative estimate of the relative width of the 3-K microwave background from measurements below the frequency of the peak.

II. THEORETICAL FOUNDATIONS

It is easy to prove, following Herrera,[5] that an unstable massless particle must decay to other massless particles moving in the original direction. Let particle A decay to several particles B_j, $j=1,2,\ldots$. Conservation of momentum and energy require

$$\vec{k}_A = \sum \vec{k}_j \,, \tag{1}$$

$$\omega_A = \sum \omega_j \,, \tag{2}$$

where A is massless, but B_j has rest mass m_j: $|\vec{k}_A| = \omega_A$, $|\vec{k}_j| = (\omega_j^2 - m_j^2)^{1/2}$. If A is incident along $+\hat{z}$ while B_j makes an angle θ_j, then taking the scalar product of (1) with $\hat{z}$,

$$\omega_A = \sum |\vec{k}_j| \cos\theta_j \le \sum |\vec{k}_j| \le \sum \omega_j \,.$$

For the equals sign to hold true, in accord with (2), all $m_j = \theta_j = 0$. This proves that the decay products are indeed massless and collinear.

In classical electrodynamics, a plane wave traveling one way in empty, flat space is stable. Its position changes but nothing else. If it has a precise wavelength λ, then it will also have a precise angular frequency $\omega = 2\pi c/\lambda$, as follows from the homogeneous wave equation. Such a monochromatic plane wave corresponds to a photon in quantum field theory.[8]

Photons having an exact wavelength and thus infinite extent are not suitable for discussing splitting as a function of distance traveled. Our "photons" must rather be spatially localized wave packets containing a range of wavelengths and frequencies. Their energy-momentum $(\omega, \vec{k})$ (in units such that $\hbar = 1$) and position $\vec{x}$ are approximately well defined at any given time t. The center of the packet moves at nearly the speed of light. When splitting occurs, one must (mentally) group the resulting harmonics into new photon wave packets, each with a small spread in ω, $\vec{k}$, and $\vec{x}$.

How large should the spread in position $L \equiv |\Delta\vec{x}|$ be? Since the packet is nearly monochromatic, the dominant wavelength $\lambda \ll L$. And since our local (tangent space) description of photons will neglect the spacetime curvature, $L \ll$ gravitational length scale. This is not restrictive: Even in the extreme case that the photon grazes the surface of a solar-mass black hole, $L \ll 3$ km.

Let us review Havas's argument[1] why an unstable particle (with or without mass) has a lifetime proportional to (angular) frequency, $\tau = A\omega$.

The quantum, emitted with four-momentum $(\omega, \vec{k})$, traverses a coordinate interval $(\Delta t, \Delta\vec{x})$ before decaying. Because these two four-vectors V^μ satisfy the same linear, homogeneous Lorentz transformation ($V'^0 = \Lambda^0{}_\alpha V^\alpha$) and because they are proportional ($\Delta\vec{x}/\Delta t = \vec{k}/\omega$), one finds that the constant of proportionality is a Lorentz scalar: $\Delta t'/\omega' = \Delta t/\omega$. (It is not a sign-changing "pseudoscalar.")

Now consider an ensemble of identical particles in the same force-free state, i.e., with the same four-momentum and spin orientation. Although the individual lifetimes vary from particle to particel, their mean $\tau \equiv \langle \Delta t \rangle$ is well defined; from the above, $\langle \Delta t/\omega \rangle = \tau/\omega$ is the same for all Lorentz frames. We denote this ratio by A. It depends only on the species of particle. The direction of motion is irrelevant because a rotation of the coordinate system is just a particular kind of Lorentz transform and must therefore leave A unchanged. A similar remark goes for the spin orientation. (In the case of massless particles, however, we must resort to the improper Lorentz group because the two distinct "helicities" $\hat{s}\cdot\hat{p} = \pm 1$ require a space-inverting parity operation to be interconverted.) Thus the constant A will have one value for photons—infinity if they are stable—and another value for neutrinos, since photons and neutrinos couple differently to virtual particles mediating the decay.

III. THE DECAY LAW

A. Flat spacetime

There are many possibilities for the decay law but physical considerations plus Lorentz covariance lead to an essentially unique choice. The simplest and most familiar type of decay is the exponential variety, described by

$$dn/dt = -\alpha n \,. \tag{3}$$

Here n might be, for example, the number density of some radioactive nucleus. Equation (3) suggests that our decay law for photons also be *linear* in the *number density*, and a *homogeneous* equation of the *first order*.

Using the number density, instead of the total number within some standard volume, permits us to write a "local" equation valid at a point. And it has the advantage over electromagnetic fields or potentials that the resulting equation can be transferred without difficulty to other massless particles.

An equation of first order is most economical. It can guarantee decay whereas one of the second order, such as $d^2n/dt^2 = \alpha^2 n$, would require an auxiliary condition to avoid growing solutions.

The equation should be linear and homogenous so that the decay of each particle is independent of the others present: Multiplying any solution by a constant yields another solution. Calculations of photon-photon scattering[6,9] suggest that nonlinear terms are only to be expected at extremely high number densities. We can safely neglect these in the systems considered here. (The last two terms on the right-hand side of $dn/dt = -\alpha n - \beta n^2 - \gamma$ exemplify what we mean by "nonlinear".)

We start our Lorentz-covariant theory[10] by defining the number-flux density four-vector $n^\mu(x,\omega,j)d\omega$. The zeroth component, $n^0(x,\omega,j)d\omega$, is the number of photons per cm^3 at $x=(t,\vec{x})$ with frequency in $(\omega, \omega+d\omega)$ that resulted from j decays. Photons that never decayed since emission have $j=0$, those due to a single decay have $j=1$, and so on. Notice that n^μ without the factor $d\omega$ is not a four-vector because $d\omega$, like ω, is not a scalar.

Now we generalize both sides of (3) in a Lorentz-covariant way. First consider photons all moving along the same direction $\hat{d}$. Replace dn/dt by the scalar

$$\partial(n^0 d\omega)/\partial t + \vec{\nabla}\cdot(\vec{n}d\omega) = (n^\mu d\omega)_{,\mu}\ .$$

The number of jth-order quanta of a certain energy decreases by their decay but increases by splitting of $(j-1)$st-order quanta. The net rate of change per unit volume $(n^\mu d\omega)_{,\mu}$ is the sum of these two terms, taking into account the appropriate mean lifetime, $\tau = A\omega$, for each:

$$[n^\mu_{(d)}(x,\omega,j)d\omega]_{,\mu} = \left[\int d\omega' n^0_{(d)}(x,\omega',j-1)(A\omega')^{-1}\right] d\omega F(\omega,\omega') - n^0_{(d)}(x,\omega,j)(A\omega)^{-1}d\omega\ . \tag{4}$$

The quantity $F(\omega,\omega')d\omega$ gives the frequency distribution of the decay products in interval $d\omega$ about ω when a photon of frequency ω' splits.

The right-hand side of (4) becomes a Lorentz scalar, like the left-hand side, if we rewrite $d\omega F(\omega,\omega')$ as $d(\omega/\omega')f(\omega/\omega')$. [Here and in the following equations $d(\omega/\omega') = d\omega/\omega'$, since *we will always first integrate over* ω/ω' *or* ω *while holding* ω' *fixed, then over* ω'. The reader should keep in mind that in all equations like (4), the integration over ω has not yet been performed—it was therefore written outside the brackets—but it will be the first integral carried out eventually in the double integral.] All terms on the right-hand side of (4) then depend on the Lorentz-invariant ratios of parallel four-vectors. To see this, recall that all photons have (up to now) the same direction and hence the same three-velocity $\vec{v}$; their differential number density $n^\mu_{(d)}d\omega = n^0_{(d)}d\omega(1,\vec{v})$ and four-momentum $k^\mu = (\omega,\vec{k}) = \omega(1,\vec{v})$ are indeed parallel, and $n^0_{(d)}d\omega/\omega$ is their invariant ratio. Similar remarks apply to ω/ω': Recalling the collinearity, we see this is also an invariant.

Defining the total number-flux density $n^\mu d\omega$ by summing over all directions $\hat{d}$, the covariant law of photon splitting in flat space becomes

$$[n^\mu(x,\omega,j)d\omega]_{,\mu} = \left[\int d\omega' n^0(x,\omega',j-1)(A\omega')^{-1}\right] d(\omega/\omega')\ f(\omega/\omega') - n^0(x,\omega,j)(A\omega)^{-1}d\omega\ . \tag{5}$$

Fragments always have lower energy than the parent particle so the lower limit of integration over ω' should be ω. But $f(x)=0$ for $x>1$ permitted us to extend the integral down to $\omega'=0$. We will see later that $\int_0^\infty f(x)dx$ = number of fragments observable in each decay of a single photon. The spins of the decay photons will not concern us.

B. Decay law in expanding universe

Next we include the effect of cosmological expansion on the radiation spectrum. The only new fact needing to be incorporated is that all photons are undergoing a steady reddening. As their frequency drops, their lifetime increases.

We do not include any effect of photon pair production by the time-varying geometry. Parker[11] has proven that there will be no such pairs produced: All conformally invariant wave equations (such as Maxwell's) give zero pair production in the conformally static Robertson-Walker (RW) universes. Even if the universe were nonstatic and antisotropic,[12] the predicted production would be negligible except immediately after the big bang.

The homogeneous, isotropic model of the universe has RW metric[13]

$$ds^2 = -dt^2 + R(t)^2[dr^2/(1-kr^2) + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2]\ , \tag{6}$$

with the value of $k=+1,0,-1$ an initial condition. We extend the decay law (5) to this geometry by: (1) replacing the partial derivative (denoted by subscript comma) by a covariant derivative (denoted by subscript semicolon) and (2) taking account of the expansion-induced reddening. The red-shift increases all wavelengths in proportion to $R(t)$, so for massless quanta with $\omega=2\pi/\lambda$, $\omega(t)\propto 1/R(t)$. (This refers to a standard observer, locally at rest, i.e., with r,θ,ϕ constant.) It is therefore convenient to define a red-shift-compensated frequency Ω by

$$\Omega\equiv R(t)\omega\,.$$

If photons were stable, the spectral distribution in Ω would not change.

Because the number density is the same whether described in terms of ω or Ω, we naturally define $n^\mu(\Omega)d\Omega\equiv n^\mu(\omega)d\omega$. Thus in the RW universe, the decay law (5) becomes

$$[n^\mu(x,\Omega,j)d\Omega]_{;\mu}=R(t)\left\{\left[\int d\Omega' n^t(x,\Omega',j-1)(A\Omega')^{-1}\right]d(\Omega/\Omega')f(\Omega/\Omega')-d\Omega n^t(x,\Omega,j)(A\Omega)^{-1}\right\}. \tag{7}$$

The extra factor $R(t)$ outside offsets another one inserted in the lifetime. Since all terms in (7) refer to the same time, $R(t)$ is effectively a constant in the differentials: $d\Omega=R(t)d\omega$. Note that the derivative on the left affects only the x, not Ω or j. Recall once again that the order of integration in (7) is firstly over (Ω/Ω'), secondly over Ω'.

To put (7) in a more useful form, we will replace n^μ by a more physical measure of photon numbers, and we will rewrite the covariant divergence as a time derivative. The treatment of the 3-K microwaves and that of quasars are very different, but the resulting equations turn out formally identical.

Consider first the 3-K microwaves. Since these are propagating isotropically, $n^r=n^\theta=n^\phi=0$. For four-vector $B^\mu\equiv n^\mu d\Omega$, putting $g=\det(g_{\mu\nu})$, we have

$$B^\mu{}_{;\mu}=(-g)^{-1/2}[(-g)^{1/2}B^\mu]_{,\mu}\,. \tag{8}$$

Inserting g from (6), we find

$$(n^\mu d\Omega)_{;\mu}=R^{-3}(t)[R^3(t)n^t d\Omega]_{,t}\,.$$

Substituting this in (7), and multiplying through by $R^3(t)$, one obtains

$$[N_{3K}(t,\Omega,j)d\Omega]_{,t}=R(t)\left\{\left[\int_0^\infty d\Omega' N_{3K}(t,\Omega',j-1)(A\Omega')^{-1}\right]d(\Omega/\Omega')f(\Omega/\Omega')-N_{3K}(t,\Omega,j)(A\Omega)^{-1}d\Omega\right\}. \tag{9}$$

Here $N_{3K}(t,\Omega,j)d\Omega$ is the total number of microwave photons (of the type specified by Ω,j) in a three-volume $\propto R^3(t)$ that shares the cosmic expansion:

$$N_{3K}(t,\Omega,j)\equiv\text{const}\times R^3(t)n^t(t,\Omega,j)\,. \tag{10}$$

An argument t replaces x because the radiation is homogeneous over space, hence independent of r,θ,ϕ. The quantity N_{3K} is the appropriate one for us because it changes only in the presence of photon splitting, whereas the density n^t decreases due to universal expansion even for stable quanta.

In studying the decaying radiation emitted by a quasar, however, we must not use a single standard observer. We wish to observe the time evolution of a fixed parcel of radiation as it propagates through space. A single observer, in contrast, continually receives new parcels of radiation, which is not what we want. Instead we must imagine a sequence of standard observers lined up like the light bulbs on a movie marquee. Each observer is at rest in r,θ,ϕ and measures the radiation pulse of interest as it goes by him.

Take $r=0$ at the center of the quasar. Follow a spherical pulse as it spreads out in all directions from $r=0$. Its isotropy ensures that $n^\theta=n^\phi=0$, but n^r and n^t do not vanish. Since a local observer (orthonormal tetrad or local Minkowskian coordinates) sees $n^t d\Omega$ quanta per cm^3, and since $(-g)^{1/2}$ converts from coordinate volume to proper volume (whether three- or four-dimensional, since $g_{tt}=-1$), the total number of (Ω,j)-type quanta in the pulse at time t is $N_Q(t,\Omega,j)d\Omega$, where

$$N_Q(t,\Omega,j)\equiv\int dr\,d\theta\,d\Phi(-g)^{1/2}n^t(x,\Omega,j)\,. \tag{11}$$

If we use (8) and multiply (7) through by $(-g)^{1/2}$, the left-hand side becomes

$$[(-g)^{1/2}n^t d\Omega]_{,t}+[(-g)^{1/2}n^r d\Omega]_{,r}\,. \tag{12}$$

Integrating over r and using the boundary condition $n^\mu=0$ on the front and back surfaces eliminates the second term. For any function $f(r,t)$ and time-dependent limits $r_j=r_j(t)$, we have the identity

$$\int_{r_1}^{r_2}f_{,t}dr=\left[\int_{r_1}^{r_2}f dr\right]_{,t}-f dr/dt\Big|_{r_1}^{r_2}.$$

This identity together with the same boundary condition permits us to bring the r integral of the first term in (12) under the time derivative. Finally integrate over θ and ϕ. The result looks exactly like (9), except that N_Q, defined in (11), takes the place of N_{3K}. We thus have the gratifying result that the decay law for photons from a localized spherical source ("quasar") is formally identical to that for an isotropic gas of photons ("3-K microwaves"). This coincidence may not be too surprising because N_{3K} and N_Q have similar interpretations: they both measure the total number of quanta, of type (Ω, j), in a volume "comoving" with the radiation, i.e., a volume with zero net flux through its walls. For the 3-K photons, the net flux vanishes because an equal number of quanta pass through the walls in opposite directions. In the next section, we derive from (9) some specific quantitative predictions.

C. Decay broadening and reddening

Photon splitting evidently produces two effects: (1) There is a progression towards lower frequencies and (2) since a single frequency breaks up into several different ones, the relative width (=dispersion/mean) of the frequency spectrum tends to increase.[14] Here we find explicit formulas for these effects taking observational selection into account.

Dropping the subscripts on N_{3K} and N_Q, the mean of any function $F(\Omega)$ at time t is

$$\langle F(\Omega); t\rangle_u = \sum_{j=0}^{\infty} \int_0^\infty d\Omega\, F(\Omega) N(t, \Omega, j). \tag{13}$$

This is unnormalized (subscript "u") because the total number of quanta $N(t) \equiv \langle 1; t\rangle_u$ increases at each splitting. The normalized mean, or mean value per quantum, is

$$\langle F(\Omega); t\rangle = \langle F(\Omega); t\rangle_u / N(t). \tag{14}$$

To include selection, we should replace $f(\Omega/\Omega')$ in (9) by $s(\Omega) f(\Omega/\Omega')$ where the detection efficiency $s(\Omega)$ lies between 0 and 1. Another, approximate, approach should be adequate for the quasar lines, assuming that the splitting is type I so that only the high-energy fragment gets detected. Then we can incorporate in $f(\Omega/\Omega')$ of (9) an additional factor

$$p(\Omega/\Omega') \simeq \begin{cases} 1, & \Omega \simeq \Omega' \\ 0, & \Omega \ll \Omega' \end{cases}.$$

By dropping to zero, it implies that low-energy fragments or their descendants will not be seen. (Remember: Ω' refers to the original photon, Ω to its daughter.)

It follows from (9) and (13) that the averaged pth power of Ω changes at a rate

$$\frac{d}{dt}\langle \Omega^p; t\rangle_u = R(t)(f_p - 1) A^{-1} \langle \Omega^{p-1}; t\rangle_u, \tag{15}$$

where the constants defined by

$$f_p \equiv \int_0^1 dx\, x^p f(x) \tag{16}$$

depend on the distribution function for daughter-photon frequencies $f(\Omega/\Omega')$. Setting $p=0$ in (15) gives the rate of increase of the total number of photons, $N(t) = \langle 1; t\rangle_u$, in the chosen volume.

Since $f(x) \geq 0$, we see that $f_0 > f_1 > f_2 > \cdots > 0$. If all decay fragments are detected (no selection), we can prove that $f_0 = 3$ and $f_1 = 1$. Consider a monochromatic beam of N photons, all of frequency ω at $t=0$. At this moment, three daughter photons are being generated for each parent dying with lifetime $A\omega$, so

$$dN(t)/dt|_0 = (3-1)N(0)/A\omega .$$

But putting

$$\langle \Omega^{-1}; 0\rangle_u = N(0)[R(0)\omega]^{-1}$$

and $p=0$, $t=0$ in (15) gives

$$dN(t)/dt|_0 = (f_0 - 1)N(0)/A\omega .$$

Comparing these, $f_0 = 3$. When there is selection, f_0 = number of fragments detected <3.

We also know that splitting conserves energy. Suppose some photons have all been emitted prior to $t = t_0$. The sum of $\Omega = R(t)\omega$ over all photons is a constant equal [apart from a factor $R(t_0)/\hbar$] to the total energy at t_0. But this sum is $\langle \Omega; t\rangle_u$. Hence the left-hand side of (15) with $p=1$ vanishes and we conclude $f_1 = 1$. If there is selection, the initial energy calculated at later times appears to decrease and $f_1 < 1$.

From (14), (15) and $N(t) = \langle 1; t\rangle_u$ the mean value per photon changes at a rate

$$\frac{d}{dt}\langle \Omega^p; t\rangle = R(t) A^{-1}[(f_p - 1)\langle \Omega^{p-1}; t\rangle - (f_0 - 1)\langle \Omega^p; t\rangle\langle \Omega^{-1}; t\rangle]. \tag{17}$$

For optical spectral lines, we only need consider type-I splitting, in which the two low-energy daughters escape detection. Other types (II and III) either smear out and destroy lines, or, if sharply peaked, produce unacceptably large red-shifts. In this case (type I), $f_0 = 1$ and the total number of quanta seen is conserved, $\langle 1; t\rangle_u$ = const. Equation (15) may be integrated exactly for all p by induction. For $p=1$,

$$\langle \Omega; t\rangle = \langle \Omega; t_0\rangle = (1 - f_1) S(t)/A , \tag{18}$$

where we define

$$S(t) \equiv \int_{t_0}^t R(u) du . \tag{19}$$

Since $f_1 < 1$, $(1-f_1)>0$. For a static universe, $R(t)=\text{const}$, (18) yields $d\langle\omega\rangle/dt = -(1-f_1)/A$. Let ϵ = the mean (apparent) fractional energy loss (due to the energy hidden in the unobserved fragments). Then $d\langle\omega\rangle/dt = -\epsilon\omega/\tau$. Comparison shows that $1-f_1=\epsilon$.

Define the variance of Ω by

$$(\Delta\Omega)^2 \equiv \langle\Omega^2\rangle - \langle\Omega\rangle^2 . \tag{20}$$

Substituting (18) in the right-hand side, we integrate (17) for $p=2$ to obtain $\langle\Omega^2\rangle$. Then we find

$$[\Delta\Omega(t)]^2 = [\Delta\Omega(t_0)]^2 + [(1+f_2-2f_1)/A] \times [\langle\Omega;t_0\rangle S(t) - (1-f_1)S^2(t)/2A] . \tag{21}$$

The combination $(1+f_2-2f_1)$ in (21) has a simple interpretation. Since $f_0 \equiv \int_0^\infty f(x)dx = 1$, we can use $f(x)$ as a normalized distribution function to calculate, e.g., the mean value of some function $q(x)$ by $\overline{q} \equiv \int_0^\infty q(x)f(x)dx$. Define the variance

$$w^2 \equiv \overline{(x^2)} - \overline{x}^2 . \tag{22}$$

From the definition (16) of f_p, this is $w^2 = f_2 - f_1^2$. Since $\epsilon = 1 - f_1$, we find

$$\epsilon^2 + w^2 = 1 + f_2 - 2f_1 . \tag{23}$$

Equation (22) shows that w is the dimensionless width for decay: When photons of frequency ω_0 split, their daughters have an rms spread in frequency of $w\omega_0$.

It has been suggested that a sequence of type-I decays in a static universe could be the real cause of the cosmological red-shift.[7] Let us show now that this is impossible because (18) predicts a frequency-dependent red-shift, contrary to observation. When $R(t)=\text{const}$, (18) gives that the red-shift $z = \langle\lambda\rangle/\langle\lambda_0\rangle - 1 = \omega_0/\langle\omega\rangle - 1$ of a sharp spectral line depends on its original frequency $\omega_0 \equiv \langle\omega;t_0\rangle$ through

$$(1+z)^{-1} = 1-(1-f_1)(t-t_0)/(A\omega_0) . \tag{24}$$

(This shows that breakup to $\langle\omega\rangle = 0$ or $z=\infty$ occurs in a finite time.) Consider the variation of red-shift with frequency for photons emitted from the same source. Holding $t-t_0$ fixed, the differential of (24) is

$$dz = -z(1+z)d\omega_0/\omega_0 . \tag{25}$$

This implies that for a quasar at $z\simeq 2$, a spectral line with source frequency 5% smaller will be observed with z larger by about 0.30. Observations conclusively contradict this: E.g., two carbon absorption lines in quasar 3C191 with $\lambda_0 \simeq 1176$ and 1549 Å, respectively, both have $z=1.947$ to three decimal places.[15] We conclude that photon splitting can only provide a *small part* of the observed frequency-independent z. Most of it presumably is cosmological.

Define the relative variance by

$$v = (\Delta\omega)^2/\langle\omega\rangle^2 = (\Delta\Omega)^2/\langle\Omega\rangle^2 . \tag{26}$$

As the last member shows v is independent of $R(t)$, so any change of v reflects the effect of splitting itself. (We neglect scattering and absorption.)

By means of (17), we find

$$dv(t)/dt = R(t)A^{-1}\langle\Omega;t\rangle^{-1}\{-(1-f_2)+2(1-f_1)[1+v(t)]+(f_0-1)[1+v(t)]\langle\Omega^{-1};t\rangle\langle\Omega;t\rangle\} . \tag{27}$$

This exact formula permits us to deduce that *the complete spectrum (no selection) always gets broader.* With $f_0=3$, $f_1=1$, (27) becomes, in an abbreviated notation,

$$dv/dt = RA^{-1}\langle\Omega\rangle^{-1}[-(1-f_2)+2(1+v)\langle\Omega\rangle\langle\Omega^{-1}\rangle] .$$

But by Schwarz's inequality[16] $\langle\Omega\rangle\langle\Omega^{-1}\rangle \geq 1$ and with $v, f_2 \geq 0$, this gives

$$dv/dt \geq RA^{-1}\langle\Omega\rangle^{-1}(1+2v) > 0 .$$

Thus the relative width steadily increases, unless photons are stable ($A=\infty$). (All photons splitting to three, each with exactly $\frac{1}{3}$ the frequency, is *not* a counterexample. Even here there is broadening because, although j decays scale all frequencies by 3^{-j}, the value of j has a statistical spread.)

The very slight deviation of the 3-K microwave spectrum from a Planckian suggests that—as with the spectral lines—splitting has played only a small role. We therefore set $\langle\Omega^p;t\rangle \simeq \langle\Omega^p;t_0\rangle$, $v(t)\simeq v(t_0)$ [but $R(t)\neq R(t_0)$] and integrate (27) to get

$$v(t)-v(t_0) \simeq S(t)A^{-1}\langle\Omega;t_0\rangle^{-1}\langle\Omega;t_0\rangle^{-1}\{-(1-f_2)+2(1-f_1)[1+v(t_0)]+(f_0-1)[1+v(t_0)]\langle\Omega;t_0\rangle\langle\Omega^{-1};t_0\rangle\} . \tag{28}$$

We will later use Eq. (28) to discuss broadening of the 3-K microwaves.

IV. COMPARISON OF THEORY WITH QUASAR OBSERVATIONS

A. Frequency-dependent red-shift and broadening

For optical observations of quasars we need only consider type-I splitting where the two low-energy fragments are not detected. Other types of splitting would produce anomalies such as excessively large red-shifts.

As shown above, the theory predicts: (a) frequency-dependent red-shift and (b) broadening. These are not observed. The absence of (a) [(b)] leads to an upper limit on ϵ/A [ϵ^2/A] where ϵ is the fractional energy loss per splitting and A determines the ratio of lifetime to frequency, $\tau = A\omega$. Smallness of ϵ/A or ϵ^2/A means then either that the energy loss is small or else the lifetime is large. By setting $\epsilon \leq 1$ one can obtain a lower limit on A, but a limit better by about three powers of ten will be obtained from the 3-K microwaves.

The quasar 3C191 is a good choice because of its large distance (permitting many decays) and large number of well-identified, sharp absorption lines. These lines have $z_{ab} = 1.947$ and a width of only 8 Å between points of half-maximum intensity (against 20–100 Å for quasar emission lines). Two widely separated carbon lines are CIII ($\lambda = 1175.7$ Å) and CIV ($\lambda = 1549.1$ Å). The observed wavelengths, larger by $1+z_{ab}$ are $\lambda = 3,465$ Å and $\lambda = 4,565$ Å.

But the identical values of z_{ab} could be partly an artifact of the way one analyzes the data. The method used to find z tends to gloss over small differences. To identify two or more lines and assign a red-shift, one proceeds by *assuming* they have the same z and then he sees what value of z will give acceptable rest wavelengths. If this does not work, he may subdivide the set of lines into smaller subgroups, and try to fit the data with a common z value within each subgroup.

Let us suppose, therefore, that the two carbon lines really have a small difference of red-shift due to photon splitting. The maximum uncertainty in z_{ab} comes from the half-line width of 4Å. The less decayed CIII line would have a $z_{ab} < 1.947$ given by

$$(1+z_{ab})^{-1} \lesssim 1175.7/(3465-4) = 0.3397\,, \qquad (29)$$

while the more decayed CIV line would have $z_{ab} > 1.947$ given by

$$(1+z_{ab})^{-1} \gtrsim 1549.1/(4565+4) = 0.3390\,. \qquad (30)$$

For comparison, (18) gives

$$\begin{aligned}(1+z_{ab})^{-1} &= (\omega/\omega_0)\\ &= (R_0/R) - [(1-f_1)S/AR]\omega_0^{-1}\,.\end{aligned}$$

Taking the difference for two lines from the same source,

$$d[(1+z_{ab})^{-1}] = -[(1-f_1)S/AR]d(\omega_0^{-1})\,. \qquad (31)$$

Or since $d[(1+z_{ab})^{-1}] \leq 7\times10^{-4}$ and $d(\omega_0^{-1}) = -2.0 \times 10^{-17}$ sec, this gives

$$(1-f_1)/A \leq 3.5\times10^{13} R/S\,\text{sec}^{-1}\,. \qquad (32)$$

For any RW universe with zero cosmological constant, the curve of $R(t)$ versus t is concave downwards [Ref. 13, p. 472, Eq. (15.1.18)]. Hence the area under the curve between t_0 (emission) and t (now), $S = \int_{t_0}^{t} R(u)du$ satisfies

$$\tfrac{1}{2}(R+R_0)(t-t_0) < S < R(t-t_0)$$

[see Fig. 2(a)]. Moreover, comparison with the linear expansion $R(t) \propto t$ gives $t_0 < R_0 t/R = t/(1+z)$ [Fig. 2(b)], where the last equality neglects the small effect of splitting on z. Hence for all expanding RW universes with $\Lambda = 0$,

$$t^{-1} < R(t)/S(t) < 2(t-t_0)^{-1} < 2(1+z^{-1})t^{-1}\,. \qquad (33)$$

Inserting (33) with $z = 1.947$ into (32), one obtains

$$(1-f_1)/A \leq 1.8\times10^{-4}(2\times10^{10}\text{ yr}/t)\text{ sec}^{-2}\,. \qquad (34)$$

To express this more clearly, let λ_0 be that wavelength with lifetime $A(2\pi c/\lambda_0) = 2\times10^{10}$ yr $= 2\pi\times10^{17}$ sec. Thus λ_0 in Ångstroms relates to A in sec^2 by

$$\lambda_0/\text{Å} = 30A/\text{sec}^2\,. \qquad (35)$$

But $1-f_1 = \epsilon$, as explained following Eq. (19). Thus (34) becomes

$$\epsilon \leq 6\times10^{-6}(\lambda_0/\text{Å})(2\times10^{10}\text{ yr}/t)\,. \qquad (36)$$

Equation (36) gives the limit imposed on photon splitting of type I by the frequency-independent red-shift. Other types of breakup are not seen in the optical. (But when we come to the 3-K microwaves, we have no *a priori* reason to restrict ourselves to one type of splitting, so all types will be included.)

So much for the effect of splitting on the mean position of the lines. Now to discuss its effect on their width. Writing $\langle\Omega;t_0\rangle = R_0\omega_0$, etc., (21) with (23) and $\epsilon = 1-f_1$ becomes

$$R^2(\Delta\omega)^2 = R_0^2(\Delta\omega_0)^2 + (\epsilon^2+w^2)A^{-1}[R_0\omega_0 S - \epsilon S^2/2A]\,. \qquad (37)$$

The second term in brackets can be neglected: Its ratio to the first is $\sim\epsilon S/A\omega_0 R_0$ which is $\ll 1$, because in (18) it represents the small fraction of red-shift due to splitting. Solving for ϵ^2/A, and setting $R_0\omega_0 \simeq R\omega$,

$$\begin{aligned}\epsilon^2/A &= [R^2(\Delta\omega)^2 - R_0^2(\Delta\omega_0)^2](R_0\omega_0 S)^{-1} - w^2/A\\ &\lesssim (R/S)(\Delta\omega)^2/\omega\,.\end{aligned} \qquad (38)$$

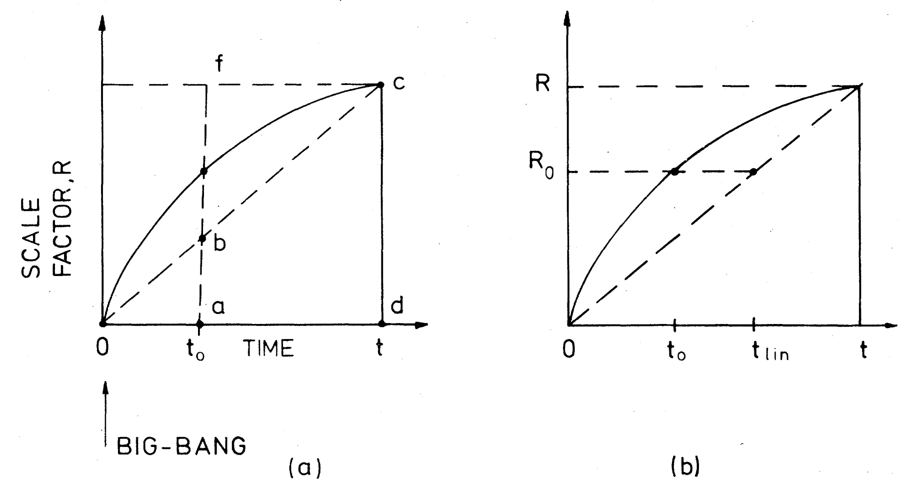

FIG. 2. Inequalities in a big-bang universe. The universal scale factor R is plotted against time. This yields the solid curve which passes through the origin, and which is always convex downwards for zero cosmological constant. Expansion begins at time 0, and the photons detected at t were emitted at t_0. (a) Shows that the area under the curve, $S = \int_{t_0}^{t} R\,dt$, from emission to detection lies between the trapezoid $abcd$ and the rectangle $afcd$. In (b), the convexity also ensures that the curve lies above the dashed line of linear expansion, which passes through the origin. Hence $t_0 < t_{\rm lin} = R_0 t/R$.

The lower limit on

$$(\Delta\omega)^2/\omega = 2\pi c(\Delta\lambda)^2/\lambda^3$$

is 5.9×10^8/sec, corresponding to $\Delta\lambda \simeq 4$ Å and $\lambda \simeq 8000$ Å, the largest visible wavelength observed. For $z\simeq 2$ of 3C191, (33) gives $R/S < 3/t$. Finally, converting from A to λ_0 by (35), (38) becomes

$$\epsilon^2 \lesssim 9.5\times10^{-11}(\lambda_0/\text{Å})(2\times10^{10}\ \text{yr}/t)\,. \qquad (39)$$

This is the limit imposed on type-I splitting by the width of spectral lines. If ϵ^2 were larger, the narrow absorption lines would have a half-width larger than 4 Å. The upper limits on ϵ imposed by (36) and (39) are plotted in Fig. 3, assuming for definiteness that $t = 2\times10^{10}$ yr. The smaller limit comes from line width, Eq. (39), if $\lambda_0 > 2.6$ Å and from frequency-independent red-shift, Eq. (36), if $\lambda_0 < 2.6$ Å. We see that if visible light can decay in 2×10^{10} yr, i.e., $\lambda_0 \simeq 5\times10^3$ Å, then the fractional energy loss per splitting must be very small: $\epsilon < 10^{-3}$, so at most 0.1% of the energy goes into the two low-energy fragments. If λ_0 is even smaller, then ϵ will be reduced still further.

B. Multiple quasar red-shifts

Multiple red-shifts are possible only in case of a sharply peaked decay curve, $w \ll \epsilon$. Otherwise the lines get smeared out beyond recognition. Different photons of the *same* frequency from a quasar may display various red-shifts because the number of decays varies statistically. In contrast, frequency-dependent red-shift refers to photons from a given quasar having different frequencies; the lower frequencies split more often and therefore display larger red-shift. Figure 4 contrasts these two kinds of red-shift.

Consider, for example, the sharp, type-I splitting of identical, frequency-ω photons in flat space. For simplicity, ignore the gradual decrease of frequency with successive breakups and set $\tau = A\omega =$ const. The probability that any photon has

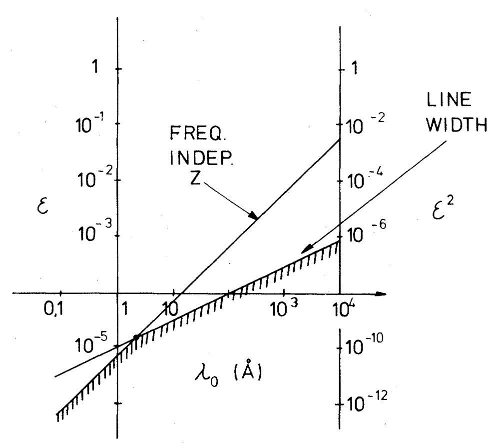

FIG. 3. Plot of fractional energy loss ϵ versus the wavelength λ_0 (Å) that decays in 2×10^{10} yr. In this figure we set the present age of the universe $t = 2\times10^{10}$ yr. These results apply only if type-I splitting is dominant. The frequency independence of quasar red-shift and the width of absorption lines give two different upper bounds on ϵ; it must lie beneath both of these lines, i.e., in the shaded region.

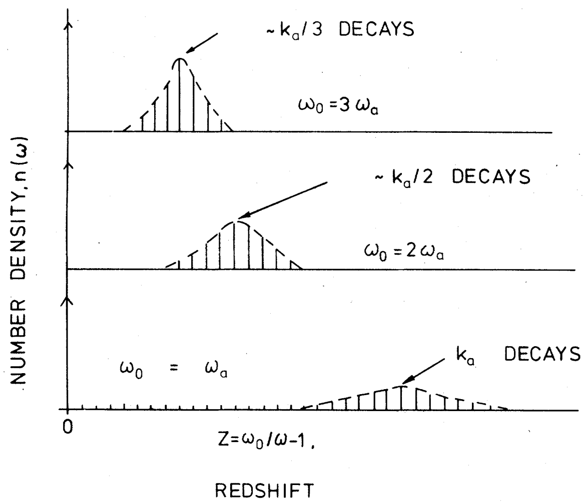

FIG. 4. Multiple versus frequency-dependent red-shift. Three monochromatic sets of photons with frequencies in the ratio 1:2:3 are emitted in flat space, and are observed again after some lapse of time. We assume that the decay is of type I, where one of the three daughters gets almost all the energy and the other two are not observed. Plotted here is the number density of these energetic daughters versus their red-shift z. The spikes in z pertain to a perfectly sharp decay law. When a photon decays, its daughter always has z one space to the right. "Multiple red-shift": statistical spread of z for any *one* initial frequency ω_0. "Frequency-dependent red-shift": low ω_0 splits more often than high ω_0. Assuming $z \ll 1$, the mean number of decays is inversely proportional to the lifetime or ω_0. "Broadening": If the decay is not very sharp, then the spikes are replaced by a smooth (dashed) envelope that spreads out with time.

undergone k decays in a time t is given by the Poisson distribution:

$$P_k(t) = (k!)^{-1}(t/\tau)^k \exp(-t/\tau)\,,$$

with equal mean and variance,

$$\langle k\rangle = t/\tau = (\Delta k)^2 \equiv \langle k^2\rangle - \langle k\rangle^2\,. \tag{40}$$

A spread in the number of decays implies a spread in the red-shift because k decays reduces the frequency by a factor $(1-\epsilon)^k$. The same thing follows from (37): Even when the decay law is razor sharp [$w=0$; see sentence after Eq. (23)] and we start with monochromatic photons [$(\Delta\omega_0)^2 = 0$], the final spectrum has a nonzero spread [$(\Delta\omega)^2 > 0$]. This can only come from a statistical variation in the number of splittings.

In Eq. (40), "multiple z" corresponds to $(\Delta k)^2 > 0$ for fixed τ, whereas "frequency-dependent z" corresponds to $\langle k\rangle$ varying with τ.

The following features should help to identify this kind of multiple z in quasar spectra:

(a) It is present both in absorption and emission.

(b) It depends only on the quasar's distance, and the number of different z's increases with distance.

(c) If a quasar displays several systems of lines $L_1, L_2, L_{33}, \ldots$ with red-shifts $z_1, z_2, z_3, \ldots$, then a large-z system will have a smaller mean rest frequency, because low-frequency lines decay more often and have larger z's. (See Fig. 4.)

(d) If a quasar displays 3 or more z's, $z_J < z_K < z_L < \cdots$, then these must be related by

$$(1+z_J):(1+z_K):(1+z_L) = (1-\epsilon)^{-j}:(1-\epsilon)^{-k}:(1-\epsilon)^{-1}\,, \tag{41}$$

where J, K, L and j, k, l are both increasing sequences of positive integers. Equation (41) follows from

$$\omega_{\text{emit}}/\omega_{\text{obs}} = 1 + z = (1-\epsilon)^{-k}(1 + z_{\text{other}})\,,$$

which states that the reddening due to Doppler shift and gravity ("other") is simply superimposed upon that due to splitting.

It might seem that (41) is a trivial requirement, because it can always be satisfied by taking ϵ small enough and j, k, l large. But this is not so because when we substitute all triplets of z, the resulting sets of integers j, k, l must be strings of consecutive integers. The ratio between any neighboring values of $1+z$ is always the same and gives $(1-\epsilon)$ uniquely.

Do we see this effect? Many quasars do have multiple red-shifts. But they are always in absorption, not emission—contrary to (a). The data is still scanty,[17] but as far as it goes it fails also on counts (b), (c), (d). We conclude that quasars do not show multiple z's from photon splitting. This is as it should be. Multiple red-shift is a special (discrete) case of broadening, occurring when the decay spikes are sharp enough not to overlap (Fig. 4). And condition (c) relies on frequency-dependent z. Consistency therefore demands that multiple red-shifts from splitting must be absent, the same as broadening and frequency-dependent red-shift.

V. OBSERVED BROADENING OF THE 3-K MICROWAVES

The Planckian spectrum of radiation decoupled from matter at time t_0 will grow ever wider under photon splitting. How does this predicted broadening compare with observations?

Taking the initial density of photons per unit volume and per unit frequency as perfectly Planckian,

$$n_0(\omega) = \omega^2[\pi^2 c^3(e^{\beta\hbar\omega} - 1)]^{-1}\,, \tag{42}$$

where $\beta = (kT)^{-1}$, we find for any integer $k \geq -1$,

$$\langle\omega^k\rangle_0 = (\beta\hbar)^{-k}[(k+2)!/2]\zeta(k+3)/\zeta(3).$$

With the tabulated values[18] for the Riemann zeta function $\zeta(k)$, this leads to the initial values

$$v_0 = 0.4187, \quad \langle\omega\rangle_0\langle\omega^{-1}\rangle_0 = 1.8482 \tag{43}$$

at time t_0 of decoupling, where $v \equiv (\Delta\omega)^2/\langle\omega\rangle^2$ measures the broadening. The advantage of this over $(\Delta\omega)^2$ is that v responds *only* to photon splitting; standard red-shifts, whether of Doppler or of curved space, leave it unaltered because they scale all frequencies by the same factor, but v is scale invariant.

The observations span a large range of wavelengths from less than 0.1 cm to about 70 cm. The corresponding drop in frequency is almost a factor of 10^3. This justifies the simplifying assumption that essentially all decay photons are seen. Then, as shown in Sec. IIIC, $f_0 = 3$ and $f_1 = 1$. Inserting these with (43) into (28), it yields for the amount of broadening

$$v(t) - v_0 = S(t)A^{-1}\langle\Omega; t_0\rangle^{-1}(4.24 + f_2). \tag{44}$$

But $\langle\Omega; t_0\rangle \simeq \langle\Omega; t\rangle$ since splitting apparently constitutes but a small perturbation, and $f_2 > 0$ always. We know also that the radiation has been highly red-shifted, $z \sim 10^3 \gg 1$, so from (33), $R(t)/S(t) < 2/t$. Equation (44) thus gives for any expanding RW universe and all types of photon splitting

$$A \geq 4.24[S(t)/R(t)]\langle\omega; t\rangle^{-1}[v(t) - v_0]^{-1}$$
$$> 2.12t\langle\omega; t\rangle^{-1}[v(t) - v_0]^{-1}.$$

Converting this by (35) to the wavelength λ_0 that decays in 2×10^{10} yr,

$$\lambda_0/\text{Å} > 63.6t\langle\omega; t\rangle^{-1}[v(t) - v_0]^{-1} \text{ sec}^{-2}. \tag{45}$$

The present-day values of t, $\langle\omega; t\rangle$, and $v(t)$ are to be substituted in (45); v_0 comes from (43). We encounter at once a difficulty: There exist very few measurements above the peak frequency and these are still subject to rather large errors. To calculate $v(t)$ and $\langle\omega; t\rangle$ we therefore extrapolate the observed spectrum to high frequencies, and we do this in the most conservative way possible in order to *minimize* any splitting that is present. We assume that above the highest frequency included in our data ($\omega = 7.17\times 10^{11}$ sec^{-1}, $\lambda = 0.263$ cm), the spectrum goes over to an exact Planckian with the average observed temperature[19] $\langle T\rangle = 2.945$ K.

The observed value of $v(t)$ shows that instead of broadening, *there is narrowing*, $v(t) - v_0 < 0$. This tends to contradict the hypothesis of photon splitting since the latter always produces broadening. Because random scattering events are hardly likely to reduce the relative frequency dispersion, one might conclude that the initial spectrum at decoupling was narrower than a Planckian. [Notice that we do not need to specify the latter's temperature because $v_0 = 0.4187$ of Eq. (43) characterizes *all* Planckians, independent of temperature.] A more fruitful hypothesis, which we adopt here, is that the value of $v(t)$ observed now only appears to be smaller than v_0 because random experimental errors result in an underestimate of its true value. We wish then to determine Δv, the standard deviation in the experimental value of $v(t)$.

Unfortunately a single curve fitted to the data points yields only one v, not a distribution of values. To get around this, we generate a whole family of curves consistent with the known experimental uncertainties, and then determine Δv for the family. First, we make a single curve $n(\omega)$ for the observed photon number per unit volume per unit frequency as follows: Neighboring experimental points are connected by straight line segments to get a polygonal graph which covers the range of frequencies from zero up to the highest observed. Then we generate a family of polygons by letting the vertices have varying n values. That is, suppose ω_k $(k = 1, 2, \ldots, 14)$ is one of the fourteen observed frequencies, and the measured value of $n(\omega_k)$ is n_{0k} with an uncertainty $\pm\Delta_k$. Then for fixed ω_k, we let $n_k = n(\omega_k)$ vary about n_{0k} with the Gaussian distribution

$$p(n_k) = (\Delta_k 2\pi)^{-1}\exp[-(n_k - n_{0k})^2/2\Delta_k^2]. \tag{46}$$

Thus there is a continuous infinity of polygonal graphs from $\omega = 0$ to $\omega = \omega_{14}$, the maximum observed frequency. At $\omega = \omega_{14}$ there is a jump discontinuity and all these curves have the identical continuation to higher frequencies, a Planckian of the mean temperature $T = \sum_{k=1}^{k=14} T_k/14 = 2.945$ K. The resulting family of curves gives for $v = (\Delta\omega)^2/\langle\omega\rangle^2$ both a "best" experimental value v_E and its rms deviation Δv

$$v_E \pm \Delta v = 0.3910 \pm 0.0265. \tag{47}$$

The data used, taken from Weinberg's complication,[20] are shown in Table I. Our appendix gives further details on the computation of (47).

We can now take the experimental uncertainty into account by saying that the "true" present-day experimental value of $v(t)$ does not exceed

$$v(t)_{\text{true}} = v_E + 3.08\Delta v = 0.4726 \tag{48}$$

at the 0.1% significance level. That is, given $v(t)_{\text{true}} = 0.4726$, only one time in a thousand would measurement error bring the observed value down to $v(t) = v_E \equiv 0.3910$ or less.

Taking into account that our sampling of frequencies is discrete improves the accuracy of (45) still more. If the microwaves had for all frequencies a blackbody spectrum at $T = 2.945$ K but we measured this exactly at the fourteen fre-

TABLE I. Observations of the 3-K microwave background. T and λ are from Weinberg (Ref. 20). All columns to the right of T are computed from ω and T or the mean value of the fourteen temperatures, $\langle T\rangle = 2.945$ K. The multiple readings (i), (ii), etc., under numbers 6, 13, 14 are combined to give a single set of values marked "av"; thereafter only this average reading is used in the subsequent statistical analysis. The value of T in a row marked "av" is the Planckian temperature corresponding to the average n_0.

Reading no.	λ (cm)	ω (10^9 sec^{-1})	T (K)	$\bar{n}$ [a] (10^{-12} sec cm^{-3})	n_+ [a] (→)	n_- [a] (→)	n_0 [b] (→)	Δ [c] (→)	$n_{\langle T\rangle}$ [d] (→)
1	73.5	2.565	3.7 ±1.2	4.65	6.164	3.138	4.651	1.513	3.699
2	49.2	3.831	3.7 ±1.2	6.935	9.196	4.678	6.937	2.259	5.515
3	21.0	8.976	3.2 ±1.0	13.96	18.37	9.551	13.96	4.410	12.84
4	20.7	9.106	2.8 ±0.6	12.37	15.06	9.688	12.37	2.686	13.02
5	7.35	25.65	3.5 ±1.0	42.88	55.48	30.29	42.88	12.60	35.89
6 av	3.2	58.90	2.71 [e]	⋯	⋯	⋯	71.97 [f]	5.01 [f]	78.88
(i)			3.0 ±0.5	80.47	94.90	66.03	80.46	14.44	⋯
(ii)			$2.69^{+0.16}_{-0.21}$	71.52	76.13	65.46	70.80	5.34	⋯
7	1.58	119.3	$2.78^{+0.12}_{-0.17}$	137.7	144.7	127.8	136.2	8.45	147.3
8	1.50	125.7	2.0 ±0.8	96.22	144.9	48.37	96.64	48.26	153.8
9	0.924	204.0	3.16±0.26	245.0	270.6	219.5	245.0	25.60	223.9
10	0.856	220.2	$2.56^{+0.17}_{-0.22}$	195.9	213.6	173.0	193.3	20.30	236.2
11	0.82	229.9	2.9 ±0.7	238.3	315.4	162.4	238.9	76.5	243.3
12	0.358	526.5	2.4 ±0.7	239.6	391.1	107.8	249.4	141.6	356.5
13 av	0.33	571.2	2.57 [e]	⋯	⋯	⋯	275.0 [f]	47.5 [f]	360.2
(i)			$2.46^{+0.40}_{-0.44}$	250.2	340.3	159.6	250.0	90.4	⋯
(ii)			2.61±0.25	283.3	340.3	228.7	284.5	55.8	⋯
14 av	0.263	716.7	3.25 [e]	⋯	⋯	⋯	438.9 [f]	38.8 [f]	355.9
(i)			3.22±0.15	430.8	472.9	389.6	431.2	41.6	⋯
(ii)			3.0 ±0.6	370.7	539.1	219.4	379.2	159.8	⋯
(iii)			3.75±0.50	583.1	733.9	439.1	586.5	147.4	⋯

[a] $\bar{n}, n_+, n_-$ are, respectively, the Planckian number densities per unit frequency [see Eq. (42)] for the given frequency and for temperatures $T, T+\Delta T_1, T-\Delta T_2$. For example, in reading 7, these three temperatures are 2.78, 2.90, and 2.61 K.

[b] $n_0 \equiv \frac{1}{2}(n_+ + n_-)$.

[c] $\Delta \equiv \frac{1}{2}(n_+ - n_-)$.

[d] $n_{\langle T\rangle}$ is the Planckian number density per unit frequency corresponding to the mean temperature 2.945 K.

[e] This temperature is calculated from the av value of n_0.

[f] The av value of n_0 is a weighted average of n_0 in (i), (ii) [and (iii)]. Likewise for Δ. See Eq. (A1) for definition of av.

quencies of Table I, we would find $v(t) = 0.4129$. This suggests replacing $v_0 = 0.4187$ by $v_0 = 0.4129$ so that in case measurements did check with a Planckian, $v(t) - 0.4129 = 0$, Eq. (45) would give $\lambda_0 = \infty$; that would mean all photons are stable, since their lifetime is $\tau = (\lambda_0/30\ \text{Å})\omega\ \text{sec}^2$.

Substituting into (45) $v(t) = 0.4726$ and $\langle \omega; t\rangle = 1.08 \times 10^{12}$ sec^{-1}, computed from the data extrapolated to high frequencies by a Planckian tail, we find for that wavelength λ_0 which decays in 2×10^{10} yr (under neglect of lifetime shortening by the cosmological red-shift),

$$\lambda_0 > 6.2\ \text{cm}\quad (t/2\times 10^{10}\ \text{yr}). \tag{49}$$

Here t is the present age of the universe, the proper time since the big bang. Since the expansion is slowing down this will be less than the present Hubble time H^{-1}.

Equation (49) is consistent with our treatment of splitting as a small perturbation, which approximation was used to find (28). The peak frequency $[dn_0(\omega)/d\omega = 0]$ of a Planckian spectrum has $\lambda = 0.9$ cm$/(T/K)$ or $\lambda = 0.3$ cm at $T = 3$ K. We see by (49) that the great majority of relic photons, which are in or near the peak, will indeed be stable against splitting for several times the age of the universe. This conclusion becomes even stronger considering that these photons were bluer in the past, hence longer-lived.

VI. CONCLUSIONS

The "totalitarian principle" of Gell-Mann, "whatever is not forbidden, is compulsory," has repeatedly proven its value by guiding particle physicists to hitherto unsuspected invariance principles. According to this principle, the possibility of photon splitting must be taken seriously. Although a detailed calculation in QED forbids it, a more direct and model-independent demonstration could be desired, considering that the number of conceivable decay channels is unlimited. The known symmetries permit splitting of a zero-mass photon provided the decay photons are collinear with the original, their number is odd, and energy is conserved.

We have here presented observational upper limits on splitting, using quasar light and the primordial cosmic microwaves. The Lorentz-covariant decay law adopted is the simplest, most plausible choice; it generalizes exponential decay to lightlike particles. In this law, any photon has a lifetime proportional to its frequency, $\tau = A\omega$.

We have shown that photon splitting cannot make a significant contribution to the cosmological red-shift because it predicts a frequency-dependent z, contrary to observation. The spectral lines of quasars fail to show any evidence of splitting. Much more sensitive limits come, however, from the 3-K microwaves both because these cover a broader frequency range and because they split about 10^4–10^6 times more often than the higher-frequency visible photons. Failure to observe "broadening" in the 3-K microwaves yields a lower limit on the lifetime τ of any photon (omitting red-shift lessening of the frequency and lifetime),

$$\tau > (6.2 \text{ cm}/\lambda)t\,, \qquad (50)$$

where λ is the wavelength and t is the present age of the universe. To obtain (50), use the definition $A\omega_0 \equiv 2\times 10^{10}$ yr to rewrite (49) as $A\omega_0 > (6.2 \text{ cm}/\lambda_0)t$, then multiply through by $\omega/\omega_0 = \lambda_0/\lambda$. Equation (50) is valid in curved spacetime provided τ and λ are the instantaneous values of lifetime and wavelength relative to any given "local observer" (or "orthonormal tetrad").

The phenomenological theory presented here should also be applicable to the splitting of neutrinos and gravitons. When observations with graviton and neutrino detectors have reached a level comparable to present-day radio and optical astonomy it may be worthwhile to test these massless particles for splitting over cosmological distances. In the absence of direct experimental evidence, attempts to explore the theoretical limitations are valuable. As an example, we mention that the idea of photon splitting leads to absurdity when applied to photons whose frequency is too low. In a positive curvature (spatially closed) Robertson-Walker universe, the normal modes of the electromagnetic field have a discrete spectrum of non-negative frequencies [Ref. 21, p. 2907, Eq. (29)]. Our theory of splitting does not apply even remotely to the low-frequency end of this discrete spectrum. But this hardly needs emphasis since we assumed at the start that wavelengths are small compared to the local radius of curvature, let alone the radius of the smoothed out cosmological background.

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APPENDIX

We give here basic steps in the calculation of the relative variance of the microwave background, Eq. (47). Since we wish to compute the spread Δv in this variance, we use only those measurements from Weinberg[20] that include an estimated error.

In generating the family of all possible experimental curves, we assume that the number density of photons per unit frequency (for the kth frequency), $n_k = n(\omega_k)$, has a symmetrical, Gaussian distribution about some average value n_{0k}. The best experimental values $\bar{n}_k$ do not always lie exactly halfway between the upper and lower limits

$n_\pm(\omega_k) = n_{0k} \pm \Delta_k$. Table I shows, however, that our assumption of symmetry is permissible because the average value n_{0k} seldom deviates by more than 1% from the $\bar{n}_k$. The next to last column in Table I gives the uncertainties Δ_k. We take this as giving the standard deviations in the Gaussian distribution (46).

Readings referring to the same frequency are grouped together. These must be reduced to a single value so that we can draw a unique polygonal line through the data points. The "average" of several uncertain readings should lie between the largest and the smallest, and it should be more accurate than any of them. Given several readings $n_k \pm \Delta_k$, a simple average which fulfills these conditions uses the reciprocal variance as a weighting factor:

$$n_{av} \equiv \left(\sum_k n_k \Delta_k{}^{-2}\right) \Big/ \left(\sum_k \Delta_k{}^{-2}\right), \qquad (\Delta_{av})^{-2} \equiv \sum_k \Delta_k{}^{-2}. \tag{A1}$$

In Table I, the averages for readings 6, 13, and 14 were deduced by means of (A1).

Given the fourteen readings, we can calculate the relative variance v_E and its spread Δv. For any ω_k, the number density n_k takes the value $n_{0k} + \epsilon_k$ with Gaussian distribution assumed independent of any other n_j. The graph of $n(\omega)$ consists of two parts: (a) a polygonal line passing through the origin and the points $(\omega_k, n_{0k} + \epsilon_k)$, $k = 1, 2, \ldots, 14$ and (b) a smooth Planckian part $n_0(\omega)$ for $\omega > \omega_{14}$ [see Eq. (42)] at the mean temperature of the fourteen readings. If we define an unnormalized average of the jth power of ω by

$$N_j = \int_0^\infty d\omega\, \omega^j n(\omega), \tag{A2}$$

then the polygonal line contributes for $0 < \omega < \omega_{14}$ and the Planckian for $\omega > \omega_{14}$. But the straight-line segment between (ω_k, n_k) and (ω_{k+1}, n_{k+1}) has ordinate

$$n(\omega) = (\omega_{k+1} - \omega_k)^{-1}[(n_{0k} + \epsilon_k)(\omega_{k+1} - \omega) + (n_{0,k+1} + \epsilon_{k+1})(\omega - \omega_k)]. \tag{A3}$$

Thus N_j contains terms linear in the set of ϵ_k. Since the normalized average is $\langle \omega^j \rangle = N_j/N_0$, we have that the relative variance (for any one curve of the family, i.e., with the ϵ_k's fixed) equals

$$v = (\Delta\omega)^2/\langle\omega\rangle^2 = N_0 N_2 (N_1)^{-2} - 1. \tag{A4}$$

Because Δ_k is usually less than 30% of n_{0k}, and because the Gaussian (46) suppresses values of $\epsilon_k > \Delta_k$, we can treat ϵ_k as a small parameter, i.e., $\epsilon_k / n_{0k} \ll 1$. We therefore expand the right-hand side of (A4) up to terms quadratic in the ϵ_k's. The average of this, $\langle\ \rangle_G$, using the multiple Gaussian density $p(\epsilon_1)p(\epsilon_2)\cdots p(\epsilon_{14})$, yields

$$v_E' \equiv \langle v \rangle_G. \tag{A5}$$

And the spread of v over the family of curves is defined by

$$\Delta v \equiv [\langle v^2 \rangle_G - \langle v \rangle_G{}^2]^{1/2}. \tag{A6}$$

We estimate that the neglected powers of ϵ_k higher than the second should make a negligible difference in $v_E' = 0.3897$ and probably alter $v = 0.0265$ by less than 1%.

We can calculate v exactly for that curve passing through the mean values (ω_k, n_{0k}) and find $v_E = 0.3910$. In our analysis of the 3-K data we use v_E rather than v_E' of (A5). This seems more natural because v_E depends directly on the original data points, whereas v_E' involves an invented family of curves.

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[1]P. Havas, Am. J. Phys. 34, 753 (1966).

[2]It might seem that one could satisfy the exclusion principle by giving the two neutrinos equal momenta but opposite spins. But this is impossible because all neutrinos are left-handed, i.e., they have helicity −1, while all antineutrinos are right-handed.

[3]More generally, Furry's theorem (see references in Havas, Ref. 1) states that when there are only external photon lines in the Feynman diagram, the process is forbidden unless there are an even number of external lines in total altogether. Thus an odd number of incoming photons must result in an odd number outgoing; we could also deduce that there are an odd number of decay photons from collinearity of the decay momenta and conservation of spin.

[4]A lower-order process $\gamma \to \nu + \bar{\nu}$ is forbidden by conservation of spin because the ν, $\bar{\nu}$, with opposite helicities, carry zero total spin.

[5]J. C. Herrera, Ph.D. dissertation, Boston University, 1964 (unpublished). The method of calculation has been published in J. C. Herrera and P. Roman, Nuovo Cimento 33, 1657 (1964).

[6]See, e.g., W. R. Yueh, Nucl. Phys. B93, 153 (1975), and references cited therein.

[7]O. Halpern, Phys. Rev. 44, 855 (1934); P. Roman, Magy. Fiz. Foly. 3, 115 (1955).

[8]See, e.g., R. P. Feynman, *Quantum Electrodynamics* (Benjamin, New York, 1962), p. 4, and J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, Mass., 1973), Chap. 2.

[9]G. Rosen and F. C. Whitmore, Phys. Lett. 14, 201 (1965); Am. J. Phys. 34, 788 (1966).

[10]Notation: Greek indices range and sum over 0, 1, 2, 3. A comma before a subscript denotes a partial de-

rivative, whereas a semicolon stands for a covariant derivative. We use units such that $G=\hbar=c=1$ unless otherwise indicated.

[11]L. Parker, Phys. Rev. D 5, 2905 (1972); 7, 976 (1973). The first paper shows that there is no backscattering for conformally-invariant classical wave equations in RW spacetimes. This corresponds to the quantum statement that no pairs are created. Reference 12 explains the relation between the classical and quantum descriptions of pair creation.

[12]Ya. B. Zel'dovich and A. A. Starobinskii, Zh. Eksp. Teor. Fiz. 61, 2161 (1971)[Sov. Phys. JETP 34, 1159 (1972)].

[13]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), p. 412.

[14]Although splitting increases the *relative* width $\Delta\omega/\langle\omega\rangle$, the same need not be true of the width $\Delta\omega$ itself. One sees that if the spreading of frequencies due to splitting itself is not very great, it will not be able to compensate for the shrinkage of $\Delta\omega$ brought about by cosmological expansion.

[15]G. Burbridge and M. Burbridge, *Quasi-Stellar Objects* (Freeman, San Francisco, 1967).

[16]For any suitable real functions $a(\Omega)$, $b(\Omega)$, and weight function $n(\Omega)>0$, define the scalar product

$$\vec{a}\cdot\vec{b}\equiv\int_0^\infty d\Omega\, n(\Omega)a(\Omega)\,b(\Omega).$$

Schwarz's inequality asserts $|\vec{a}\cdot\vec{b}|^2\leq|\vec{a}|^2|\vec{b}|^2$. With $a(\Omega)=\Omega^{1/2}$ and $b(\Omega)=\Omega^{-1/2}$, this gives $1\leq\langle\Omega\rangle\langle\Omega^{-1}\rangle$.

[17]J. J. Perry, lectures at Winter School on High-Energy Astrophysics, Tata Institute of Fundamental Research, Bombay, 1976 (unpublished).

[18]I. S. Gradstein and I. M. Ryshik, *Summen-Product- und Integral-Tafeln* (VEB Deutscher Verlag der Wissenschaft, Berlin, 1963).

[19]Our mean value $T=2.945$ K for the cosmic microwaves agrees closely with the temperature of 2.96 K reported by R. Muller [Ninth Texas Symposium on Relativistic Astrophysics, Ann. N. Y. Acad. Sci. (to be published)] for precise measurements on the high-frequency side of the peak. He noted that this differs systematically, for reasons still unknown, from the $T=2.8$ K blackbody temperature of the low-frequency tail. The higher of Muller's two temperatures is certainly the more appropriate for us because the large number density and high frequencies near the peak, dominate our statistics.

[20]S. Weinberg, Ref. 13, p. 512.

[21]L. Parker, Phys. Rev. D 5, 2905 (1972).