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The pion and an improved static bag model

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Quark-model calculations involve an extended static object localized in space. We introduce new methods, involving momentum-space wave packets, which account for this localization. These methods have little effect on heavy states, whose sizes are large compared to their Compton size 1/m, but are very important for light particles such as the pion. In this treatment the pion's mass is naturally very small, and, in order to connect with a spontaneously broken chiral symmetry, we require that m_π vanish when the light quarks are massless. Expanding about this limit (and also readjusting the fit to other hadrons), we obtain $m_q=(m_u+m_d)/2=33$ MeV. We calculate $F_\pi \approx 145$ MeV (using a normalization such that $F_\pi|_{\rm exp}=93$ MeV), $F_K/F_\pi \approx 1$, and various corrections to static properties of baryons. In addition we explore the relationship of our methods with chiral perturbation theory, deriving the formula $m_\pi{}^2=(m_u+m_d)\langle\pi(p)|\bar{q}(0)q(0)\,|\pi(p)\rangle$ in the appropriate approximation and commenting on the quark mass obtained from the nucleon's σ term. Finally we discuss the bag model's use of the scalar density $\bar{q}q$ as an order parameter describing the separation of the spontaneously broken vacuum phase from the perturbative vacuum of the bag's interior.

I. INTRODUCTION

The static bag model applied to hadrons made up of low-mass quarks has had a considerable phenomenological success in its reproduction of hadron masses and other parameters in terms of a few fundamental constants.[1-4] The most notable exception to this success is the π meson. It is the purpose of this paper to develop a simple improvement to the static approximation which will play a relatively minor role for the more massive states, but will be of crucial importance for the pion. In particular, it will enable us to show that it may be possible to resolve the apparent dichotomy between the quark-model pion and the PCAC (partial conservation of axial-vector current) pion.[5]

The bag model is a formulation which incorporates in a local and relativistic framework the following features which have been abstracted from the observed properties of hadrons:

(1) Hadrons are composed of quarks which move relatively independently within a single hadron. Consistent with this is the following:

(2) The effective interaction between the quarks at short distances is governed by quantum chromodynamics (QCD) *treated perturbatively*.

These features are included in a model which allows the hadrons to consist only of color-singlet combinations of quarks.

In this model it is hypothesized that the region of space within a hadron is the usual perturbation-theory vacuum where quark interactions are relatively weak. The true (and complex) vacuum outside of hadrons is unspecified except that it has an energy per unit volume B lower than that inside. For light quarks the location of the boundary between these phases is governed by the scalar density $\bar{q}q$, with the bag action derivable from the Lagrangian[6]

$$\mathcal{L}=(\mathcal{L}_{\rm QCD}-B)\theta(\bar{q}q)\,. \tag{1}$$

As a further approximation the static bag model assumes that the region of space which contains the quarks is a fixed spherical cavity. In this paper we shall develop a method to correct for the effects of "nailing" the quark wave functions to a fixed origin in space. The corrections are of interest by themselves but as we stated above, they will be of particular importance for the pion.

The quark-model description of the pion is that of a quark-antiquark bound state with properties not much different from other hadrons. When spin-dependent forces due to colored-gluon exchange are taken into account perturbatively, the pion emerges naturally as the lightest state.[3] Spin forces provide a rather large interaction energy which subtracts from the unperturbed energy. However, despite this, it is extremely difficult to obtain a pion with $m_\pi=140$ MeV. For example, the bag fit of Ref. 3 quotes a pion mass of 280 MeV.

The pion of the quark model appears superficially quite different from the pion of PCAC.[7,8] With vanishing up- and down-quark masses the fundamental QCD theory has a chiral SU(2)×SU(2) symmetry. This symmetry is assumed to be spontaneously broken, with the pion as the associated massless Goldstone boson. For finite but small quark masses, the pion deviates only slightly from this description, picking up a small mass but retaining the couplings of a collective excitation.

The connection between the PCAC pion and the pion of the quark model remains a persistent question.

However, even within the quark model, the pion has a property which distinguishes it from other hadrons: It is the only particle whose "size" is smaller than its Compton size ($1/m_\pi$). Quark-model techniques have evolved from the bound-state treatments of atomic and nuclear physics. In these latter situations one deals with a heavy system whose spatial extent is much larger than its Compton size. This property is also true for most quark bound states. For example, the proton has a radius of about 1 fm while its Compton size is $\frac{1}{5}$ fm. However, the pion's Compton size is 1.4 fm and estimates of its radius are near $\frac{2}{3}$ fm. In this paper we use this as a basis for a reconsideration of the pion in the quark model.

This size distinction requires that the naive static-bag techniques be modified in order to properly treat the pion. The type of correction we have in mind is known generally as a "center-of-mass" correction, but it has some novel aspects in a relativistic theory. As expected, the new methods produce very little change in the properties of heavy states. However, the pion emerges lighter than before. In particular, it is possible to make the pion massless. We will do this in the limit of vanishing quark mass in an attempt to establish a connection with the pion of PCAC. Some interesting results emerge. We study the pion mass for small quark masses, and make a connection with chiral perturbation theory. The pion decay constant F_π is calculated, and our treatment is such that it remains finite in the limit $m_\pi \to 0$, as is required by the chiral-symmetric theory. While we are far from uniting the two treatments, our work indicates that they are not as disjoint as previously thought.

In Sec. II we present the new tools and illustrate them by a calculation of F_π. Section III is devoted to applying our new methods to compute corrections to the pion's energy and a discussion of the pion mass. The methods developed require some less significant changes in the other hadron states also, so in Sec. IV we redo the fit to hadron masses. We show the relationship between our description and chiral perturbation theory in Sec. V. Concluding remarks are made in Sec. VI.

II. BAG WAVE PACKETS, THE PION DECAY CONSTANT, AND OTHER CORRECTIONS TO THE NAIVE STATIC-BAG CALCULATIONS

In this section we use the calculation of F_π and some corrections to the static-bag approximation for nucleons to demonstrate the construction of a bag wave packet. The crucial observation is that the static bag state (which we will denote simply by use of the particle symbol with the subscript B, i.e., $|\pi\rangle_B$) is not a momentum eigenstate [in usual notation $|\pi(p)\rangle$]. However, we shall show that it can be related to a superposition of such states, or wave packet. We define, for a covariantly normalized momentum eigenstate $\langle\pi(p)|\pi(p')\rangle = (2\pi)^3\delta^{(3)}(p-p')2\omega_p$,

$$|\pi\rangle_B = \int d^4p\,\phi(p)e^{ip\cdot x}\delta(p^2+m^2)\theta(p^0)\,|\pi(p)\rangle$$
$$= \int \frac{d^3p}{2\omega_p}\,\phi(p)e^{ip\cdot x}|\pi(p)\rangle\,. \qquad (2)$$

Here, $\phi(p)$ is a wave packet which describes the localization of the particle around the position x. We shall assume that it is possible to choose an appropriate wave packet ϕ so that the momentum spread approximates that of the total momentum of the quarks in a static bag. We shall describe how this wave packet is determined below. For baryons, a more convenient convention is

$$|B,M\rangle_B = 2m\int d^4p \sum_\lambda \chi_\lambda^M(p)\delta(p^2+m^2)\theta(p^0) \times e^{ip\cdot x}|B(p),\lambda\rangle$$
$$= \int d^3p\,\frac{m}{E_p}\sum_\lambda \chi_\lambda^M(p)e^{ip\cdot x}|B(p),\lambda\rangle\,, \qquad (3)$$

where here we have the usual

$$\langle B(p),\lambda|B(p'),\lambda'\rangle = (2\pi)^3\delta^{(3)}(p-p')(E/m)\delta_{\lambda\lambda'}\,.$$

The bag states are normalized in the conventional way

$$_B\langle\pi|\pi\rangle_B = 1\,,$$
$$_B\langle B,M|B,M'\rangle_B = \delta_{MM'}\,, \qquad (4)$$

which corresponds to the wave-packet normalizations

$$\int\frac{d^3p}{2\omega_p}(2\pi)^3|\phi(p)|^2 = 1\,, \qquad (5a)$$

$$\int d^3p\,\frac{m}{E_p}(2\pi)^3\sum_\lambda \chi_\lambda^M(p)^*\chi_\lambda^{M'}(p) = \delta_{MM'}\,. \qquad (5b)$$

For simplicity in this paper we shall assume that it is possible to take as the wave packet for a spin-$\frac{1}{2}$ baryon, $\chi_\lambda^M(p)$, a form

$$\sum_\lambda u_\lambda(p)\chi_\lambda^M(p) = u^M(p)\chi(p)\,, \qquad (6)$$

where $\chi(p)$ is a scalar function, and $u^M(p)$ is the covariantly normalized Dirac spinor with the spin quantized along the same spatial axis as that of the fixed cavity state. With this convention Eq. (5b) becomes

$$\int d^3p\,\frac{m}{E_p}|\chi(p)|^2(2\pi)^3 = 1\,. \qquad (7)$$

For particles whose extension is larger than their

Compton size, the momentum spread of the quarks in the static bag is small in comparison with the mass. The reverse holds for the pion.

For heavy states this leads to the usual static-bag-model methods for calculating matrix elements. We can illustrate this by considering the axial-vector coupling constant $g_A = g_A(0)$, defined by

$$\langle \lambda', p(p') | \bar{u}(x)\gamma^\mu\gamma_5 d(x) | \lambda, n(p)\rangle = \bar{u}_{\lambda'}(p')\gamma^\mu\gamma_5 u_\lambda(p) g_A((p-p')^2) e^{ix\cdot(p-p')} + \cdots . \tag{8}$$

Transforming to a bag located at $X=0$,

$$_B\langle M', p | \bar{u}(x)\gamma^\mu\gamma_5 d(x) | M, n\rangle_B = \int d^3p\, d^3p' \frac{m^2}{E_p E_{p'}} e^{ix\cdot(p-p')} \bar{u}_{M'}\gamma^\mu\gamma_5 u_M \chi(p)\chi^*(p') g_A((p-p')^2) + \cdots \tag{9}$$

or

$$_B\left\langle M', p \right| \int d^3x\, \bar{u}\gamma^\mu\gamma_5 d \left| M, n\right\rangle_B = \int d^3p \left(\frac{m}{E_p}\right)^2 (2\pi)^3 |\chi(p)|^2 g_A(0) \bar{u}_{M'}(p)\gamma^\mu\gamma_5 u_M(p) + \cdots . \tag{10}$$

Now let us assume that the wave-packet spread is small in comparison to the nucleon mass, $\langle p^2\rangle \ll M^2$. Then to first order, if we expand the right-hand side of Eq. (10) we find

$$g_A^{\text{static bag}} = g_A\left(1 - \frac{1}{3}\frac{\langle p^2\rangle}{m^2}\right), \tag{11}$$

where

$$\langle p^2\rangle = \int d^3p \frac{m}{E_p} (2\pi)^3 |\chi(p)|^2 p^2 \tag{12}$$

is the mean momentum spread in the wave packet. In the limit of the naive static-bag model, we recover the standard result $(g_A) = (g_A^{\text{static bag}})$. The first correction is

$$g_A = g_A^{\text{static bag}}\left(1 + \frac{1}{3}\frac{\langle p^2\rangle}{m^2}\right). \tag{13}$$

In the case of vector density, there is no correction to the static-bag result for the total charge. Clearly this is necessary for the consistency of our method. When we go beyond the integrated quantitiy to obtain, for example, the mean square radius or magnetic moment, we obtain $\langle p^2\rangle / m^2$ corrections. For example,

$$\mu = \mu^{\text{static bag}}\left(1 + \frac{1}{2}\frac{\langle p^2\rangle}{m^2}\right). \tag{14}$$

The charge radius is more subtle and is treated in the Appendix, yielding

$$\langle r^2\rangle = \langle r^2\rangle_{\text{bag}}^{\text{static}}\left(1 + \frac{1}{3}\frac{\langle p^2\rangle}{m^2}\right), \tag{15}$$

where

$$\langle r^2\rangle_{\text{bag}}^{\text{static}} = \int d^3r\, r^2 \rho_{\text{bag}}^{\text{static}}(r). \tag{16}$$

Clearly, all of these corrections imply a commitment to a specific wave packet, and we shall now turn to our proposal for that by studying the meson state, in particular the pion.

For the pion, we shall begin with the vacuum-to-π matrix element, that is, a calculation of F_π, defined by

$$\langle 0 | \bar{u}(x)\gamma^\mu\gamma_5 d(x) | \pi(p)\rangle = i\sqrt{2}\, F_\pi p^\mu e^{ip\cdot x}. \tag{17}$$

Taking the time component and transforming to the bag state

$$\langle 0 | \bar{u}(x)\gamma^0\gamma_5 d(x) | \pi\rangle_B = i\frac{F_\pi}{\sqrt{2}} \int d^3p\, e^{ip\cdot x} \phi(p). \tag{18}$$

Using the static-bag-model wave functions, the left-hand side is given by

$$\langle 0 | \bar{u}(x)\gamma^0\gamma_5 d(x) | \pi\rangle_B = i\sqrt{6}[u^2(x) - l^2(x)], \tag{19}$$

where u (l) is the upper (lower) component in the static-bag wave function. With this identification we can solve for the bag wave packet $\phi(p)$. This is our crucial step. We identify the wave packet which describes the total momentum spread of the quarks in the static bag with that which comes from the static-bag wave functions applied to the total annihilation of the particles. Thus, we find

$$\phi(p) = \frac{2\sqrt{3}}{F_\pi} \int \frac{d^3x}{(2\pi)^3} e^{-ix\cdot p}(u^2(x) - l^2(x)). \tag{20}$$

F_π is then determined by the normalization condition Eq. (5a). With $m_\pi = 0$, this results in

$$F_\pi = 0.501/R_\pi. \tag{21}$$

An interesting feature of this calculation is that F_π has a finite limit as $m_\pi \to 0$, as long as the pion radius R_π remains finite. This is as would be hoped for in theories where m_π is small, and indeed is necessary for our calculation to be consistent with chiral perturbation theory. In later sections we will determine the pion's radius to be in the range $R_\pi = 3.3$–$3.5\ \text{GeV}^{-1}$, which corresponds to a value

$$F_\pi = 140\text{–}150\ \text{MeV} \tag{22}$$

to be compared with the experimental $F_\pi = 93$ MeV.

There is one consideration that we have neglected above which would lower the value of F_π. We have calculated the amplitude for the removal of two quarks from a bag. This, however, leaves an "empty bag," not the true vacuum. Thus, there is an extra factor in Eq. (19) which describes the overlap of the inside, perturbative vacuum of the bag with the true vacuum. One might anticipate, because of the spatial homogeneity of the vacuum state, that this overlap would take the form $\exp(-B^{3/4}V_\pi X)$, with V_π the volume of the pion bag, and X a dimensionless parameter. However, without a theory of the true QCD vacuum we are unable at present to calculate the overlap. Since $B^{3/4}V_\pi \approx 0.4$ is not large, one might hope that this factor is not the dominating one. We can also compute F_K, where if $V_K \sim V_\pi$, the vacuum overlap should be the same. That is, the ratio F_K/F_π should be insensitive to this. We shall return to this calculation below.

The wave-packet amplitude $\phi(p)$ will play a considerable role in subsequent discussions. It has the desirable feature that it is insensitive to the bag boundary surface, since (u^2-l^2) vanishes there. Alternative choices of the wave packet (for example, that gotten by considering the annihilation amplitude associated with the operator $\bar{u}\gamma_5 d$) often have a discontinuity at $r=R$, which produce very-high-momentum components in $\phi(p)$. We are using in $\phi(p)$ the cavity wave functions for the state with no gluons. The gluons introduced perturbatively produce a sizable energy shift in the pion, and could produce wave-function corrections also. However, the boundary conditions at the surface of the bag are independent of the coupling. Since the $\phi(p)$ is a smooth function whose main property is that associated with its scale, we believe that the properties of $\phi(p)$ derived above are quite reasonable, and we will use Eq. (20) throughout the paper.

We can at this time apply the same formulation to the kaon, by using strange-particle wave functions for one of the quarks in $\phi(p)$. The same procedure with $m_K=0.5$ GeV and $m_s \simeq 0.3$ GeV yields

$$F_K/F_\pi = 1.01\, R_\pi/R_K\,. \tag{23}$$

Since $R_K \approx R_\pi$ this produces $F_K \approx F_\pi$. As we have remarked already, we believe that this result is more accurate than our absolute calculation of either F_π or F_K since the "empty-bag," true vacuum amplitude should cancel in the ratio F_K/F_π. We recall that the experimental ratio $F_K/F_\pi \simeq 1.20$.

III. MASS OF THE PION

The localization of the quarks described above also produces a modification in the calculation of masses in the quark model. In momentum space we have the definition (using the proton to be specific)

$$\langle P(p)|H|P(p)\rangle = E\,, \tag{24}$$

where $E=(p^2+m^2)^{1/2}$, so that for a proton at rest,

$$\langle P(0)|H|P(0)\rangle = m\,. \tag{25}$$

However, when we convert to bag states,

$$E_{\rm bag} = {}_B\langle P|H|P\rangle_B = \langle E\rangle\,, \tag{26}$$

where $\langle E\rangle$ is the average of $(p^2+m^2)^{1/2}$ using the appropriate wave packet. For heavy states, with $R \gg 1/m$, $\langle E\rangle \approx m$, in zeroth approximation yielding the usual equality of bag energy and mass

$$E_{\rm bag} = m\,. \tag{27}$$

In fact, even the first correction to this has been accounted for in the earlier static-bag calculations.[3] Expanding the energy, one obtains

$$\langle E\rangle = m + \frac{1}{2m}\langle P^2\rangle\,. \tag{28}$$

Thus, in a state with independent particles, $\langle p^2\rangle \propto n/R^2$, where n is the number of quarks. Since the zeroth approximation to the mass in the bag model is $m=\frac{4}{3}n(2.04/R)$ (with massless quarks) there is therefore an approximate relation

$$m = E_{\rm bag} - C/R \tag{29}$$

with C independent of n.

However, this correction has the same form as the "zero-point energy" term $(-Z_0/R)$ which was included in the hadron masses calculated in Ref. 3. We now observe that part of Z_0 is accounted for by the momentum fluctuation effect. The phenomenologically determined value of Z_0 is about 1.8. We estimate that C would be 0.6–0.8. Hence we still assume that $E_{\rm bag}$ contains a term $-Z_0/R$, but where we shall now expect $Z_0 \sim 1$.

The static bag states have $\langle \vec{p}\rangle = 0$, but $\langle p^2\rangle \neq 0$. This correction to the energy can therefore also be described as a "center-of-mass" correction since it removes the effect of nonzero $\langle p^2\rangle$ from the static-bag calculation. For light states such as the pion the "fluctuation" momentum $(\langle p^2\rangle)^{1/2}$ is not small in comparison to the mass, so equating the mass and the bag energy is clearly not correct. Rather, to compute the mass one should use

$$E_{\rm bag} = \langle (p^2+m^2)^{1/2}\rangle\,. \tag{30}$$

Even massless states have $\langle (p^2)^{1/2}\rangle \neq 0$ so that the bag energy need not vanish. The important general feature of Eq. (30) is that, for a given $E_{\rm bag}$, m is smaller than would be obtained from $E_{\rm bag}=m$, since some of the energy goes into localizing the quarks in a bag at a fixed point in space. This makes a low-energy bag state correspond to an even-lower-mass particle; a light pion becomes even lighter.

When using a wave packet to estimate the contribution to the bag's energy of the total momentum of the localized quarks, several alternatives can be considered. A close parallel between our treatment and chiral perturbation theory exists if we base our calculation on a fixed wave packet with zero particle mass in the normalization condition and zero quark mass in the wave function (see Sec. V). The discussion in the remainder of this section is also based on such a wave packet. An alternate proposal, which does not lead to such a close parallel, is to use a wave packet whose form is self-consistently fitted to the mass of the particle and the masses of the quarks. This is the method used in Sec. IV to determine the light quark mass as part of the overall fit to the hadronic spectrum. It turns out that the numerical values of the quark mass obtained in Sec. IV and Sec. V do not differ greatly. This would not be the case if the pion mass were much smaller than $1/R$. We have not made comparisons of which of these proposals gives the more accurate estimation of the center-of-mass correction by using exact models. When making a variational treatment of the mass spectrum based upon the bag's size, the proposal of a fixed wave packet, with both quark masses and particle masses set equal to zero, leads to the simplest discussion, which is why we adopt it in the remainder of this section and in Sec. V.

Before proceeding further we should discuss the spin-dependent force due to gluons since it has an important role in making the pion a low-mass particle. By considering single-gluon exchange between quarks there is a contribution to the pion energy of the form

$$E_{\text{gluon}} = -\frac{\alpha_s(R)}{R} G_0 , \tag{31}$$

where G_0 is a pure number ($G_0 = 0.7$) which comes from an integration over the quark wave functions.[3] We have written the coupling constant as a function of R to indicate the dependence of the effective coupling on the scale size in the problem. With massless quarks the mass scale is set by a/R (with $a \sim 2$), which provides a low-momentum cutoff on the gluon propagator. Combined with the idea of asymptotic freedom this suggests that the effective $\alpha_s(R)$ should get weaker at small R and stronger at large R. The specific form of $\alpha_s(R)$ will not be needed until Sec. IV, but the general variation of $\alpha_s(R)$ is important in what follows.

The precise predictions of the previous bag model fit[3] do not apply in the new framework, since we must now adjust for the center-of-mass effect and the running coupling constant. However, when one makes such adjustments the pion mass calculated from $\langle (p^2+m_\pi^2)^{1/2}\rangle = E_{\text{bag}}$ naturally falls close to zero. Therefore, the opportunity arises to put in by hand a connection with the PCAC pion associated with a spontaneously broken chiral symmetry. It is not difficult to constrain the bag model to yield $m_\pi = 0$ when the quark masses vanish. To do this, expand

$$(p^2+m_\pi^2)^{1/2} = p + \frac{m_\pi^2}{2p} \tag{32}$$

and evaluate $\langle p\rangle$ and $\langle 1/2p\rangle$ with the wave packet $\phi(p)$ determined with $m_\pi = 0$ and $m_q = 0$. This yields

$$\langle p\rangle = A/R , \tag{33}$$

$$\langle 1/2p\rangle = R/C , \tag{34}$$

with

$$A = \tfrac{1}{2}R \int d^3p [\phi(p)]^2 (2\pi)^3 = 2.3 \tag{35}$$

and

$$C = R\left\{ \int d^3p [\phi(p)]^2 (2\pi)^3 \left(\frac{1}{2\omega_p}\right)^2 \right\}^{-1} = 2.9 . \tag{36}$$

The bag pion's mass is then given by the following function of the radius:

$$m_\pi^2 = \frac{C}{R}(E_{\text{bag}} - A/R) . \tag{37}$$

Here E_{bag} contains

(1) the quark kinetic energy

$$E_{\text{kin}} = 2x/R \tag{38}$$

($x = 2.04\ldots$ for massless quarks);

(2) the gluon-exchange energy

$$E_{\text{gluon}} = -\frac{\alpha_s(R)}{R} G_0 , \tag{39}$$

where $G_0 \simeq 0.7$ for massless quarks;

(3) the volume and "zero-point" energy,

$$E_{\text{vac}} = \frac{4\pi}{3}BR^3 - Z_0/R . \tag{40}$$

The usual procedure is to minimize the energy as a function of R to find the actual state. Here we shall minimize the mass,

$$\left.\frac{dm_\pi^2}{dR}\right|_{R=R_\pi} = 0 . \tag{41a}$$

We shall also demand

$$m_\pi^2 \big|_{R=R_\pi} = 0 , \tag{41b}$$

which may be regarded as a constraint on the parameter "Z_0." We note first that if α_s were a constant, no solution to both (41a) and (41b) would exist with $R_\pi > 0$. Without the constraint (41b), a solution in general would exist with Z_0 less than some critical value, but at the minimum we would find $m_\pi^2 > 0$. If we try to enforce (b), we should find that Z_0 would be at the critical value, and there $R_\pi = 0$. A zero radius pion is not satisfactory because consistency with chiral perturbation theory requires that F_π remain finite when $m_\pi=0$, $m_q=0$. Since $F_\pi \sim 1/R_\pi$, this means $R_\pi \neq 0$. If we try to enforce both (a) and (b), it is required that $\partial\alpha_s/\partial R > 0$, i.e., asymptotic freedom is necessary for our treatment of the pion to be possible. The enforcement of a condition on Z_0 to make the pion massless is not a "natural" condition in the context of the bag model; nevertheless, we believe that it could be here where the bag model as a phenomenological version of QCD with a spontaneously broken chiral symmetry must be constrained. The total volume and "zero-point" term in $E_{\rm bag}$ refers to the energy of a bubble of perturbative vacuum embedded in the true vacuum. If these states differ by a spontaneous breakdown of chiral symmetry, then it would be natural that Z_0 would be such that the pion should be massless. Since we do not have a microscopic theory of this spontaneous symmetry breakdown, it is necessary for us to enforce this requirement by hand. Equations (41a) and (41b) translate into two relations, one of which determines R_π in terms of B, G_0, x, and Z_0, and the other is the simple relation

$$G_0 R \frac{d}{dR}\alpha_s(R)\bigg|_{R=R_\pi} = \frac{16\pi}{3} B R_\pi^{\ 4}. \tag{42}$$

This equation clearly expresses the fact noted above that without asymptotic freedom our treatment of the pion would not be possible, since $m_\pi = 0$ would also require $R_\pi = 0$. The fact that both sides of Eq. (42) are governed by the scale parameter of the strong interaction means that such a relationship is not absurd. From a phenomenological point of view we shall regard Eq. (42) as an equation for R_π if we are given B and a form of α_s. It turns out that the forms of α used in Sec. IV all yield $R_\pi = 3.3-3.5\ \mathrm{GeV}^{-1}$.

The physical pion mass is not zero, but $m_\pi = 0.14$ GeV. This can be accommodated by giving the quarks a mass. The pion mass can be solved for by calculating

$$\langle (p^2+m_\pi^{\ 2})^{1/2}\rangle = E_{\rm bag}(m_q)\,, \tag{43}$$

where the left-hand side is obtained with a wave packet that is independent of both m_π and m_q. At each value of the radius, for fixed m_q, this equality determines a value of m_π. For small m_q we can expand $E_{\rm bag}(m_q)$ as

$$\begin{aligned} E_{\rm bag}(m_q) &= E_{\rm bag}(0) + m_q \frac{\partial}{\partial m_q} E_{\rm kin} + m_q \frac{\partial}{\partial m_q} E_{\rm gluon} \\ &= E_{\rm bag}(0) + \frac{m_q}{x_0 - 1} + 0.23 m_q R E_{\rm gluon}\,, \end{aligned} \tag{44}$$

where the dependence of the quark's kinetic energy,

$$\frac{x}{R} = \frac{x_0}{R} + \frac{m_q}{2(x_0-1)} + \cdots\,, \quad x_0 = 2.04\ldots \tag{45}$$

and Fig. 3 of Ref. 3 have been used.

The most important contribution is the change in kinetic energy, and this lessens the sensitivity to the form of $\alpha_s(R)$. The fits of Sec. IV all give the observed pion mass for a quark mass

$$m_q = 33 \pm 2\ \mathrm{MeV}\,, \tag{46}$$

where the ± 2 indicates the range of possible values.

IV. FIT

The new methods which we have introduced require a redetermination of the phenomenological quark parameters. The purpose of this section is to provide such an evaluation. With the exception of Z_0, the bag parameters determined below are quite similar to those of a previous fit in Ref. 3. As mentioned before, Z_0 changes because some portion of the phenomenologically determined Z_0 is now accounted for by the center-of-mass correction.

We do not have the wave packets $\phi(p)$ or $\chi(p)$ for all the hadrons. Here we will employ the same form of $\phi(p)$ for all particles, and use Eq. (20), and the mass-dependent normalization of Eq. (5a). We do this for simplicity and because this form has several attractive features described above. In addition, we feel that the precise form of $\phi(p)$ should be only important for the pion.[9]

The contributions to the bag energy are discussed in detail in Ref. 3. The only change which we make is the introduction of a running effective coupling $\alpha_s(R)$ in place of the fixed one. We of course do not know the precise form of $\alpha_s(R)$. When R is small, one would expect that $\alpha_s(R)$ would change logarithmically as determined by the renormalization group. However, a naive identification of the lowest-order QCD result with the bag coupling, i.e.,

$$\alpha_s(R) = \frac{2\pi}{9\ln(R_0/R)} \tag{47}$$

for three flavors (R_0 is the scale parameter analogous to Λ in momentum space), has an unfortunate consequence. This formula diverges at $R=R_0$. There are some states, such as the nucleon and the pion, which receive a negative contribution to their energy of the form $-\alpha_s(R)M_0/R$. The mass of these particles exhibits an instability as one approaches R_0, becoming arbitrarily small or negative. This clearly is unphysical, except that it in some way corresponds to the instability of the perturbation-theory vacuum. Since this instability in α corresponds to low-momentum fluctuations,

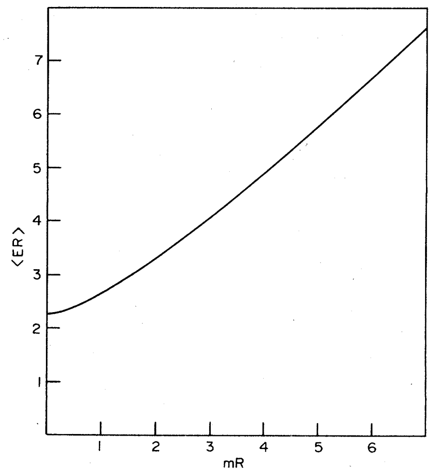

FIG. 1. The average value of $\langle ER\rangle$ vs mR using the wave function $\phi(p)$.

"bag" effects will make $\alpha(R)$ finite except as $R\to\infty$. For the pion one could ignore the problem because R_π remains safely away from R_0. However, the proton has a larger radius, and for this form of α_s it is not possible to both fit the N-Δ mass difference and retain a stable nucleon in lowest order.

To overcome this, we introduce a set of forms of $\alpha_s(R)$ which do not suffer from this problem. We require that they yield Eq. (47) at small R, but be smoother at large R. This can be obtained by setting

$$\alpha_s(R)=\frac{2\pi}{9\ln(A+R_0/R)}\,. \tag{48}$$

For $A\geq 1$ there is no singularity. For $A\geq 0.3$ there remains a divergence in $\alpha_s(R)$, but it is moved sufficiently far away from the region of interest that no problems arise. We will use $0.3\leq A\leq 1$ to obtain a feel for the dependence of our results on the form of $\alpha_s(R)$.

We now proceed by setting

$$\langle E\rangle=\langle(m^2+p^2)^{1/2}\rangle=E_{\text{bag}}\,. \tag{49}$$

TABLE I. The bag parameters obtained for various forms of $\alpha_s(R)=2\pi/9\ln(A+R_0/R)$.

A	R_0	Z_0	$B^{1/4}$	R_N	R_Δ
1	2.13	1.01	0.135	5.47	5.62
0.9	2.68	1.10	0.137	5.40	5.43
0.8	3.22	1.16	0.139	5.35	5.28
0.7	3.74	1.22	0.141	5.33	5.16
0.6	4.24	1.26	0.142	5.31	5.05
0.5	4.74	1.30	0.143	5.31	4.59
0.4	5.23	1.34	0.145	5.32	4.87
0.3	5.71	1.36	0.146	5.35	4.79

TABLE II. A comparison of the bag parameters of the previous fit (Ref. 3) with those obtained here (using $A=1$).

	Ref. 3	This paper ($A=1$)
$B^{1/4}$	0.145 GeV	0.135 GeV
Z_0	1.84	1.01
$\alpha_s(R_N)$	2.2	2.1
R_N	5 GeV^{-1}	5.5 GeV^{-1}
R_Δ	5.5 GeV^{-1}	5.6 GeV^{-1}
$\alpha_s(R_\pi)$	2.2	1.5
R_π	3.3 GeV^{-1}	3.5 GeV^{-1}

The values of the dimensionless quantity $\langle E\rangle R$ as a function of mR are given in Fig. 1. For large mR, $\langle E\rangle R\to mR$, while for $m\to 0$, $\langle E\rangle R\to 2.3$. The equality $\langle E\rangle=E_{\text{bag}}$ then determines $m(R)$ whose minimum value fixes the particle mass.

The basic input to the fit will be m_N, m_Δ, and the requirement that m_π vanish when $m_q=0$. The dependence of the parameters on the input is of course coupled. However, to a large extent m_N+m_Δ determines the bag constant B, $m_\Delta-m_N$ determines α_s (or in this case R_0), and the condition on m_π fixes Z_0 (and predicts R_π). The results of such a procedure are given in Table I with a quark mass of 33 MeV. That mass was chosen by requiring that the pion be reproduced. Table II gives a brief comparison of the parameters from Ref. 3 with those for $A=1$. The gluon coupling constant is somewhat weaker, and radii are slightly changed, but otherwise the results are rather similar.

The ρ meson differs from the pion in the sign and magnitude of the gluon-induced spin-spin interaction. It it were not for this, the ρ would be degenerate with the pion as in the SU(6) models. There are no extra parameters to be determined, and the mass of the ρ is a prediction of the model. It is listed in Table III. We see that gluon exchange does provide a large splitting of the ρ and pion, placing the ρ close to its experimental mass, although a little low. It is not surprising that the

TABLE III. The mass and radii of the ρ meson predicted from our fit, for various values of A.

A	R_π (GeV^{-1})	m_ρ (MeV)	R_ρ (GeV^{-1})
1	3.5	704	4.4
0.9	3.5	682	4.2
0.8	3.5	663	4.0
0.7	3.5	647	3.8
0.6	3.4	634	3.7
0.5	3.4	620	3.6
0.4	3.3	607	3.5
0.3	3.3	597	3.4

ρ mass should be sensitive to the form of $\alpha_s(R)$ since that is the parameter which most directly governs the ρ-π splitting. The results favor $A=1$.

Particles containing strange quarks may be accommodated by determining the strange-quark mass. For strange baryons, $m_s = 0.28$ GeV (the same as in Ref. 3) reproduces the $\Delta S = 1$ mass differences quite well. This value then predicts a kaon mass $m_K = 0.44$ GeV $(A=0.3)$–0.46 GeV $(A=1)$, which is quite reasonable. Alternatively one could use the kaon mass to determine m_s, yielding $m_s = 0.33 \pm 0.01$ for the various forms of $\alpha_s(R)$.

The basic static properties of the nucleon with the above parameters are[3,4]

$$g_A^{\text{static bag}} = 1.13\,, \tag{50a}$$

$$2m_p \mu_p^{\text{static bag}} = 2.1\,, \tag{50b}$$

$$\langle r^2\rangle^{\text{static bag}} = (0.77)^2\,\text{fm}^2\,. \tag{50c}$$

The momentum fluctuation inherent in the wave function which we are using corresponds to

$$\frac{\langle p^2\rangle}{m^2} \simeq \frac{10}{(mR)^2} \simeq 0.36\,. \tag{51}$$

The first corrections to the static limit, given in Sec. II, can therefore be computed, with the result

$$g_A = 1.27\,, \tag{52}$$

$$2m_p \mu_p = 2.5\,, \tag{53}$$

$$\langle r^2\rangle = (0.82)^2\,\text{fm}^2\,. \tag{54}$$

The most serious discrepancy in the static properties calculated in Ref. 3 was the low value of the nucleons' gyromagnetic ratio $(2m_p\mu_p = 1.9)$. The combination of a slightly larger proton radius and the fluctuation correction improves this considerably.

V. THE RELATIONSHIP WITH CHIRAL PERTURBATION THEORY

A massless pion is an exact consequence of a spontaneously broken chiral symmetry. In theories such as QCD with small, explicit breaking of chiral symmetry in the form of quark masses, the pion is an approximate Goldstone boson, and a chiral perturbation theory has been developed.[8]

In this scheme, one adds to the chirally invariant Lagrangian the mass terms

$$\Delta\mathcal{L} = m_u \bar{u}u + m_d \bar{d}d\,. \tag{55}$$

The pion mass in lowest-order perturbation theory is

$$\begin{aligned} m_\pi^2 &= m_u \langle \pi | \bar{u}u | \pi\rangle + m_d \langle \pi | \bar{d}d | \pi\rangle \\ &= (m_u + m_d)\langle \pi | \bar{q}q | \pi\rangle\,, \end{aligned} \tag{56}$$

where the matrix elements are evaluated with $m_\pi = 0$. The mass parameters here are those of "current quarks."

Equation (56) also arises in an approximation to our treatment. To make this clear, we shall first establish the connection between the field-theory matrix element $\langle \pi(p) | \bar{q}(x)q(x) | \pi(p')\rangle$ and the bag-model matrix element $\langle \pi | \bar{q}(x)q(x) | \pi\rangle_B$ using the fixed wave packet ϕ with $m_\pi = 0$, and zero quark masses as discussed in Sec. II. This is

$$_B\langle \pi | \bar{q}(x)q(x) | \pi\rangle_B = \int d^3p\, d^3p' \,\frac{\phi^*(p)}{2p}\frac{\phi(p')}{2p'} e^{ix\cdot(p-p')}\langle \pi(p) | \bar{q}(0)q(0) | \pi(p)\rangle\,, \tag{57}$$

so in particular

$$\begin{aligned} {}_B\left\langle \pi \left| \int d^3x\, \bar{q}(x)q(x) \right| \pi \right\rangle_B &= \int d^3p\,(2\pi)^3 \frac{[\phi(p)]^2}{(2p)^2}\langle \pi(p) | \bar{q}(0)q(0) | \pi(p)\rangle \\ &= \langle 1/2p\rangle \langle \pi(p) | \bar{q}(0)q(0) | \pi(p)\rangle\,. \end{aligned} \tag{58}$$

Equation (58) will be basic in what follows. Now we can do perturbation theory in the bag model, again with the assumption that when the quarks are massless $m_\pi = 0$. Then we have (as above)

$$E_{\text{bag}} = \langle (p^2 + m_\pi^2)^{1/2}\rangle = \langle p\rangle + m_\pi^2 \langle 1/2p\rangle\,, \tag{59}$$

so

$$\langle p\rangle + m_\pi^2 \left\langle \frac{1}{2p}\right\rangle = E_{\text{bag}}(0) + m_q \frac{\partial}{\partial m_q} E_{\text{bag}} \tag{60}$$

or

$$m_\pi^2 \left\langle \frac{1}{2p}\right\rangle = m_q \frac{\partial}{\partial m_q} E_{\text{bag}}\,. \tag{61}$$

If we assume that in the bag model we compute $\partial/\partial m(E_{\text{bag}})$ in lowest order from the quark mass term, we obtain

$$\begin{aligned} \frac{\partial E_{\text{bag}}}{\partial m_q} &= \frac{\partial E_{\text{kin}}}{\partial m_q} = 2 \int d^3x\, \bar{q}_0(x) q_0(x) \\ &= 2\, {}_B\left\langle \pi \left| \int d^3x\, \bar{q}(x)q(x) \right| \pi \right\rangle_B\,, \end{aligned} \tag{62}$$

where q_0 is the zero-quark-mass static bag wave function. With the basic relation (58), and Eqs. (61) and (62) we find the same expression for m_π^2, i.e.,

$$m_\pi^2\left\langle\frac{1}{2p}\right\rangle=2m_q{}_B\left\langle\pi\left|\int d^3x\,\bar{q}(x)q(x)\right|\pi\right\rangle_B$$
$$=2m_q\left\langle\frac{1}{2p}\right\rangle\langle\pi(p)|\bar{q}(0)q(0)|\pi(p)\rangle\,. \qquad (63)$$

In particular, numerically using Eq. (4) and Eq. (60),

$$m_\pi^2\frac{R_\pi}{C}=m_\pi^2\frac{R_\pi}{2.9}=\frac{m_q}{x_0-1}, \qquad (64)$$

and for later reference,

$$\langle\pi(p)|\bar{q}(0)q(0)|\pi(p)\rangle=1.4/R_\pi\,. \qquad (65)$$

The quark masses in the bag model have always been light, more like "current" quark masses than those of "constituent" quarks which appear in nonrelativistic treatments. This derivation makes clear the identification of the bag-model quark mass with that of the current quarks. Of course, absolute quark masses only make sense when the prescription used to renormalize them is spelled out. The use of the bag wave function in evaluating m_q is one reasonable method.

Actually, the derivation of the chiral perturbation formula Eq. (56) does not depend on the bag model. It is independent of the precise form of $\phi(p)$, provided the wave packet itself is independent of m_π. Note, however, that the wave packet used in Sec. IV involved the hadron mass via Eq. (5a), and cannot be used to obtain Eq. (63), since the expansion of Eq. (59) does not apply to such a wave packet. The important ingredients are (1) the equality of $\langle E\rangle$ with the total quark energy $E_{\rm bag}$, (2) the vanishing of m_π when $m_q=0$, and in both cases, first-order perturbation theory.

Direct evaluation of Eq. (64) yields $m_q=22$ MeV. This corresponds to a chiral parameter

$$\langle 0|\bar{q}q|0\rangle=-F_\pi^2\langle\pi|\bar{q}q|\pi\rangle=-(0.15\text{ GeV})^3$$

for $R_\pi=3.3-3.5$ GeV. The mass m_q is not meant to be compared directly to masses from current algebra. The current-algebra study of pseudoscalar masses yields only mass ratios. Weinberg has proposed a measure of absolute size.[10] If $\langle H|\bar{q}q|H\rangle=ZN_q$, where Z is a renormalization constant and N_q is the number of quarks in H, then

$$m_q^*=\frac{\langle H|m_q\bar{q}q|H\rangle}{N_q}=m_qZ \qquad (66)$$

can be defined. $\Delta S=1$ mass differences can be used to indicate $m_s^*\approx150$ MeV (this is true in the bag model also since there $m_s=280-330$ MeV, and $Z\approx0.5$). Current-algebra mass ratios[10,11] then suggest $m_q^*=\frac{1}{2}(m_u^*+m_d^*)=6$ MeV. The renormalization constant Z is readily calculable in the bag (see above). In fact, it is only the product m_qZ which enters the calculation of m_π through Eq. (62). The result to be compared with the chiral perturbation theory mass is $m_q^*=12$ MeV (or as in Sec. IV, more accurately, $m_q^*=17$ MeV). The bag model's values of the up- and down-quark masses are higher than those of chiral SU(3) by a factor of 2 or 3.

The quark masses determined in our method from the pion mass are consistent with those found by a study of the nucleon's σ term defined from[12]

$$[F_i^5,\partial_\mu A_j^\mu(0)]=\delta_{ij}\sigma$$

or $\qquad (67)$

$$\sigma=m_q(\bar{u}u+\bar{d}d)\,.$$

Taking matrix elements of σ between nucleon states yields $m_q^*=m_qZ$ directly[13]:

$$m_q^*=\tfrac{1}{3}\sigma_{\pi N}(1-\langle p^2\rangle/2m_p^2)\,. \qquad (68)$$

Estimates of $\sigma_{\pi N}$ range from 51 to 70 MeV or $m_q^*=14-19$ MeV. This translates to quark masses $m_q=25-37$ MeV, consistent with the quark mass found in Sec. IV. Jaffee has recently discussed the σ term in his discussion of scale-dependent light-quark masses.[14]

The perturbation method developed in this section, Eq. (64), yields $m_q=22$ MeV, while the alternative method based on a wave packet satisfying Eq. (5a) gave a value $m_q=33$ MeV in Sec. IV. The disagreement is due to two effects. First, we have included gluon effects in Sec. IV, but omitted them from Eq. (62). Second, the mass dependence of $\phi(p)$ is included in Sec. IV (but not in Secs. III and V) and introduces terms proportional to $m_\pi^2\ln(m_\pi R_\pi)$ in the expansion analogous to Eq. (59). But they are not a major source of discrepancy in practice, since the fit of Sec. IV gave a value $m_\pi R_\pi\approx0.5$. In chiral perturbation theory corrections to the lowest-order result also involve $m_\pi^2\ln m_\pi^2$, although their effect is numerically different from ours.

If we naively extended the bag-model version of chiral perturbation theory to include the kaon, we would obtain the usual chiral-SU(3) × SU(3) current-algebra ratios (neglecting isospin breaking)

$$\frac{m_q}{m_q+m_s}=\frac{m_\pi^2}{2m_K^2}=\frac{1}{26} \qquad (69)$$

or

$$m_q/m_s\simeq\tfrac{1}{25}\,. \qquad (70)$$

Instead, the correct bag procedure, using as input m_π and m_K, yields

$$m_q/m_s \cong \tfrac{1}{10}. \tag{71}$$

In this application SU(3) is not a good symmetry, and significant differences in the treatment of pions and kaons are seen. This explains why we agree with Weinberg's value of m_s^*, but not m_q^*.

VI. CONCLUSIONS

We have shown how the static bag model can be corrected to take into account in a simple way the effects associated with the localization of the particle at a fixed point in space. Although the modifications are small for hadrons whose size is large in comparison to the scale set by the mass, they have a very substantial effect on the lightest hadron, the pion. By incorporating these corrections, we have shown how it is possible for the traditional quark-model state with a substantial extended size to coincide with the PCAC pion, which is a "collective state" associated with the spontaneous breakdown of chiral symmetry. Although we have by no means gone all the way to reconcile these two seemingly very different physical pictures of the pion, we have at least indicated how they could be compatible with each other. Our methods have allowed us to estimate with some success the π and K decay constants, and to derive a connection with chiral perturbation theory.

Finally perhaps it would be interesting if we remark on a possibly significant similarity between the chiral field theory and the bag-model description of hadrons composed principally of light quarks. In field theory, the ratio

$$\frac{\langle 0|\bar{q}q|0\rangle}{\langle\pi(p)|\bar{q}q|\pi(p)\rangle} = -F_\pi^{\,2} \tag{72}$$

in the limit of massless pions. If we say (as in the bag model) that the interior of extended hadrons is accurately described by a perturbative vacuum (that is, a chirally symmetric state), then at "edge" where the effects of valence quarks should be lowest one might expect that $\bar{q}q \approx 0$ (as in the bag model). Inside, with the presence of the valence quarks $\bar{q}q > 0$. Consequently $\langle\pi(p)|\bar{q}q|\pi(p)\rangle > 0$. Hence, because of Eq. (72), $\langle 0|\bar{q}q|0\rangle < 0$, that is, $\bar{q}(x)q(x)$ changes discontinuously from zero to less than zero at the boundary of the hadron. In this way it is similar to the order parameter at the boundary between two phases which differ by a first-order change. Viewed in this way, the bag model for light quarks corresponds to a model of a "two-phase" vacuum with the order parameter $\bar{q}q > 0$ defining the chirally symmetric phase and $\bar{q}q < 0$, the spontaneously broken state where the chiral symmetry is spontaneously broken, it cannot itself be a chirally symmetric model. It is, however, natural that the bag model's chiral density finds its source only on the bag boundary.

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APPENDIX: NONSTATIC MATRIX ELEMENTS

For static quantities such as axial-vector coupling constants or magnetic moments, the methods derived in the main text are sufficient. However, for nonstatic quantities, such as charge radius contained in the form factor $F(q)$, a further improvement is needed. This appendix provides the necessary treatment.

When a photon couples to a particle it transfers to it a finite momentum q. Therefore we must necessarily go beyond a purely static treatment, so rather than considering only fixed static bags, let us consider a moving bag as a wave packet with a nonzero net momentum $\vec{q}$. We do this by transforming each of the momentum eigenstates in the wave packet by $\vec{q}$, i. e., for mesons

$$|M,q\rangle_B = \int \frac{d^3p}{(2\omega_p 2\omega_{p+q})^{1/2}}\phi(p)|M(p+q)\rangle. \tag{A1}$$

The normalization factor is such that

$${}_B\langle M,q|M,q\rangle_B = 1. \tag{A2}$$

This superposition produces a normalized bag state, centered around the origin, with an overall momentum $\vec{q}$. This wave packet can be written in a more convenient fashion by redefining the integration variable.

$$|M,q\rangle_B = \int \frac{d^3p}{(2\omega_p 2\omega_{p-q})^{1/2}}\phi(p-q)|M(p)\rangle. \tag{A3}$$

To calculate the form factor $F(q)$ defined by

$$\langle M(p')|J_\mu(x)|M(p)\rangle = (p+p')^\mu F(p-p')e^{2x\cdot(p-p')}, \tag{A4}$$

we transform to a matrix element between one static-bag state and one moving state:

$${}_B\langle M,q|J_0(x)|M,0\rangle_B = \int \frac{d^3p'd^3p}{(2\omega_{p'}2\omega_{p'-q})^{1/2}}\frac{1}{2\omega_p}(\omega_p+\omega_{p'})\phi^*(p'-q)\phi(p)F(p-p')e^{ix\cdot(p-p')}. \tag{A5}$$

If we now integrate this with a factor $e^{-iq\cdot x}$ we find

$$F_{\rm bag}(q)=\int\frac{d^3p}{2\omega_p}[\phi(p)]^2[(\omega_p/\omega_{p+q})^{1/2}+(\omega_{p+q}/\omega_p)^{1/2}](2\pi)^3F(q^2-(\omega_{p+q}-\omega_p)^2)\,, \tag{A6}$$

where, dropping small kinematic terms of order q^2/m^2, we identify

$$F_{\rm bag}(q)=\int d^3x\,e^{-iq\cdot x}\langle M,q|J_0(x)|M,0\rangle \tag{A7}$$

with the Fourier transform of the static bag charge density

$$F_{\rm bag}=\int d^3x\,e^{-iq\cdot x}\rho_{\rm bag}^{\rm static}(x)\,. \tag{A8}$$

Here we wish to compute corrections to the form factor of a heavy particle, the proton, so again we shall neglect such small kinematic quantities of order q^2/m^2. In this case, the spin-dependent effects are also absent and the formulas for mesons and baryons coincide. If we express the result in terms of the charge radius defined by

$$F(q^2)=1-\tfrac{1}{6}q^2\langle r^2\rangle_{\rm exp}\,, \tag{A9}$$

we find

$$\langle r^2\rangle_{\rm exp}=\langle r^2\rangle_{\rm bag}^{\rm static}\,(1+\tfrac{1}{3}\langle p^2\rangle/m^2)\,, \tag{A10}$$

which is also Eq. (15).

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