Renormalization of an SU(3) linear σ model with mesons and baryons in the one-loop approximation

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A linear SU(3) σ model with mesons and baryons is demonstrated to be renormalizable in the one-loop approximation. The mesons and baryons are assigned to the $(3,3^*)\oplus(3^*,3)$ and the $[(3,3^*),\ (3^*,3)]$ representations of chiral $SU(3)\times SU(3)$, respectively. The model incorporates both spontaneous and explicit symmetry breaking. The baryon symmetry-breaking terms are chosen to allow the model to describe the N-Ξ mass difference. Higher-order renormalization is also considered.

I. INTRODUCTION

The σ model[1,2] has been used extensively in exploring the implications of chiral symmetry in low-energy hadron dyanmics. Most of these investigations have employed the SU(2) model with mesons[3] and nucleons[4] and the SU(3) model with mesons only.[5] More recently, the SU(4) meson model has been studied.[6] In this paper we describe a linear SU(3) model with mesons and baryons and demonstrate, in the one-loop approximation, that the theory is renormalizable. We employ a form of symmetry breaking that can incorporate the Ξ-N mass difference.

The classic version of the SU(2) meson σ model was developed by Gell-Mann and Lêvy,[1] who considered both its linear and nonlinear forms. The linear model was extended to SU(3) by Lêvy.[7]

The SU(3) σ model is of interest for several reasons: The Lagrangian currents obey the chiral $SU(3)\times SU(3)$ current algebra; depending on the choice of the symmetry-breaking Lagrangian, operator PCAC (partial conservation of axial-vector current) may be incorporated as an identity; in the appropriate limit (as the scalar masses $\to\infty$, which effectively gives the nonlinear model), the tree-approximation calculations reproduce the soft-meson current-algegra–PCAC theorems[8]; the Lagrangian can be constructed to be nearly SU(3) $\times$SU(3) invariant. The approximate chiral SU(3) symmetry of the Hamiltonian may be the only reasonable way to explain the successful current-algebra–PCAC results[9]; finally, the effects of spontaneous symmetry breaking can be seen at the tree-approximation level. Indeed, solutions have been found in both the tree and one-loop approximations that exhibit a Nambu-Goldstone symmetry realization.

Numerical work in the one-loop approximation in the SU(2), SU(3), and SU(4) linear meson models and the SU(2) nonlinear meson-nucleon models indicates that the second-order corrections to the tree-approximation results are usually in the range of 10–20% or less. The difference between the second-order calculated values and their physical counterparts is also within this limit. Differences of this magnitude are acceptable in the spirit of perturbation theory. One expects the SU(3) model with mesons and baryons also to be within the acceptable numerical limits.

The SU(2) meson model incorporating symmetry breaking that is linear in the fields has been shown to be renormalizable by Lee[10] and by Symanzik.[11] Lee and Gervais[12] considered the SU(2) model with fermion fields included. Crater[13] explicitly demonstrated the renormalization of the $SU(3)\times SU(3)$-invariant meson model without spontaneous symmetry breaking in the one-loop approximation. Chan and Haymaker[14] extended this to the SU(3) model with spontaneous and explicit linear symmetry breaking. The SU(n) meson model for $n\geq 4$ incorporating both spontaneous and explicit linear symmetry breaking in the one-loop approximation has been investigated by Geddes.[15] In all the above models, the divergences can be canceled using only the counterterms of the symmetric Lagrangian. In the model outlined in this paper the coefficients in the symmetry-breaking Lagrangian also acquire divergent parts.

In the SU(3) model, the addition of the baryons poses several problems. First there is the choice of the SU(3) representation for the baryons, i.e., octet or nonet. We choose the nonet form to preserve the SU(3) current-algebra structure; however, this requires a reinterpretation of the SU(3) singlet. Second, we must specify the form of the symmetric Lagrangian. Only nonderivative couplings are allowed. Finally, there is the choice of the meson and baryon symmetry-breaking terms. The latter must describe the Ξ-N mass difference. It turns out that the choice of the baryon symmetry-breaking term imposes severe constraints on the allowable form for the meson sector of the Lagrangian.

The paper is organized as follows: In Sec. II we consider the choice of the meson and baryon fields and construct the basic Lagrangian; Sec. III restructures this Lagrangian into a form that is useful for calculation; the model is demonstrated to be renormalizable in the one-loop approximation in Sec. IV; higher-order renormalization is briefly discussed in Sec. V; our results are summarized in Sec. VI.

II. CHOICE OF FIELDS AND LAGRANGIAN

In this section we first consider the choice of the basic fields and the structure of the chiral-invariant Lagrangian. We then discuss the composition of the symmetry-breaking Lagrangian.

The meson fields are chosen so that the currents obey the SU(3) current algebra and the axial-vector current divergences have a PCAC-type structure. As a result we assign the nonets of pseudoscalar (π, K, η, η') and scalar $(\epsilon, \kappa, \sigma, \sigma')$ mesons to the $(3, 3^*) \oplus (3^*, 3)$ representation of chiral $SU(3)\times SU(3)$. To this end, consider the operators $M^{0a}_{\bar{b}}$ and $M^{0\bar{a}}_{b}$ $(a, b = 1, 2, 3)$ which transform as the $(3, 3^*)$ and $(3^*, 3)$ representations, respectively. The upper (lower) indices denote the 3 (3^*) representation of SU(3) and the unbarred (barred) indices denote the left- (right-) hand space of chiral $SU(3)\times SU(3)$, respectively. The superscript 0 will be used to indicate unrenormalized fields and Lagrangian parameters.

These operators have the equal-time commutation relations

$$[F_i^+, M^{0a}_{\bar{b}}] = -\tfrac{1}{2}\lambda^i_{ac} M^{0c}_{\bar{b}} \quad (i = 1, \ldots, 8), \tag{2.1}$$

$$[F_i^-, M^{0a}_{\bar{b}}] = \tfrac{1}{2}\lambda^i_{cb} M^{0a}_{\bar{c}}, \tag{2.2}$$

$$[F_i^+, M^{0\bar{b}}_{a}] = \tfrac{1}{2}\lambda^i_{ca} M^{0\bar{b}}_{c}, \tag{2.3}$$

and

$$[F_i^-, M^{0\bar{b}}_{a}] = -\tfrac{1}{2}\lambda^i_{bc} M^{0\bar{c}}_{a}, \tag{2.4}$$

with F^+ and F^- the generators of $SU(3)\times(3)$ which act on the left- and right-hand spaces, respectively. These generators are related to F and F^5, the vector and axial-vector charges, respectively, via

$$F^{\pm} = \tfrac{1}{2}(F \pm F^5). \tag{2.5}$$

The fields also obey the Hermiticity relation

$$(M^{0a}_{\bar{b}})^\dagger = M^{0\bar{b}}_{a} \tag{2.6}$$

and transform under parity as

$$P M^{0a}_{\bar{b}}(\vec{x}, t) P^{-1} = M^{0\bar{a}}_{b}(-\vec{x}, t). \tag{2.7}$$

These relations allow the reduction of $M^{0a}_{\bar{b}}$ and $M^{0\bar{a}}_{b}$ to operators of definite parity as

$$M^{0a}_{\bar{b}} = \Sigma^{0a}_{b} + i\Phi^{0a}_{b} \tag{2.8}$$

and

$$M^{0\bar{a}}_{b} = \Sigma^{0a}_{b} - i\Phi^{0a}_{b}, \tag{2.9}$$

where Σ and Φ denote nonets of scalar and pseudoscalar fields, respectively. Finally, for matrix notation we identify

$$M^{0a}_{\bar{b}} = (M^0)_{ab} = \Sigma^0_{ab} + i\Phi^0_{ab} \tag{2.10}$$

and

$$M^{0\bar{a}}_{b} = (M^{0\dagger})_{ab} = \Sigma^0_{ab} - i\Phi^0_{ab}. \tag{2.11}$$

Chiral-invariant operators can now be constructed from the M's by contracting indices in the left- and right-hand spaces; for example,

$$I_1^{0M} = M^{0a}_{\bar{b}}\, M^{0\bar{b}}_{a} = \mathrm{Tr}(M^0 M^{0\dagger}). \tag{2.12}$$

There are four independent, even-parity, chiral-invariant operators that can be built using the M's. The others are

$$I_2^{0M} = \mathrm{Tr}(M^0 M^{0\dagger} M^0 M^{0\dagger}), \tag{2.13}$$

$$I_3^{0M} = \mathrm{Tr}(M^0 M^{0\dagger} M^0 M^{0\dagger} M^0 M^{0\dagger}), \tag{2.14}$$

and

$$I_4^{0M} = \tfrac{1}{6}\epsilon_{abc}\epsilon^{\bar{d}\bar{e}\bar{f}} M^{0a}_{\bar{d}} M^{0b}_{\bar{e}} M^{0c}_{\bar{f}} + \tfrac{1}{6}\epsilon_{\bar{a}\bar{b}\bar{c}}\epsilon^{def} M^{0\bar{a}}_{d} M^{0\bar{b}}_{e} M^{0\bar{c}}_{f} \tag{2.15}$$

$$= \det M^0 + \det M^{0\dagger}. \tag{2.16}$$

I_1^{0M}, I_2^{0M}, and I_3^{0M} are invariant under the full $U(3)\times U(3)$ group, while I_4^{0M} is invariant only under the $SU(3)\times SU(3)$ subgroup. Consequently, the most general renormalizable form for a chiral-invariant meson Lagrangian is

$$\mathcal{L}_M = \tfrac{1}{2}\mathrm{Tr}(\partial_\mu M^0 \partial^\mu M^0) - \tfrac{1}{2}\mu^{0\,2} I_1^{0M} + f_1^0 (I_1^{0M})^2 + f_2^0 I_2^{0M} + g^0 I_4^{0M}. \tag{2.17}$$

To accommodate the $\frac{1}{2}^+$ baryon octet we can consider both the $[(3, 3^*), (3^*, 3)]$ and $[(8, 1), (1, 8)]$ representations.[16] We chose the former for three reasons: First, it is the only one that allows the currents to obey the SU(3) current algebra[7]; second, it allows the Goldberger-Treiman relation to be given directly; third, it gives a D-type axial-vector current rather than the F-type corresponding to the $[(8, 1), (1, 8)]$ representation. The ninth baryon in the $[(3, 3^*), (3^*, 3)]$ representation can be interpreted as a $\frac{1}{2}^-$ object,[17] perhaps the $\Lambda(1405)$.

The baryon-nonet operator B^{0a}_{b} can be decomposed into left- and right-hand components under $SU(3)\times SU(3)$ via

$$\begin{pmatrix} L^0 \\ R^0 \end{pmatrix}^a_b = \tfrac{1}{2}(1 \pm \gamma_5) B^{0a}_{b}, \tag{2.18}$$

where $L^{0a}_{\bar{b}}$ and $R^{0\bar{a}}_{b}$ transform as the (3, 3*) and (3*, 3) representations, respectively. These fields obey commutation relations analogous to Eqs. (2.1)–(2.4). Under parity the operators transform, for example, as

$$PL^{0a}_{\bar{b}}(\vec{x}, t)P^{-1}=\gamma_0 R^{\bar{a}}_{b}(-\vec{x}, t). \tag{2.19}$$

They also obey a Hermiticity relation of the form

$$L^{0a\dagger}_{\bar{b}} = L^{0\dagger\bar{b}}_{a}=\tfrac{1}{2}B^{0\dagger b}_{a}(1+\gamma_5), \tag{2.20}$$

which gives

$$\bar{L}^{0\bar{b}}_{a}=\tfrac{1}{2}\bar{B}^{0b}_{a}(1-\gamma_5). \tag{2.21}$$

For matrix notation we set

$$B^{0a}_{b}=(B^0)_{ab} \tag{2.22}$$

and

$$B^{0\dagger a}_{b}=(B^{0\dagger})_{ab}. \tag{2.23}$$

Chiral-invariant operators can now be constructed following the prescription used in the meson case. The simplest invariant is

$$I^{0B}_1 = L^{0\dagger\bar{a}}_{b} L^{0b}_{\bar{a}} + R^{0\dagger a}_{\bar{b}} R^{0\bar{b}}_{a} \tag{2.24}$$

$$=\mathrm{Tr}(B^{0\dagger}B^0), \tag{2.25}$$

which is ineligible for the Lagrangian. As is well known, the baryon mass term $\mathrm{Tr}(\bar{B}^0 B)$ is not chiral invariant.

The chiral-invariant operator

$$I^{0B\mu}_2=\bar{L}^{0\bar{a}}_{b}\gamma^\mu L^{0b}_{\bar{a}}+\bar{R}^{0a}_{\bar{b}}\gamma^\mu R^{0\bar{b}}_{a} \tag{2.26}$$

$$=\mathrm{Tr}(\bar{B}^0\gamma^\mu B^0) \tag{2.27}$$

is used to construct the baryon kinetic-energy term. Higher-order baryon invariants are not renormalizable. Consequently, the most general renormalizable chiral-invariant baryon Lagrangian is just the kinetic-energy term

$$\mathfrak{L}_B = i\ \mathrm{Tr}(\bar{B}^0\gamma\cdot\partial B^0). \tag{2.28}$$

Next consider the meson-baryon sector. The only renormalizable nonderivative chiral-invariant coupling is

$$I^{0MB}=\tfrac{1}{6}\epsilon_{\bar{a}\bar{b}\bar{c}}\epsilon^{def}\bar{L}^{0\bar{a}}_{d}M^{0\bar{b}}_{e}R^{0\bar{c}}_{f}+\tfrac{1}{6}\epsilon_{abc}\epsilon^{\bar{d}\bar{e}\bar{f}}\bar{R}^{0a}_{\bar{d}}M^{0\,b}_{\;\bar{e}}L^{0\,c}_{\;\bar{f}}. \tag{2.29}$$

Thus, the Lagrangian coupling is

$$\mathfrak{L}_{MB}=h^0\tfrac{1}{6}\epsilon_{abc}\epsilon_{def}\bar{B}^0_{ad}(\Sigma^0_{be}+i\gamma_5\Phi^0_{be})B^0_{cf}. \tag{2.30}$$

The symmetric Lagrangian is now complete. Before considering the symmetry-breaking Lagrangian, however, it is useful to restructure the symmetric Lagrangian into a nine-component form employing the decomposition

$$\begin{pmatrix}\Sigma\\ \Phi\\ B\end{pmatrix}_{ab}=\frac{1}{\sqrt{2}}\lambda^i_{ab}\begin{pmatrix}\sigma\\ \phi\\ b\end{pmatrix}\quad (i=0,\ldots,8). \tag{2.31}$$

These fields obey the linear commutation relations[18]

$$[F_i, \phi^0_j]=if_{ijk}\phi^0_k\quad (i=1,\ldots,8), \tag{2.32}$$

$$[F_i, \sigma^0_j]=if_{ijk}\sigma^0_k, \tag{2.33}$$

$$[F^5_i, \phi^0_j]=id_{ijk}\sigma^0_k, \tag{2.34}$$

$$[F^5_i, \sigma^0_j]=-id_{ijk}\phi^0_k, \tag{2.35}$$

$$[F_i, b^0_a]=if_{iab}b^0_b, \tag{2.36}$$

and

$$[F^5_i, b^0_a]=-d_{iab}\gamma_5 b^0_b. \tag{2.37}$$

Using the standard SU(3) tensor reductions,[19] the symmetric Lagrangian transforms to

$$\begin{aligned}\mathfrak{L}_{\mathrm{sym}}=&\tfrac{1}{2}\partial_\mu\sigma^0_i\partial^\mu\sigma^0_i+\tfrac{1}{2}\partial_\mu\phi^0_i\partial^\mu\phi^0_i+i\bar{b}^0_a\gamma\cdot\partial b^0_a\\ &-\mu^{0\,2}(\sigma^0_i\sigma^0_i+\phi^0_i\phi^0_i)\\ &+\tfrac{1}{3}F^0_{ijkl}(\sigma^0_i\sigma^0_j\sigma^0_k\sigma^0_l+\phi^0_i\phi^0_j\phi^0_k\phi^0_l)\\ &+2\hat{F}^0_{ij,kl}\phi^0_i\phi^0_j\sigma^0_k\sigma^0_l+G^0_{ijk}(\sigma^0_i\sigma^0_j\sigma^0_k-3\phi^0_i\phi^0_j\sigma^0_k)\\ &+H^0_{ab,i}(\bar{b}^0_a b^0_b\sigma^0_i+i\bar{b}^0_a\gamma_5 b^0_b\phi^0_i),\end{aligned} \tag{2.38}$$

where

$$F^0_{ijkl}=f^0_1 J^1_{ijkl}+\frac{f^0_2}{2}J^2_{ijkl}, \tag{2.39}$$

$$\hat{F}^0_{ij,kl}=f^0_1\delta_{ij}\delta_{kl}+\frac{f^0_2}{2}J^3_{ijkl}, \tag{2.40}$$

$$G^0_{ijk}=g^0 J^4_{ijk}, \tag{2.41}$$

$$H^0_{ab,i}=\frac{h^0}{2}J^4_{abi}, \tag{2.42}$$

$$J^1_{ijkl}=\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}, \tag{2.43}$$

$$J^2_{ijkl}=d_{ijm}d_{mkl}+d_{ikm}d_{mjl}+d_{ilm}d_{mjk}, \tag{2.44}$$

$$J^3_{ijkl}=d_{ijm}d_{mkl}+f_{ikm}f_{mjl}+f_{ilm}f_{mjk}, \tag{2.45}$$

and

$$\begin{aligned}J^4_{ijk}=&\frac{\sqrt{2}}{3}d_{ijk}-\frac{1}{\sqrt{3}}(\delta_{i0}\delta_{jk}+\delta_{j0}\delta_{ik}+\delta_{k0}\delta_{ij})\\ &+\sqrt{3}\delta_{i0}\delta_{j0}\delta_{k0}.\end{aligned} \tag{2.46}$$

The symmetry-breaking Lagrangian is constructed from three separate parts. The first part is built from the $(3,3^*)\oplus(3^*,3)$ representation linear in the meson fields and explicitly is

$$\mathfrak{L}^1_{\mathrm{SB}}=-c^0_i\sigma^0_i, \tag{2.47}$$

where c^0_i is nonvanishing only for the $I=Y=0$ operators. This class of symmetry breaking allows an operator PCAC type of structure for the axial-vector current divergence.

The second part, included to enable a description of the Ξ-N mass difference, is

$$\mathcal{L}_{SB}^2 = -ie^0 f_{8ab}\bar{b}_a^0 b_b^0 . \tag{2.48}$$

Other baryon symmetry-breaking contributions such as $\bar{b}_a^0 b_a^0$ and $d_{8ab}\bar{b}_a^0 b_b^0$ are not included as they render the theory nonrenormalizable.

The final part consists of various bilinear meson terms that are required to permit the inclusion of the baryon symmetry-breaking term. The reasoning behind this particular choice of terms is discussed in Sec. IV. Here we merely state that

$$\begin{aligned}\mathcal{L}_{SB}^3 = &-J_{ij}^5(a_s^0\sigma_i^0\sigma_j^0 + a_\phi^0\phi_i^0\phi_j^0)\\ &- d_1^0 d_{8ij}(\sigma_i^0\sigma_j^0 + \phi_i^0\phi_j^0)\\ &- (d_2^0 J_{ij}^6 + d_3^0 J_{ij}^7)(\sigma_i^0\sigma_j^0 - \phi_i^0\phi_j^0),\end{aligned} \tag{2.49}$$

where

$$J_{ij}^5 = f_{i8k}f_{k8j} - \frac{3\sqrt{3}}{14} d_{8ij} , \tag{2.50}$$

$$J_{ij}^6 = \delta_{ij} - \frac{4\sqrt{3}}{7} d_{8ij} , \tag{2.51}$$

and

$$J_{ij}^7 = \delta_{i0}\delta_{j0} - \frac{2\sqrt{2}}{7}(\delta_{i0}\delta_{j8} + \delta_{i8}\delta_{j0}) . \tag{2.52}$$

On the basis of previous numerical work with σ models, this symmetry-breaking term is expected to be small. The complete Lagrangian is now

$$\mathcal{L} = \mathcal{L}_{sym} + \mathcal{L}_{SB}^1 + \mathcal{L}_{SB}^2 + \mathcal{L}_{SB}^3 . \tag{2.53}$$

III. THE RESTRUCTURING OF THE LAGRANGIAN

Several modifications must be made to the Lagrangian before calculations are possible. These include allowing a Nambu-Goldstone symmetry realization, introducing the wave-function and Lagrangian-parameter renormalization constants, and outlining the type of perturbation theory to be employed.

To permit a Nambu-Goldstone symmetry realization[20] we define the vacuum expectation value of the scalar fields as

$$\langle 0|\sigma_i^0|0\rangle = v_i^0 . \tag{3.1}$$

A new scalar field with a vanishing vacuum expectation value is next defined as

$$s_i^0 = \sigma_i^0 - v_i^0 . \tag{3.2}$$

These fields are then introduced into the Lagrangian; however, owing to problems inherent in this translation, we do not normal order this translated Lagrangian.[10]

After the translation the Lagrangian can be written as

$$\begin{aligned}\mathcal{L} = &\tfrac{1}{2}\partial_\mu s_i^0\partial^\mu s_i^0 + \tfrac{1}{2}\partial_\mu\phi_i^0\partial^\mu\phi_i^0 + i\bar{b}_a^0\gamma\cdot\partial b_a^0 - \tfrac{1}{2}m_{ij}^{0\phi^2}\phi_i^0\phi_j^0\\ &- \tfrac{1}{2}m_{ij}^{0s^2}s_i^0 s_j^0 - m_{ab}^0\bar{b}_a^0 b_b^0\\ &+ \tfrac{1}{3}F_{ijkl}^0(s_i^0 s_j^0 s_k^0 s_l^0 + \phi_i^0\phi_j^0\phi_k^0\phi_l^0)\\ &+ 2\hat{F}_{ij,kl}^0\phi_i^0\phi_j^0 s_k^0 s_l^0 + G_{ijk}^{0s}s_i^0 s_j^0 s_k^0 - 3G_{ij,k}^{0\phi}\phi_i^0\phi_j^0 s_k^0\\ &+ H_{ab,i}^0(\bar{b}_a^0 b_b^0 s_i^0 + i\bar{b}_a^0\gamma_5 b_b^0\phi_i^0) - E_i^0 s_i^0 ,\end{aligned} \tag{3.3}$$

where

$$\begin{aligned}m_{ij}^{0s^2} = &\mu^{0\,2}\delta_{ij} - 6G_{ij\alpha}^0 v_\alpha^0 - 4F_{ij\alpha\beta}^0 v_\alpha^0 v_\beta^0\\ &+ 2a_s^0 J_{ij}^5 + 2d_1^0 d_{8ij}\\ &+ 2d_2^0 J_{ij}^6 + 2d_3^0 J_{ij}^7 ,\end{aligned} \tag{3.4}$$

$$\begin{aligned}m_{ij}^{0\phi^2} = &\mu^{0\,2}\delta_{ij} + 6G_{ij\alpha}^0 v_\alpha^0 - 4\hat{F}_{ij,\alpha\beta}^0 v_\alpha^0 v_\beta^0\\ &+ 2a_\phi^0 J_{ij}^5 + 2d_1^0 d_{8ij} - 2d_2^0 J_{ij}^6 - 2d_3^0 J_{ij}^7 ,\end{aligned} \tag{3.5}$$

$$m_{ab}^0 = -H_{ab,\alpha}^0 v_\alpha^0 + ie^0 f_{8ab} , \tag{3.6}$$

$$G_{ijk}^{0s} = G_{ijk}^0 + \tfrac{4}{3}F_{ijk\alpha}^0 v_\alpha^0 , \tag{3.7}$$

$$G_{ij,k}^{0\phi} = G_{ijk}^0 - \tfrac{4}{3}\hat{F}_{ij,k\alpha}^0 v_\alpha^0 , \tag{3.8}$$

and

$$\begin{aligned}E_i^0 = &c_i^0 + \mu^{0\,2}v_i^0 - 3G_{i\alpha\beta}^0 v_\alpha^0 v_\beta^0 - \tfrac{4}{3}F_{i\alpha\beta\gamma}^0 v_\alpha^0 v_\alpha^0 v_\gamma^0\\ &+ 2(a_s^0 J_{i\alpha}^5 + d_1^0 d_{8i\alpha} + d_2^0 J_{i\alpha}^6 + d_3^0 J_{i\alpha}^7)v_\alpha^0 .\end{aligned} \tag{3.9}$$

Perturbation theory is defined as an expansion in the powers of λ which is introduced via

$$\mathcal{L}(M,B,\lambda) = (1/\lambda^2)\mathcal{L}(\lambda M, \lambda B) . \tag{3.10}$$

λ is used exclusively for power counting and is set to unity at the end of the calculations. This is, in effect, an expansion in the number of closed loops in the Feynman diagrams contributing to a given process. The symmetry properties of the Lagrangian are preserved order by order in this expansion.[21]

We next introduce an "intermediate" renormalization,[10] which we use to mean the renormalization procedure that leads to a finite S matrix without at the same time leading to the conventional asymptotic renormalization of the fields. A final finite renormalization is needed for this. Consequently, we introduce the chiral-invariant wave-function renormalization constants Z_M and Z_B via

$$(\phi_i^0, s_i^0, v_i^0) = Z_M^{1/2}(\phi_i, s_i, v_i) \tag{3.11}$$

and

$$b_a^0 = Z_B^{1/2} b_a . \tag{3.12}$$

Renormalization constants are also introduced for each parameter in the Lagrangian; for example,

$$f_1^0 = Z_{f_1} f_1 / Z_M{}^2 \tag{3.13}$$

and

$$e^0 = Z_e e / Z_B . \tag{3.14}$$

The Lagrangian can finally be rewritten as

$$\begin{aligned}\mathcal{L} = {} & \tfrac{1}{2}\partial_\mu s_i \partial^\mu s_i + \tfrac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i + i\bar{b}_a \gamma \cdot \partial b_a - \tfrac{1}{2} m_{ij}^{\phi^2} \phi_i \phi_j - \tfrac{1}{2} m_{ij}^{s^2} s_i s_j - m_{ab} \bar{b}_a b_b \\ & + \lambda^2 F_{ijkl}(s_i s_j s_k s_l + \phi_i \phi_j \phi_k \phi_l) + 2\lambda^2 \hat{F}_{ij,kl} \phi_i \phi_j s_k s_l + \lambda G^s_{ijk} s_i s_j s_k - 3\lambda G^\phi_{ij,k} \phi_i \phi_j s_k \\ & + \lambda H_{ab,i}(\bar{b}_a b_b s_i + i\bar{b}_a \gamma_5 b_b \phi_i) - (1/\lambda) E_i S_i + (Z_M - 1)\tfrac{1}{2}(\partial_\mu s_i \partial^\mu s_i + \partial_\mu \phi_i \partial^\mu \phi_i) + i(Z_B - 1)\bar{b}_a \gamma \cdot \partial b_a ,\end{aligned} \tag{3.15}$$

where only the wave-function renormalization-constant counterterms have been explicitly written. The couplings have also been restructured; for example,

$$F_{ijkl} = Z_{f_1} f_1 J^1_{ijkl} + \tfrac{1}{2} Z_{f_2} f_2 J^2_{ijkl} . \tag{3.16}$$

The Feynman rules for this Lagrangian are given in Fig. 1. The vector and axial-cector currents are

$$\begin{aligned} V_i^\mu = {} & \tfrac{1}{2} f_{ijk} Z_M (s_j \overleftrightarrow{\partial}^\mu s_k + \phi_j \overleftrightarrow{\partial}^\mu \phi_k) \\ & + f_{ijk} Z_M v_j \partial^\mu s_k - i f_{iab} Z_B \bar{b}_a \gamma^\mu b_b \end{aligned} \tag{3.17}$$

and

$$A_i^\mu = d_{ijk} Z_M (\phi_j \overleftrightarrow{\partial}^\mu s_k - v_j \partial^\mu \phi_k) + d_{iab} Z_B \bar{b}_a \gamma^\mu \gamma_5 b_b , \tag{3.18}$$

respectively.

All parameters in the Lagrangian have contributions to each order in perturbation theory. For example, to second order we write

$$Z_{f_1} f_1 = f_1 + \lambda^2 \delta f_1 , \tag{3.19}$$

where the counterterm is denoted by δ. The counterterms can be separated into divergent (D) and finite (Δ) parts, i.e.,

$$\delta = D + \Delta . \tag{3.20}$$

Similarly, v_i has contributions to all orders. To second order we set

$$v_i = \xi_i + \lambda^2 \delta \xi_i . \tag{3.21}$$

In this paper we are concerned only with the divergent parts of counterterms.

In Sec. IV we demonstrate that a consistent renormalization is possible with $D\xi_i = 0$ and evaluate the divergent counterterms to second order.

Finally, we note that in the definitions of the masses and coupling constants of the final Lagrangian all the basic Lagrangian constants appeared linearly, except the v_i. To ensure that the symmetry of the Lagrangian is maintained, only terms to a given order of λ can be retained in the counterterms. Thus, for example, δE_i to second order is

$$\delta E_i = E_i(\delta\mu^2, \delta f_1, \delta f_2, \delta g, \delta c, \delta a_s, \delta d) + m_{ij}^{s^2} \delta \xi_j . \tag{3.22}$$

IV. RENORMALIZATION OF THE ONE-LOOP AMPLITUDES

In this section we demonstrate that the model is renormalizable in the one-loop approximation and evaluate the counterterms. We set

$$D\xi_i = 0 \tag{4.1}$$

and, at the conclusion, it is clear that the remaining counterterms are sufficient to cancel all second-order divergences. Each proper vertex is analyzed in turn and is shown to be finite. In subsections A, B, C, and D below we consider the four-, three-, two-, and one-point amplitudes, respectively. In subsection E a non-S-matrix type of divergence from the current-field vertex is analyzed.

A. Four-point amplitudes

Three four-point amplitudes need to be considered. First consider the four-point scalar vertex. The diagrams to be evaluated to second order containing divergent terms are presented in Fig. 2. Evaluating these diagrams and requiring that the divergent parts cancel, one finds

$$\begin{aligned} & 8DF_{ijkl} - 32F_{ijmn} F_{klm'n'} DB^{ss}_{mm'nn'}((q_1+q_2)^2) - 32\hat{F}_{ij,mn}\hat{F}_{kl,m'n'} DB^{\phi\phi}_{mm',nn'}((q_1+q_2)^2) \\ & \quad + \tfrac{1}{2} H_{ah,i} H_{bc,j} H_{de,k} H_{fg,l} D[D_{ab,cd,ef,gh}(q_1,q_1;q_3,q_4) + D_{hg,fe,dc,ba}(q_4,q_3;q_2,q_1)] \\ & \quad + \tfrac{1}{2} H_{ah,i} H_{bc,j} H_{de,l} H_{fg,k} D[D_{ab,cd,ef,gh}(q_1,q_2;q_4,q_3) + D_{hg,fe,dc,ba}(q_3,q_4;q_2,q_1)] + \text{crossed terms} = 0 , \end{aligned} \tag{4.2}$$

where

$$B_{ij,kl}(p^2)=i\int\frac{d^4l}{(2\pi)^4}D((l-p)^2)_{ij}D(l^2)_{kl} \tag{4.3}$$

and

$$D_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4)=i\int\frac{d^4l}{(2\pi)^4}\mathrm{Tr}[D(l+q_1)_{ab}D(l+q_1+q_2)_{cd}D(l-q_4)_{ef}D(l)_{gh}]\,. \tag{4.4}$$

To isolate the divergent parts of the Feynman integrals, we expand the meson propagators $D^{s,\phi}((l-p)^2)_{ij}$ about the point $p^2=0$ and the arbitrary chiral-invariant mass $m^2=\nu^2$, and expand the baryon propagators $D(l-p)_{ab}$ in a similar fashion about $p=0$ and $m=\nu$. Explicitly, one then has

$$D((l-p)^2)_{ij}=\frac{\delta_{ij}}{l^2-\nu^2}+\frac{m_{ij}^2-(p^2-2l\cdot p+\nu^2)\delta_{ij}}{(l^2-\nu^2)^2}+\cdots \tag{4.5}$$

and

$$D(l-p)_{ab}=\frac{\delta_{ab}}{\not{l}-\nu}+\frac{1}{(\not{l}-\nu)^2}[m_{ab}+(\not{p}-\nu)\delta_{ab}]+\cdots\,. \tag{4.6}$$

This expansion is valid whether or not there is particle mixing and allows the divergent parts of the integrals to be easily identified. When evaluating the integrals, the trace of the γ matrices must be computed before performing the integrations.

Applying this prescription to the above integrals gives

$$DB_{ij,kl}(p^2)=i\int\frac{d^4l}{(2\pi)^4}\frac{\delta_{ij}\delta_{kl}}{(l^2-\nu^2)^2} \tag{4.7}$$

$$\equiv\delta_{ij}\delta_{kl}B(\nu^2) \tag{4.8}$$

and

$$DD_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4)=4\delta_{ab}\delta_{cd}\delta_{ef}\delta_{gh}B(\nu^2)\,. \tag{4.9}$$

Convergent integrals can then be defined as

$$\overline{B}_{ij,kl}(p^2)=B_{ij,kl}(p^2)-\delta_{ij}\delta_{kl}B(\nu^2) \tag{4.10}$$

and

$$\overline{D}_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4)=D_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4)-4\delta_{ab}\delta_{cd}\delta_{ef}\delta_{gh}B(\nu^2)\,. \tag{4.11}$$

Equation (4.2) can now be rewritten as

$$\begin{aligned}DF_{ijkl}=\;&4(F_{ijmn}F_{klmn}+F_{ikmn}F_{jlmn}+F_{ilmn}F_{jkmn}\\&+\hat{F}_{ij,mn}\hat{F}_{kl,mn}+\hat{F}_{ik,mn}\hat{F}_{jl,mn}\\&+\hat{F}_{il,mn}\hat{F}_{jk,mn})B(\nu^2)\\&-\tfrac{1}{2}(H^4_{ijkl}+H^4_{ikjl}+H^4_{iljk}+H^4_{ijlk}\\&+H^4_{iklj}+H^4_{ilkj})B(\nu^2)\,,\end{aligned} \tag{4.12}$$

where the crossed terms have been explicitly in-

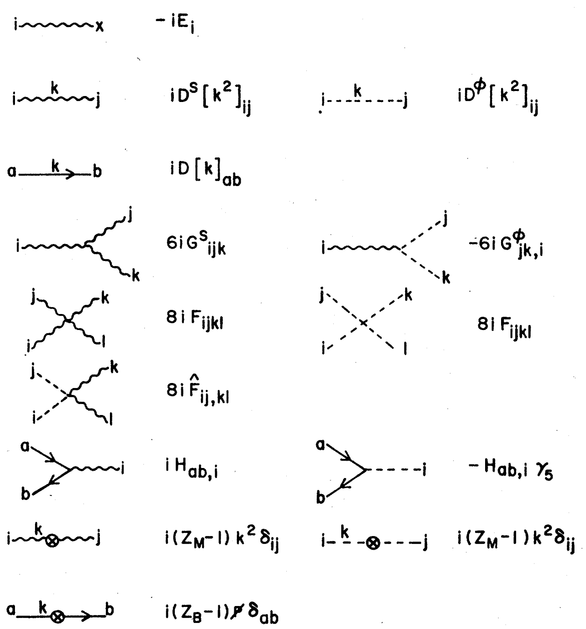

FIG. 1. Feynman rules for the Lagrangian of Eq. (3.15). Wavy lines, dashed lines, and solid lines represent scalar, pseudoscalar, and baryon fields, respectively.

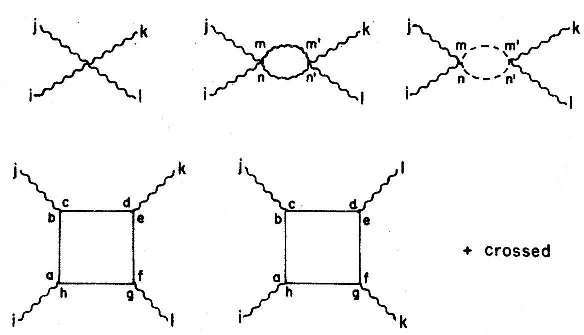

FIG. 2. Diagrams containing divergent contributions to the four-point proper scalar vertex to second order.

cluded and

$$H^4_{ijkl} = H_{ab,i}H_{bc,j}H_{cd,k}H_{da,l}\,. \quad (4.13)$$

The product terms can be evaluated in a straightforward manner using standard SU(3) tensor identities[19]; for example,

$$H^4_{ijkl} = \frac{h^4}{1296}(d_{ijm}d_{mkl} - d_{ikm}d_{mjl} + d_{ilm}d_{mjk} + \delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk})\,. \quad (4.14)$$

Evaluating and summing these terms gives

$$DF_{ijkl} = J^1_{ijkl}\left[8(13f_1^2 + 12f_1f_2 + 3f_2^2) - \frac{h^4}{648}\right]B(\nu^2) + J^2_{ijkl}\left[24f_2(f_1+f_2) - \frac{h^4}{1296}\right]B(\nu^2)\,. \quad (4.15)$$

From Eq. (2.39) we also have

$$DF_{ijkl} = Df_1 J^1_{ijkl} + \tfrac{1}{2}Df_2 J^2_{ijkl}\,. \quad (4.16)$$

Consequently, the required counterterms are

$$Df_1 = \left[8(13f_1^2 + 12f_1f_2 + 3f_2^2) - \frac{h^4}{648}\right]B(\nu^2) \quad (4.17)$$

and

$$Df_2 = \left[48f_2(f_1+f_2) - \frac{h^4}{648}\right]B(\nu^2)\,. \quad (4.18)$$

With these values for the divergent second-order counterterms the four-point proper scalar vertex is finite in the one-loop approximation. These counterterms also remove the second-order divergences in the four-point proper pseudoscalar vertex in a similar calculation. The only significant difference in this case is that the baryon loop diagrams involve the integral

$$D^{5555}_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4) = i\int\frac{d^4l}{(2\pi)^4}\mathrm{Tr}[\gamma_5 D(l+q_1)_{ab}\gamma_5 D(l+q_1+q_2)_{cd}\gamma_5 D(l-q_4)_{ef}\gamma_5 D(l)_{gh}]\,, \quad (4.19)$$

for which the divergent part is again

$$DD^{5555}_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4) = 4\delta_{ab}\delta_{cd}\delta_{ef}\delta_{gh}B(\nu^2)\,. \quad (4.20)$$

The diagrams containing divergences to be evaluated to second order for the four-point proper scalar-pseudoscalar vertex are given in Fig. 3. Evaluating these diagrams and setting the divergent part of the amplitude to zero gives

$$\begin{aligned}&8D\hat{F}_{ij,kl} - 32\hat{F}_{ij,mn}F_{klm'n'}DB^{ss}_{mm',nn'}((q_1+q_2)^2) - 32F_{ijmn}\hat{F}_{kl,m'n'}DB^{\phi\phi}_{mm',nn'}((q_1+q_2)^2)\\ &-\tfrac{1}{2}H_{ha,i}H_{bc,j}H_{de,k}H_{fg,l}D[D^{5500}_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4) + D^{5500}_{ba,hg,fe,dc}(q_2,q_1;q_4,q_3)]\\ &-\tfrac{1}{2}H_{ha,i}H_{bc,j}H_{de,l}H_{fg,k}D[D^{5500}_{ab,cd,ef,gh}(q_1,q_2;q_4,q_3) + D^{5500}_{ba,hg,fe,dc}(q_2,q_1;q_3,q_4)] + \text{crossed terms} = 0\,. \end{aligned} \quad (4.21)$$

This amplitude contains baryon loop integrals of the type

$$D^{5500}_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4) = i\int\frac{d^4l}{(2\pi)^4}\mathrm{Tr}[\gamma_5 D(l+q_1)_{ab}\gamma_5 D(l+q_1+q_2)_{cd}D(l-q_4)_{ef}D(l)_{gh}] \quad (4.22)$$

and

$$D^{5050}_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4) = i\int\frac{d^4l}{(2\pi)^4}\mathrm{Tr}[\gamma_5 D(l+q_1)_{ab}D(l+q_1+q_2)_{cd}\gamma_5 D(l-q_4)_{ef}D(l)_{gh}]\,, \quad (4.23)$$

which have divergent parts

$$DD^{5500(5050)}_{ab,cd,ef,gh}(q_1,q_2;q_3,q_4) = +(-)\delta_{ab}\delta_{cd}\delta_{ef}\delta_{gh}B(\nu^2)\,. \quad (4.24)$$

Isolating the divergent parts of the integrals and including the crossed terms, one finds

$$\begin{aligned}D\hat{F}_{ij,kl} &= 4(\hat{F}_{ij,mn}F_{klmn} + F_{ijmn}\hat{F}_{kl,mn} + 2\hat{F}_{im,kn}F_{jm,ln} + 2\hat{F}_{im,ln}\hat{F}_{jm,kn})B(\nu^2)\\ &\quad - \tfrac{1}{2}(H^4_{ijkl} + H^4_{ijlk} + H^4_{iklj} + H^4_{ilkj} - H^4_{ikjl} - H^4_{iljk})B(\nu^2)\,. \end{aligned} \quad (4.25)$$

This reduces to

$$D\hat{F}_{ij,kl} = \left[8(13f_1^2 + 12f_1f_2 + 3f_2^2) - \frac{h^4}{648}\right]\delta_{ij}\delta_{kl}B(\nu^2) + \left[24f_2(f_1+f_2) - \frac{h^4}{1296}\right]J^3_{ijkl}B(\nu^2)\,. \quad (4.26)$$

From Eq. (2.40) $D\hat{F}_{ij,kl}$ can also be written as

$$D\hat{F}_{ij,kl} = Df_1\delta_{ij}\delta_{kl} + \tfrac{1}{2}Df_2 J^3_{ijkl}\,. \quad (4.27)$$

Consequently, the counterterms of Eqs. (4.17) and (4.18) also render this vertex finite.

B. Three-point amplitudes

Four three-point amplitudes need to be considered. The diagrams containing divergences to be evaluated to second order for the three-point scalar-meson vertex are given in Fig. 4. Requiring the divergent part of the amplitude to vanish gives

$$6DG^s_{ijk} - 24G^s_{imn}F_{jkm'n}DB^{ss}_{mm',nn'}(q^2) + 24G^{\phi}_{mn,i}\hat{F}_{jk,m'n'}DB^{\phi\phi}_{mm',nn'}(q^2) - H_{fa,i}H_{bc,j}H_{de,k}D[C_{ab,cd,ef}(q_1,q_2,q_3) + C_{fe,dc,ba}(q_1,q_3,q_2)] + \text{crossed terms} = 0\,. \quad (4.28)$$

The divergent part of

$$C_{ab,cd,ef}(q_1,q_2,q_3) = i\int \frac{d^4l}{(2\pi)^4}\mathrm{Tr}[D(l+q_1)_{ab}D(l-q_3)_{cd}D(l)_{ef}] \quad (4.29)$$

has the form

$$DC_{ab,cd,ef}(q_1,q_2,q_3) = 4(\delta_{ab}\delta_{cd}m_{ef} + \delta_{ab}\delta_{ef}m_{cd} + \delta_{cd}\delta_{ef}m_{ab})B(\nu^2)\,. \quad (4.30)$$

Isolating the divergent parts of the integrals, one has

$$DG^s_{ijk} = 4(G^s_{imn}F_{jkmn} - G^{\phi}_{mn,i}\hat{F}_{jk,mn} + \text{crossed terms})B(\nu^2) + \tfrac{2}{3}[H_{fa,i}H_{ac,j}H_{ce,k}(m_{ef}+m_{fe}) + H_{de,k}H_{ea,i}H_{ac,j}(m_{cd}+m_{dc}) + H_{ac,j}H_{ce,k}H_{eb,i}(m_{ab}+m_{ba})]B(\nu^2)\,. \quad (4.31)$$

Rewriting this using Eqs. (3.6)–(3.8) for m, G^s, and G^{ϕ} gives

$$DG^s_{ijk} = 4[G_{imn}(F_{jkmn} - \hat{F}_{jk,mn}) + G_{jmn}(F_{ikmn} - \hat{F}_{ik,mn}) + G_{kmn}(F_{ijmn} - \hat{F}_{ij,mn}) + \tfrac{4}{3}(F_{i\alpha mn}F_{jkmn} + F_{j\alpha mn}F_{ikmn} + F_{k\alpha mn}F_{ijmn} + \hat{F}_{i\alpha,mn}\hat{F}_{jk,mn} + \hat{F}_{j\alpha,mn}\hat{F}_{ik,mn} + \hat{F}_{k\alpha,mn}\hat{F}_{ij,mn})\xi_\alpha]B(\nu^2) - \tfrac{2}{3}(H^4_{\alpha ijk} + H^4_{\alpha ikj} + H^4_{\alpha kij} + H^4_{\alpha jik} + H^4_{\alpha jki} + H^4_{\alpha kji})\xi_\alpha B(\nu^2)\,. \quad (4.32)$$

But DG^s is also given by

$$DG^s_{ijk} = DG_{ijk} + \tfrac{4}{3}DF_{ijk\alpha}\xi_\alpha\,, \quad (4.33)$$

where we have assumed that $D\xi_\alpha$ vanishes. The terms containing ξ_α in the above relation reproduce Eq. (4.12) of the four-point scalar vertex calculation. The remaining constraint reduces to

$$DG_{ijk} = DgJ^4_{ijk} = 24g(f_1 - f_2)J^4_{ijk}B(\nu^2)\,. \quad (4.34)$$

Consequently, the required counterterm is

$$Dg = 24g(f_1 - f_2)B(\nu^2)\,. \quad (4.35)$$

From Eq. (4.31) it is clear that symmetric baryon symmetry-breaking terms such as $\bar{b}_a b_a$ or $d_{8ab}\bar{b}_a b_b$ are not acceptable as they would contribute symmetry-breaking effects to the right-hand side, and thus render the model nonrenormalizable.

The calculation for the three-point proper scalar-pseudoscalar-meson vertex parallels the one above. The diagrams are given in Fig. 5. Part of the evaluation duplicates that of the four-point scalar-pseudoscalar vertex and the remainder reproduces the calculation of Dg in Eq. (4.34). This amplitude requires the integral

$$C^{55}_{ab,cd,ef}(q_1,q_2,q_3) = i\int\frac{d^4l}{(2\pi)^4}\mathrm{Tr}[\gamma_5 D(l+q_1)_{ab}\gamma_5 D(l-q_3)_{cd}D(l)_{ef}]\,, \quad (4.36)$$

which has a divergent part given by

$$DC^{55}_{ab,cd,ef}(q_1,q_2,q_3) = 4(\delta_{cd}\delta_{ef}m_{ab} - \delta_{ab}\delta_{cd}m_{ef} - \delta_{ab}\delta_{ef}m_{cd})B(\nu^2)\,. \quad (4.37)$$

The diagrams containing divergences for the scalar meson-baryon proper vertex are given in Fig. 6. Evaluating the diagrams, setting the divergent part of the amplitude to zero, and neglecting spinors gives

$$DH_{ab,i} + H_{ac,k}H_{de,i}H_{fb,j}D\mathcal{C}_{cd,ef,jk}(q_1,q_3;q_2) - H_{ac,k}H_{de,i}H_{fb,j}D\mathcal{C}^{505}_{cd,ef,jk}(q_1,q_3;q_2)\,, \quad (4.38)$$

where

$$\mathcal{C}_{ab,cd,ij}(q_1,q_2;q_3) = i\int\frac{d^4l}{(2\pi)^4}D(l+q_1)_{ab}D(l-q_3)_{cd}D^s(l^2)_{ij}\,, \quad (4.39)$$

$$\mathcal{C}^{505}_{ab,cd,ij}(q_1,q_2;q_3) = i\int\frac{d^4l}{(2\pi)^4}\gamma_5 D(l+q_1)_{ab}D(l-q_3)_{cd}\gamma_5 D^{\phi}(l^2)_{ij}\,, \quad (4.40)$$

and

$$D\mathcal{C}_{ab,cd,ij}(q_1, q_2; q_3) = D\mathcal{C}^{505}_{ab,cd,ij}(q_1, q_2; q_3) = \delta_{ab}\delta_{cd}\delta_{ij}B(\nu^2) . \tag{4.41}$$

Equation (4.38) reduces to

$$DH_{ab,i} = 0 . \tag{4.42}$$

From Eq. (2.42) the resultant counterterm is

$$Dh = 0 . \tag{4.43}$$

The calculation for the pseudoscalar-meson–baryon three-point vertex is similar to the one above. The diagrams are given in Fig. 7. In this case we encounter the integrals

$$\mathcal{C}^{5}_{ab,cd,ij}(q_1, q_2; q_3) = i\int \frac{d^4l}{(2\pi)^4} D(l+q_1)_{ab}\gamma_5 D(l-q_3)_{cd}D^s(l^2)_{ij} \tag{4.44}$$

and

$$\mathcal{C}^{555}_{ab,cd,ij}(q_1, q_2; q_3) = i\int \frac{d^4l}{(2\pi)^4} \gamma_5 D(l+q_1)_{ab}\gamma_5 D(l-q_3)_{cd}\gamma_5 D^{\phi}(l^2)_{ij} \tag{4.45}$$

with divergent parts

$$D\mathcal{C}^{5}_{ab,cd,ij}(q_1, q_2; q_3) = D\mathcal{C}^{555}_{ab,cd,ij}(q_1, q_2; q_3) = -\gamma_5\delta_{ab}\delta_{cd}\delta_{ij}B(\nu^2) . \tag{4.46}$$

C. Two-point amplitudes

In this subsection we consider the renormalization of the masses. These calculations are much more tedious then those above as both the meson and baryon explicit symmetry-breaking terms manifest themselves at this level.

First consider the scalar mass. The diagrams for the scalar two-point function are given in Fig. 8. Setting the divergent part of this amplitude to zero gives

$$(Z_M - 1)p^2\delta_{ij} - Dm^{s2}_{ij} + 4F_{ijmn}DA^s_{mn} + 4\hat{F}_{ij,mn}DA^{\phi}_{mn} - 18G^s_{imn}G^s_{jm'n'}DB^{ss}_{mm',nn'}(p^2)$$
$$- 18G^{\phi}_{mn,i}G^{\phi}_{m'n',j}DB^{\phi\phi}_{mm',nn'}(p^2) + H_{ad,i}H_{bc,j}D[B_{ab,cd}(p) + B_{ba,dc}(p)] = 0 , \tag{4.47}$$

where

$$A_{ij} = i\int \frac{d^4l}{(2\pi)^4} D(l^2)_{ij} \tag{4.48}$$

and

$$B_{ab,cd}(p) = i\int \frac{d^4l}{(2\pi)^4} \mathrm{Tr}[D(l-p)_{ab}D(l)_{cd}] . \tag{4.49}$$

These integrals have the divergent parts

$$DA_{ij} = A(\nu^2)\delta_{ij} + (m^2_{ij} - \nu^2\delta_{ij})B(\nu^2) \tag{4.50}$$

and

$$DB_{ab,cd}(p) = 4A(\nu^2)\delta_{ab}\delta_{cd} + 4[m_{ab}m_{cd} + \delta_{ab}m_{ce}m_{ed} + \delta_{cd}m_{ae}m_{eb} - (p^2/2 + \nu^2)\delta_{ab}\delta_{cd}]B(\nu^2) , \tag{4.51}$$

which involve both meson and baryon masses. $A(\nu^2)$ is defined by

$$A(\nu^2) = i\int \frac{d^4l}{(2\pi)^4} \frac{1}{l^2 - \nu^2} . \tag{4.52}$$

Equation (4.47) can now be rewritten as

$$Dm^{s2}_{ij} - (Z_M - 1)p^2\delta_{ij} = 4(F_{ijmm} + \hat{F}_{ij,mm} + 2H_{ab,i}H_{ab,j})A(\nu^2)$$
$$+ 4[F_{ijmn}m^{s2}_{mn} + \hat{F}_{ij,mn}m^{\phi 2}_{mn} - \nu^2(F_{ijmm} + \hat{F}_{ij,mm})]B(\nu^2) - 18(G^s_{imn}G^s_{jmn} + G^{\phi}_{mn,i}G^{\phi}_{mn,j})B(\nu^2)$$
$$+ 4H_{ad,i}H_{bc,j}[m_{ab}m_{cd} + m_{ba}m_{dc} + \delta_{ab}(m_{ce}m_{ed} + m_{de}m_{ec})$$
$$+ \delta_{cd}(m_{ae}m_{eb} + m_{be}m_{ea}) - 2(p^2/2 + \nu^2)\delta_{ab}\delta_{cd}]B(\nu^2) . \tag{4.53}$$

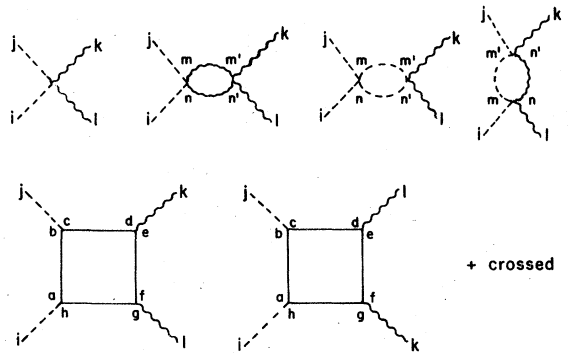

FIG. 3. Diagrams containing divergent contributions to second order to the four-point proper scalar-pseudoscalar amplitude.

From the reduction

$$H_{ab,i}H_{ab,j}=\frac{h^2}{9}\delta_{ij} \tag{4.54}$$

one immediately finds

$$Z_M+1+\tfrac{4}{9}B(\nu^2)\,. \tag{4.55}$$

The remainder we write as

$$Dm^{s2}_{ij}=T^{s1}_{ij}A(\nu^2)+T^{s2}_{ij}B(\nu^2)\,. \tag{4.56}$$

A straightforward evaluation gives

$$T^{s1}_{ij}=16\left[5f_1+3f_2+\frac{h^2}{18}\right]\delta_{ij}. \tag{4.57}$$

T^{s2}_{ij} can be further decomposed via

$$T^{s2}_{ij}=T^{s0}_{ij}+T^{sM}_{ij}+T^{sB}_{ij} \tag{4.58}$$

into parts that are obtained exclusively from the symmetric and symmetry-breaking parts of the meson and baryon masses, respectively. The

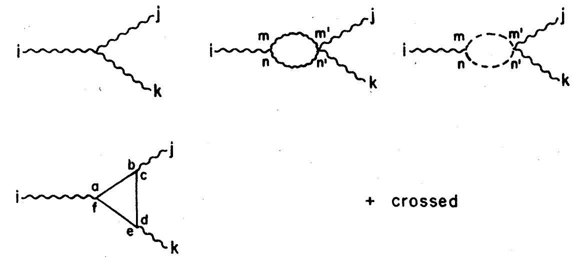

FIG. 4. Diagrams containing divergent contributions to the three-point proper scalar vertex in the one-loop approximation.

evaluation of T^{s0}_{ij} largely reproduces calculations shown above; in particular, after using Eqs. (3.4)–(3.8) for the masses (isolating the symmetric part), G^s and G^ϕ, one essentially duplicates Eqs. (4.15) and (4.34). The remaining portion is a δ_{ij} component. The final result is

$$T^{s0}_{ij}=\left[16(\mu^2-\nu^2)(5f_1+3f_2)-16g^2-\frac{8h}{9}\nu^2\right]\delta_{ij} \\ -6DG_{ij\alpha}\xi_\alpha-4DF_{ij\alpha\beta}\xi_\alpha\xi_\beta\,. \tag{4.59}$$

Next consider the contribution of the baryon symmetry-breaking terms. From Eq. (4.53) this is

$$T^{sB}_{ij}=-\frac{2e^2h^2}{9}(7f_{i8m}f_{m8j}-\tfrac{1}{2}3\sqrt{3}\,d_{8ij}-3\delta_{ij})\,. \tag{4.60}$$

The meson symmetry-breaking term must be able to accommodate this structure; however, before analyzing it in detail we consider the pseudoscalar-meson mass.

The diagrams for the pseudoscalar-meson two-point function are given in Fig. 9. Requiring the divergent part of the amplitude to vanish gives

$$(Z_M-1)p^2\delta_{ij}-Dm^{\phi\,2}_{ij}+\hat{F}_{ij,mn}DA^{s}_{mn}+F_{ijmn}DA^{\phi}_{mn}-36G^{\phi}_{im,n}G^{\phi}_{jm',n'}DB^{s\phi}_{mm',nn'}(p^2) \\ -H_{ad,i}H_{bc,j}D[B^{55}_{ab,cd}(p)+B^{55}_{ba,dc}(p)]=0\,, \tag{4.61}$$

where

$$B^{55}_{ab,cd}(p^2)=i\int\frac{d^4l}{(2\pi)^4}\mathrm{Tr}[\gamma_5D(l-p)_{ab}\gamma_5D(l)_{cd}] \tag{4.62}$$

and

$$DB^{55}_{ab,cd}(p)=-4\delta_{ab}\delta_{cd}A(\nu^2)+4[m_{ab}m_{cd}-\delta_{ab}m_{ce}m_{ed}-\delta_{cd}m_{ae}m_{eb}+(p^2/2+\nu^2)\delta_{ab}\delta_{cd}]B(\nu^2)\,. \tag{4.63}$$

This can be rewritten as

$$Dm^{\phi\,2}_{ij}-(Z_M-1)p^2\delta_{ij}=4(F_{ijmm}+\hat{F}_{ij,mm}+2H_{ab,i}H_{ab,j})A(\nu)^2 \\ +4[F_{ij,mn}m^{s\,2}_{mn}+F_{ijmn}m^{\phi\,2}_{mn}-\nu^2(F_{ijmm}+\hat{F}_{ij,mm})]B(\nu^2)-36G^{\phi}_{im,n}G^{\phi}_{jm,n}B(\nu^2) \\ +4H_{ad,i}H_{bc,j}[-m_{ab}m_{cd}-m_{ba}m_{dc}+\delta_{ab}(m_{ce}m_{ed}+m_{de}m_{ec}) \\ +\delta_{cd}(m_{ae}m_{eb}+m_{be}m_{ea})-2(p^2/2+\nu^2)\delta_{ab}\delta_{cd}]B(\nu^2)\,. \tag{4.64}$$

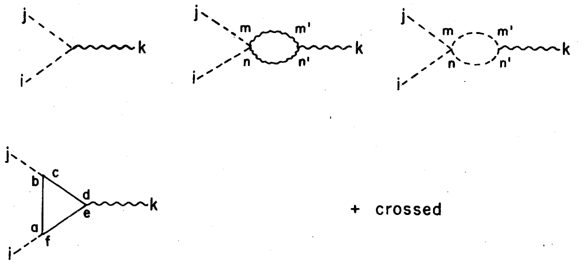

FIG. 5. One-loop approximation diagrams containing divergences for the three-point proper scalar-pseudoscalar amplitude.

The p^2 term reproduces Eq. (4.55). The remaining part is

$$Dm^{\phi 2}_{ij} = T^{\phi 1}_{ij} A(\nu^2) + (T^{\phi 0}_{ij} + T^{\phi m}_{ij} + T^{\phi B}_{ij}) B(\nu^2), \qquad (4.65)$$

paralleling Eqs. (4.56) and (4.58) for the scalar mass. One immediately finds

$$T^{\phi 1}_{ij} = T^{s1}_{ij}. \qquad (4.66)$$

The symmetric mass calculation partly resembles those for the three- and four-point scalar-pseudoscalar-meson vertices and gives

$$T^{\phi 0}_{ij} = \left[16(\mu^2 - \nu^2)(5f_1 + 3f_2) - 16g^2 - \frac{8h^2}{9}\nu^2\right]\delta_{ij} + 6DG_{ij\alpha}\xi_\alpha - 4D\hat{F}_{ij,\alpha\beta}\xi_\alpha\xi_\beta. \qquad (4.67)$$

The baryon contribution in this case is

$$T^{\phi B}_{ij} = -\frac{2e^2h^2}{9}\left(-3f_{i8m}f_{m8j} - \frac{\sqrt{3}}{2}d_{8ij} - \delta_{ij}\right). \qquad (4.68)$$

The meson symmetry-breaking terms must be chosen to accommodate both T^{sB} and $T^{\phi B}$. This choice is complicated by the fact that the meson symmetry-breaking terms feed back into Dm^2_{ij} via the m^2_{mn} contributions in Eqs. (4.53) and (4.64). Consequently, there is a minimal set of tensors allowed. This set governed the choice of $\mathcal{L}^3_{SB}$ in Eq. (2.49). Using Eqs. (3.4) and (3.5), T^{sM}_{ij} is now given by

$$T^{sM}_{ij} = 8[F_{ijmn}(a_s J^5_{mn} + d_1 d_{8mn} + d_2 J^6_{mn} + d_3 J^7_{mn}) + \hat{F}_{ij,mn}(a_\phi J^5_{mn} + d_1 d_{8mn} - d_2 J^6_{mn} - d_3 J^7_{mn})]. \qquad (4.69)$$

$T^{\phi M}_{ij}$ has F and $\hat{F}$ interchanged.

Evaluating these expressions and isolating the

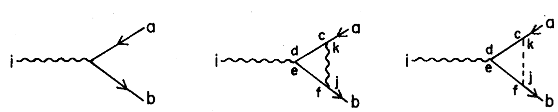

FIG. 6. Diagrams containing divergences to second order for the proper scalar-baryon vertex.

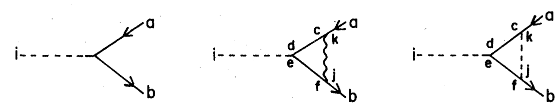

FIG. 7. Diagrams containing divergent contributions to the three-point proper pseudoscalar-baryon amplitude to second order.

counterterms using Eqs. (4.56) and (4.65) gives

$$D\mu^2 = 16\left(5f_1 + 3f_2 + \frac{h^2}{18}\right)A(\nu^2) + 4\left[4(\mu^2 - \nu^2)(5f_1 + 3f_2) - 4g^2 - \frac{2h^2\nu^2}{9} - (3f_1 + 2f_2)(a_s + a_\phi) + \tfrac{1}{9}e^2h^2\right]B(\nu^2), \qquad (4.70)$$

$$Da_s = \left[8a_s f_1 + 4f_2(a_s - a_\phi) - \tfrac{7}{9}e^2h^2\right]B(\nu^2), \qquad (4.71)$$

$$Da_\phi = [8a_\phi f_1 - 4f_2(a_s - a_\phi) + \tfrac{1}{3}e^2h^2]B(\nu^2), \qquad (4.72)$$

$$Dd_1 = 4\left[2d_1(f_1 + 3f_2) - \frac{\sqrt{3}}{7}f_2(a_s + a_\phi) + \frac{\sqrt{3}}{63}e^2h^2\right]B(\nu^2), \qquad (4.73)$$

$$Dd_2 = [8f_1 d_2 + 2f_2(a_s - a_\phi) + \tfrac{8}{3}f_2 d_3 + \tfrac{1}{9}e^2h^2]B(\nu^2), \qquad (4.74)$$

and

$$Dd_3 = 2[4f_1 d_3 - 3f_2(a_s - a_\phi) + 12f_2 d_2]B(\nu^2). \qquad (4.75)$$

From the nature of these counterterms, it is clear that the complex structure of $\mathcal{L}^3_{SB}$ is necessary. For a nonvanishing e, all five meson bilinear symmetry-breaking terms are required.

The diagrams for the baryon two-point function are given in Fig. 10. Neglecting spinors and setting the divergent part of the amplitude to zero gives

$$-Dm_{ab} + (Z_B - 1)\not{p}\delta_{ab} - H_{ac,i}H_{db,j}D[\not{B}^s_{cd,ji}(p) - \not{B}^\phi_{cd,ji}(p)] = 0, \qquad (4.76)$$

where

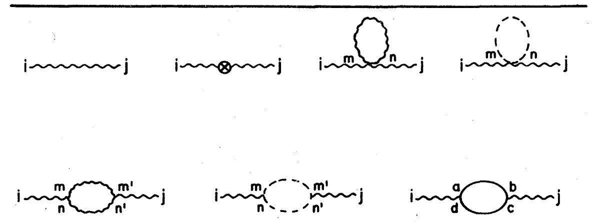

FIG. 8. Diagrams for the two-point scalar amplitude to second order.

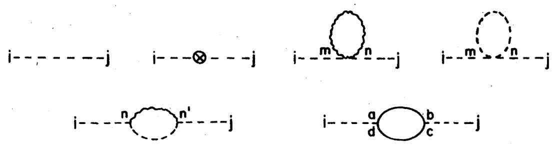

FIG. 9. Diagrams for the two-point pseudoscalar amplitude in the one-loop approximation.

$$\not{B}^s_{ab,ij}(p)=i\int\frac{d^4l}{(2\pi)^4}D(l)_{ab}D^s((l-p)^2)_{ij}\,, \tag{4.77}$$

$$\not{B}^\phi_{ab,ij}(p)=i\int\frac{d^4l}{(2\pi)^4}\gamma_5 D(l)_{ab}\gamma_5 D^\phi((l-p)^2)_{ij}\,, \tag{4.78}$$

$$D\not{B}^s_{ab,ij}(p)=(\not{p}/2\delta_{ab}+m_{ab})\delta_{ij}B(\nu^2)\,, \tag{4.79}$$

and

$$D\not{B}^\phi_{ab,ij}(p)=(-\not{p}/2\delta_{ab}+m_{ab})\delta_{ij}B(\nu^2)\,. \tag{4.80}$$

Consequently, we have

$$Dm_{ab}-(Z_B-1)\not{p}\delta_{ab}=-\not{p}H_{ac,i}H_{cb,i}B(\nu^2)\,, \tag{4.81}$$

which affords the counterterms

$$De=0 \tag{4.82}$$

and

$$Z_B=1+\frac{h^2}{9}B(\nu^2)\,. \tag{4.83}$$

D. One-point amplitude

In this section we consider the vacuum expectation value of the scalar field. The diagrams for this amplitude are given in Fig. 11. Evaluating these diagrams and setting the divergent part of the amplitude to zero gives

$$DE_i-6G^s_{imn}DA^s_{mn}+6G^\phi_{mn,i}DA^\phi_{mn}+H_{ab,i}D(A^B_{ab}+A^B_{ba})=0\,, \tag{4.84}$$

where

$$A^B_{ab}=i\int\frac{d^4l}{(2\pi)^4}\mathrm{Tr}[D(l)_{ab}] \tag{4.85}$$

and

$$DA^B_{ab}=4m_{ab}A(\nu^2)+4(m_{ac}m_{cd}m_{db}-m_{ab}\nu^2)B(\nu^2)\,. \tag{4.86}$$

The evaluation of this amplitude parallels earlier scalar vertex calculations. This amplitude provides the counterterms

$$Dc_0=2\sqrt{3}\,g(a_s-a_\phi-4d_2+\tfrac{4}{3}d_3)B(\nu^2) \tag{4.87}$$

and

$$Dc_8=\frac{2\sqrt{2}}{7}Dc_0\,. \tag{4.88}$$

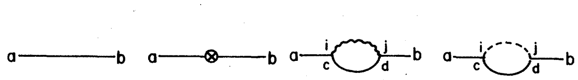

FIG. 10. One-loop-approximation diagrams for the baryon two-point function.

FIG. 11. Diagrams for the vacuum expectation value of the scalar field to second order.

E. Current-field amplitude

The current-field loop diagrams of Fig. 12 contain a potential divergence not removed by the above S-matrix program. This amplitude contains the integral

$$R^\mu(p,x^2,y^2)=i\int\frac{d^4l}{(2\pi)^4}\frac{(2l-p)^\mu}{(l^2-x^2)[(l-p)^2-y^2]} \tag{4.89}$$

for the meson loop.

Formally this integral is linearly divergent; however, explicit evaluation gives a finite result. The only manifestation of the formal divergence is a surface term that contributes to the finite result. This term cannot be retained as it violates the Ward-Takahashi identities. A regularization procedure is used to remove the surface term.[10]

The baryon loop integral in this amplitude is logarithmically divergent; however, the divergent part contains the factor

$$(m_{ab}-m_{ba})\delta_{cd}+(m_{cd}-m_{dc})\delta_{ab}\,.$$

This antisymmetric factor renders the amplitude finite when combining with the symmetric vertex tensors.

At this stage all amplitudes are finite to second order. As indicated at the outset, the ξ_i do not acquire divergent second-order parts.

V. HIGHER-ORDER RENORMALIZATION

It is clear from Sec. IV that the bilinear meson and baryon symmetry-breaking terms severely complicate the renormalization procedure. The question of the implications of these terms in higher-order calculations then arises. In this section we outline a nonrigorous proof that the

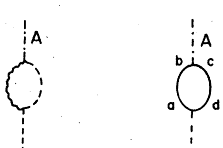

FIG. 12. Two diagrams for the axial-vector-current–pseudoscalar-field amplitude. The dot-dashed line represents the axial-vector current.

theory is renormalizable to all orders. We assume that the symmetric theory is fully renormalizable. The divergent parts of integrals will be isolated using the propagator expansions of Eqs. (4.5) and (4.6), as in the one-loop approximation.

The superficial divergence D of any proper diagram in this model is given by

$$D=4-E-V_3+I_B-V_B\,, \tag{5.1}$$

where E, V_3, I_B, and V_B are the number of external lines, three-point meson vertices, internal baryon lines, and three-point meson-baryon vertices, respectively. Consequently, all vertices with $E>4$ are superficially covergent.

For meson vertices the maximum divergence is then

$$D=4-E\,. \tag{5.2}$$

Thus the four-point proper meson vertex is logarithmically divergent. This divergence can be removed by the counterterms of the symmetric Lagrangian.

The three-point proper meson vertex contains a logarithmic divergence that is linear in the baryon mass. Circling the baryon loops in both directions will then cancel the antisymmetric part of the baryon mass. Consequently, the divergence can also be removed by the counterterms of the symmetric theory.

The meson two-point function is quadratically divergent. However, all quadratic subgraph divergences in any order for both meson and baryon loops correspond to lower-order mass renormalization and thus do not present a problem. Consequently, we need only consider the overall quadratic divergence. The quadratic divergence itself can be removed employing the counterterms of the symmetric theory. The logarithmic divergence contains meson and baryon mass squared terms. The symmetric part of this divergence can again be removed as in the symmetric theory. The remaining symmetry-breaking part corresponds to a nonsymmetric tensor (as in the one-loop case) contracted with a symmetric tensor. The result will have the same form as the one-loop case. Consequently, only the five bilinear meson symmetry-breaking terms of the present model will be needed. The Lagrangian symmetry-breaking parameters will acquire divergent counterterms.

The meson one-point proper vertex is quadratically divergent with cubic baryon mass terms and quadratic meson mass terms. With the $I=Y=0$ symmetry-breaking operators in the model, only the E_i with $i=0,8$ will be nonvanishing. These can contain divergences not present in the symmetric model, but they can be removed employing counterterms available from the linear meson symmetry-breaking terms.

The vertices with external baryon lines have a maximum divergence of

$$D=3-E\,. \tag{5.3}$$

Thus all vertices with external baryon lines and $E>3$ are superficially convergent. The three-point proper meson-baryon vertex is logarithmically divergent. Consequently, it can be renormalized using the counterterms of the symmetric theory.

The baryon two-point function contains a logarithmic divergence linear in the baryon masses. The subgraph divergences can be treated as in the meson-two-point function. We need only consider the overall logarithmic divergence. The symmetric part presents no problem. The symmetry-breaking part can be renormalized using the counterterms from the baryon bilinear symmetry-breaking term.

Consequently, assuming the symmetric theory is renormalizable, this model will also be renormalizable to all orders of perturbation theory.

VI. SUMMARY

We have considered a linear SU(3) σ model incorporating both meson and baryon fields with spontaneous and explicit symmetry breaking. The chiral-symmetric Lagrangian contains the most general renormalizable nonderivative couplings. The symmetry-breaking Lagrangian contains bilinear baryon terms to describe the N-Ξ mass difference, and linear and bilinear meson terms.

The bilinear meson symmetry-breaking terms are required to construct a renormalizable theory, but are expected to have a small numerical effect so that the meson sector reproduces the success of the linear $(3,3^*)\oplus(3^*,3)$ symmetry-breaking meson model. These successes include a good description of the meson mass spectrum and an approximate chiral $SU(2)\times SU(2)$ Lagrangian symmetry.

The model is demonstrated explicitly to be renormalizable in the one-loop approximation and the counterterms are evaluated. The model is expected to be renormalizable to all orders.

ACKNOWLEDGMENTS

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