PHYSICAL REVIEW D VOLUME 21, NUMBER 7 1 APRIL 1980

Reggeization of elementary fermions in arbitrary renormalizable gauge theories

Marcus T. Grisaru and Howard J. Schnitzer
Department of Physics, Brandeis University, Waltham, Massachusetts 02254
(Received 10 December 1979)

An arbitrary, renormalizable non-Abelian gauge theory with gauge group $\mathcal{G}$ and arbitrary scalar and fermion structure is considered. Necessary conditions for all the fermions of the theory to lie on Regge trajectories are derived. If no fermion transforms as a singlet under $\mathcal{G}$, then all fermions Reggeize. If some fermions transform as singlets under $\mathcal{G}$, then certain group-theoretic constraints on the fermion mass matrix must be satisfied if all fermions are to Reggeize. These methods are applied to the Weinberg-Salam and grand unified theories. As a by-product of this study, kinematical-singularity-free, signatured helicity amplitudes are constructed which allow for P and CP violation. Such amplitudes may be useful for applications outside the context of the Regge program.

I. INTRODUCTION

It is the purpose of this paper to discuss the Regge behavior of elementary fermions near $J \simeq \frac{1}{2}$ in an arbitrary renormalizable gauge theory. This analysis should be considered a sequel to our study[1] [which we call (I)] of the Reggeization of the gauge vector mesons in arbitrary gauge theories. The reader should consult (I) for a detailed discussion of the motivation and setting of the question of Reggeization within the context of Lagrangian field theory. Here, as in (I), we stress possible applications of our results to unified and grand unified (GU) field theories. In particular, if the Higgs-meson mass exceeds 1 TeV and/or if fermion masses (either quark or lepton) exceed 300 GeV, then partial-wave unitarity in the electroweak sector is violated in perturbation theory,[2] and the weak interactions *must* become strong at energies exceeding 1 TeV. In this case one anticipates that the unified SU(2) × U(1) theory will exhibit resonances and Regge recurrences of intermediate vector bosons and elementary fermions, and other rich phenomena usually associated with the strong interactions. We hope that our analysis will contribute to the understanding of these aspects of unified theories, as well as to a general appreciation of the elegance of GU theories, at least within the context of our discussion.

In this paper we show that the presence of elementary fermions transforming as either right-handed or left-handed singlet representations of an arbitrary, non-Abelian gauge group $\mathcal{G}$ implies that some fermions may not Reggeize unless certain necessary conditions are satisfied. On the other hand, if *no* fermion transforms as a singlet under $\mathcal{G}$, then *all* fermions lie on Regge trajectories. Our criteria are as follows: Necessary conditions to be satisfied for all fermions to lie on a Regge trajectory are

$$\left.\begin{aligned} &\bar{t}^L_A m \hat{P}_R = 0 \qquad &(1.1a)\\ &\text{and} \\ &\bar{t}^R_A m \hat{P}_L = 0 \qquad &(1.1b)\end{aligned}\right\} \text{ for all } A\,,$$

where m is the fermion mass matrix, and $\bar{t}^L_A$ $(\bar{t}^R_A)$ is the left-handed (right-handed) fermion representation matrix (in general reducible) in the basis in which the gauge-vector-meson mass matrix is diagonal, with A labeling a particular gauge meson. One forms the Casimir operators for the left- and right-handed fermion representations, i.e.,

$$C_L = \sum_A \bar{t}^L_A \bar{t}^L_A$$

and (1.2)

$$C_R = \sum_A \bar{t}^R_A \bar{t}^R_A .$$

Then $\hat{P}_L$ and $\hat{P}_R$ are the projection operators onto the null spaces of C_L and C_R, respectively. That is,

$$\hat{P}_L C_L = C_L \hat{P}_L = 0$$

and (1.3)

$$\hat{P}_R C_R = C_R \hat{P}_R = 0 .$$

These requirements provide a considerable extension of earlier work,[3] which led to the erroneous conjecture that all elementary fermions always Reggeized in a renormalizable gauge theory. We provide specific counterexamples to this false conjecture in Sec. IV, in that we present models which fail to satisfy Eqs. (1.1). If Eq. (1.1) is satisfied for some but not all vector-meson states A, then some but not all fermions may lie on Regge trajectories. It should be emphasized that all our results are natural in the technical sense, being independent of the magnitude of coupling constants, and valid for arbitrary reducible or irreducible scalar-meson or fermion multiplets, for arbitrary (renormalizable) scalar self-couplings, and arbitrary gauge group $\mathcal{G}$.

In Sec. IV we analyze the Weinberg-Salam theory from the point of view of requirements (1.1). We show that if the lepton sector is made of several generations of left-handed doublets and right-handed singlets, with massless neutrinos, then all leptons lie on Regge trajectories. On the other hand, the quarks of the $SU(2)\times U(1)$ Weinberg-Salam theory do not Reggeize. If we then consider the standard model with the $SU(2)\times U(1)\times SU(3)$ group, we show that the colored gauge bosons enable all fermions in the standard model to Reggeize. In a GU theory[4] such as SU(5), O(10), or E_6, all quarks and leptons also Reggeize, since it is possible to avoid singlet representations of $\mathcal{G}$ in model building. A typical example is provided by the simplest SU(5) model, where each generation of fermions transforms as a $5+\overline{10}$ under $\mathcal{G}$. Combining the conclusions of (I) with those of this paper, we find that all gauge vector mesons and all elementary fermions lie on Regge trajectories if the gauge group $\mathcal{G}$ is semisimple (with no Abelian subgroups), and if all right- and left-handed fermions transform nontrivially under $\mathcal{G}$. Therefore, *all* gauge bosons and *all* fermions Reggeize in typical GU theories, a property which is in sharp contrast to that of the Weinberg-Salam model. Therefore, the analytic S matrix of the $SU(2)\times U(1)$ electroweak theory is dramatically different from that of a GU theory, with the distinction being natural in the technical sense.

It is not possible to compute the detailed behavior of the fermion (boson) Regge trajectories outside the neighborhood of $J=\frac{1}{2}$ ($J=1$) by currently available methods. However, e.g., the slope of the electron or muon trajectories at $J=\frac{1}{2}$ are of the order of $\Delta J/\Delta m \sim 1/(2\ \text{TeV})$, which is the same order as the slopes of the intermediate-vector-boson Regge trajectories. Thus, if perturbation theory fails for the unified electroweak theory above $1\ \text{TeV}^2$ (due, say, to a heavy Higgs meson), and if the Regge trajectories are straight lines, then Regge recurrences of the intermediate vector bosons and elementary fermions are expected in the 2-to-4-TeV region.

In Sec. II of this paper we formulate vector-fermion scattering in an arbitrary gauge theory, as is required for a study of Regge behavior near $J=\frac{1}{2}$. Much of the notation and strategy, which is to be found in (I), is not repeated here in the interest of brevity. Section III is devoted to an analysis of the factorization condition, which is a necessary condition for Reggeization [see Eq. (2.15) of (I)]. This eventually leads to the criteria presented in Eqs. (1.1)–(1.3) above. Several examples which illustrate the general criteria are given in Sec. IV.

In Appendix A we extend the usual formulation of kinematical-singularity-free, signatured, natural- and unnatural-parity helicity amplitudes to the most general situation in which neither parity nor CP is conserved. The usual formulation[3] assumes P and CP conservation; hence it is restricted to the strong interactions, and is not adequate for applications to unified and GU theories. The material in this Appendix is new, and is probably useful for a variety of applications outside the context of this paper. Appendix B contains some details of the calculation of vector-fermion helicity amplitudes.

II. VECTOR-FERMION SCATTERING

Our program requires the computation of the kinematical-singularity-free helicity amplitudes near $J=\frac{1}{2}$ for vector-fermion scattering in an arbitrary, non-Abelian gauge theory. We use the notation and formulation of (I) and denote equations from that paper with the prefix I. The Lagrangian in unitary gauge is

$$\mathcal{L}=\mathcal{L}_{\text{gauge}}+\mathcal{L}_{\text{boson}}+\mathcal{L}_{\text{fermion}}, \tag{2.1}$$

where $\mathcal{L}_{\text{gauge}}$ and $\mathcal{L}_{\text{boson}}$ are given in Eqs. (I-3.2) and (I-3.7) respectively, while

$$\mathcal{L}_{\text{fermion}}=i\bar{\psi}\gamma^{\mu}D_{\mu}\psi-\bar{\psi}m_0\psi-\bar{\psi}(\Gamma,\phi)\psi, \tag{2.2}$$

(Recall we are following the general theory of broken symmetry given by Weinberg,[5] but with Bjorken and Drell conventions.) In (2.2) m_0 is the bare-mass matrix of the fermions. The gauge-covariant derivative of the spin-$\frac{1}{2}$ field $\psi_n(x)$ is

$$(D_{\mu}\psi)_n=\partial_{\mu}\psi_n+i(t_{\alpha})_{nm}\psi_m A_{\alpha\mu}, \tag{2.3}$$

where t_{α} is the matrix representation of T_{α} in the (reducible) representation D_F of $\mathcal{G}$ furnished by the fermion fields. Thus,

$$[t_{\alpha},t_{\beta}]=iC_{\alpha\beta\gamma}t_{\gamma}. \tag{2.4}$$

The representation matrices may be decomposed into left- and right-handed parts, i.e.,

$$t_{\alpha}=t_{\alpha}^{L}+t_{\alpha}^{R}, \tag{2.5}$$

where

$$t_{\alpha}^{L}=\tfrac{1}{2}(1-\gamma_5)t_{\alpha}$$

and

$$t_{\alpha}^{R}=\tfrac{1}{2}(1+\gamma_5)t_{\alpha}. \tag{2.6}$$

As a consequence,

$$[t_{\alpha}^{L},t_{\beta}^{L}]=iC_{\alpha\beta\gamma}t_{\gamma}^{L},$$
$$[t_{\alpha}^{R},t_{\beta}^{R}]=iC_{\alpha\beta\gamma}t_{\gamma}^{R}, \tag{2.7}$$

and

$$[t^L_\alpha, t^R_\beta] = 0 .$$

Similarly,

$$\psi = \psi^L + \psi^R , \tag{2.8}$$

where

$$\psi^L = \tfrac{1}{2}(1-\gamma_5)\psi , \quad \psi^R = \tfrac{1}{2}(1+\gamma_5)\psi . \tag{2.9}$$

Since $i(\bar\psi\gamma^\mu D_\mu \psi)$ is invariant under $\mathcal{G}$,

$$(t^L_\alpha)^\dagger = t^L_\alpha \quad \text{and} \quad (t^R_\alpha)^\dagger = t^R_\alpha . \tag{2.10}$$

The bare-mass term in (2.2) is also invariant under $\mathcal{G}$, which implies that

$$t^R_\alpha m_0 - m_0 t^L_\alpha = 0 \quad \text{and} \quad t^L_\alpha m_0 - m_0 t^R_\alpha = 0 . \tag{2.11}$$

Further,

$$m_0{}^\dagger = \gamma^0 m_0 \gamma^0 . \tag{2.12}$$

After a suitable redefinition of the fields ψ, we eliminate all terms proportional to γ_5 in m_0.

The invariance of the Yukawa interaction under $\mathcal{G}$ leads to

$$[t_\alpha, \gamma^0\Gamma_p] = -(\theta_\alpha)_{pq}(\gamma^0\Gamma_q) , \tag{2.13}$$

where θ_α is the matrix representation of T_α in the representation D_B of $\mathcal{G}$ [see (I-3.7)–(I-3.10)]. One can decompose

$$\Gamma_p = \tfrac{1}{2}(1+\gamma_5)\Gamma^{RL}_p + \tfrac{1}{2}(1-\gamma_5)\Gamma^{LR}_p , \tag{2.14}$$

so that (2.13) and (2.14) give

$$t^L_\alpha \Gamma^{LR}_p - \Gamma^{LR}_p t^R_\alpha = -(\theta_\alpha)_{pq}\Gamma^{LR}_q \quad \text{and} \quad t^R_\alpha \Gamma^{RL}_p - \Gamma^{RL}_p t^L_\alpha = -(\theta_\alpha)_{pq}\Gamma^{RL}_q . \tag{2.15}$$

Since the Yukawa interaction is Hermitian,

$$\Gamma^\dagger_p = \gamma^0\Gamma_p\gamma^0 \quad \text{and} \quad (\Gamma^{LR}_p)^\dagger = (\Gamma^{RL}_p) . \tag{2.16}$$

According to the program outlined in Sec. II of (I), in order to verify the Reggeization of the fermions near $J=\frac{1}{2}$, we must compute the Born S matrix for vector-fermion scattering. Therefore it is best to work in a basis in which both the vector-meson and fermion mass matrices are diagonal. This is described for the vector mesons in Eqs. (I-3.18)–(I-3.22). When one diagonalizes the vector-meson mass matrix, the representation matrix of the fermions becomes

$$\bar t_A = C_{\alpha A} t_\alpha , \tag{2.17}$$

where $\{C_{\alpha A}\}$ is the set of orthonormal vectors defined in (I-3.19)–(I-3.20). Equation (2.17) is decomposed as in (2.10). One shifts the scalar fields

$$\phi_p = \lambda_p + \phi'_p \tag{I-3.13}$$

according to (I-3.11)–(I-3.17). Then

$$\mathcal{L}_{\text{fermion}} = i\bar\psi\gamma^\mu D_\mu\psi - \bar\psi m\psi - \bar\psi(\Gamma, \phi')\psi , \tag{2.18}$$

where

$$D_\mu\psi = \partial_\mu\psi + i\sum_N (\bar t_N)\bar A_{N\mu}\psi , \tag{2.19}$$

and where the zero-order fermion matrix is

$$m = m_0 + (\Gamma, \lambda) , \tag{2.20}$$

with matrix elements

$$m_{ij} = m_i \delta_{ij} \tag{2.21}$$

in the basis in which m is diagonal. [This does not mean $(\Gamma_p)_{ij}$ is diagonal, since in general $(m_0)_{ij}$ is not diagonal in this basis.] In this representation, the shifted Lagrangian is given in unitary gauge by (I-3.23) and (2.18), from which we extract the Feynman rules for a lowest-order calculation.

Let us consider the vector-fermion scattering processes

$$V_A(k_A) + F_i(p_1) \to V_B(k_B) + F_j(p_2) , \tag{2.22}$$

where p_1 and p_2 are the four-momenta of the fermions of type i and j, respectively, in the basis defined by (2.21), and k_A and k_B are the four-momenta of the vector mesons A and B in the basis defined by (I-3.19)–(I-3.21). In lowest order there are four diagrams contributing to the process (2.22): two fermion exchanges, one vector exchange, and one scalar exchange. These diagrams give amplitudes

$$D_1 = -i\bar u(p_2)\left[\gamma_\nu(\bar t_B)\frac{\gamma\cdot(p_1+k_A)+m}{(s-m^2)}\gamma_\mu(\bar t_A)\right] \times u(p_1)\epsilon_\mu(k_A)\epsilon^*_\nu(k_B) , \tag{2.23}$$

$$D_2 = -i\bar u(p_2)\left[\gamma_\mu(\bar t_A)\frac{\gamma\cdot(p_1-k_B)+m}{(u-m^2)}\gamma_\nu(\bar t_B)\right] \times u(p_1)\epsilon_\mu(k_A)\epsilon^*_\nu(k_B) , \tag{2.24}$$

$$D_3 = \sum_G \frac{\bar u(p_2)\gamma_\sigma(\bar t_G)u(p_1)}{(t-\mu_G{}^2)}\left[g_{\sigma\lambda} - \frac{(k_A-k_B)_\lambda(k_A-k_B)_\sigma}{\mu_G{}^2}\right] \times \bar C_{ABG}[g_{\mu\nu}(k_A+k_B)_\lambda + g_{\nu\lambda}(k_A-2k_B)_\mu + g_{\lambda\mu}(k_B-2k_A)_\lambda]\epsilon_\mu(k_A)\epsilon^*_\nu(k_B) , \tag{2.25}$$

and

$$D_4 = -i[\bar{u}(p_2)(\Gamma_q)u(p_1)]\left(\frac{\Pi}{t-M^2}\right)_{qp} \times [(\bar{\theta}_B\lambda\bar{\theta}_A)+(\bar{\theta}_A\lambda\bar{\theta}_B)]_p g_{\mu\nu}\epsilon_\mu(k_A)\epsilon_\nu^*(k_B), \quad (2.26)$$

respectively. In diagram D_4, the projection operator Π_{pq} is given by (I-3.26), with the $\bar{\theta}_A$ defined by (I-3.20). In (2.23)–(2.26) s, t, u denote the usual Mandelstam variables, i.e.,

$$s=(p_1+k_A)^2, \quad t=(p_1-p_2)^2, \quad (2.27)$$

and

$$u=(p_1-k_B)^2,$$

with

$$s+t+u=\mu_A{}^2+\mu_B{}^2+m_i{}^2+m_j{}^2. \quad (2.28)$$

The complete Feynman amplitude for process (2.22) is

$$D=D_1+D_2+D_3+D_4. \quad (2.29)$$

In the center-of-mass system, the four-momenta of the boson and fermion states are

$$\begin{aligned} k_A&=(E_A,0,0,K_A),\\ k_B&=(E_B,K_By,0,K_Bz),\\ p_1&=(\mathcal{E}_i,0,0,-K_A),\\ p_2&=(\mathcal{E}_j,-K_By,0,-K_Bz), \end{aligned} \quad (2.30)$$

where $z=\cos\theta$ and $y=\sin\theta$, with θ the center-of-mass scattering angle. Also we define

$$W=\sqrt{s}=(E_A+\mathcal{E}_i)=(E_B+\mathcal{E}_j). \quad (2.31)$$

Further,

$$t=m_i{}^2+m_j{}^2-2\mathcal{E}_i\mathcal{E}_j+2K_AK_Bz \quad (2.32a)$$

and

$$u=m_i{}^2+\mu_B{}^2-2\mathcal{E}_iE_B-2K_AK_Bz \quad (2.32b)$$

in the center-of-mass system.

In computing the J-plane behavior of the amplitudes near $J=\frac{1}{2}$, it is sufficient to examine the large-z form of the helicity amplitudes.[3] This fact leads to some algebraic simplification because it allows one to replace, in the denominators of D_2, D_3, and D_4, the $u-m^2$, $t-\mu^2$, and $t-M^2$ factors by $-2K_AK_Bz$, $2K_AK_Bz$, and $2K_AK_Bz$, respectively, after which the numerators may be combined:

$$D=D_1+\frac{1}{2K_AK_Bz}(-N_2+N_3+N_4), \quad (2.33)$$

where N_i are the numerators of the D_i in Eqs. (2.24)–(2.26). After some algebra, which involves commuting $\gamma\cdot\epsilon^*$ to the extreme left and $\gamma\cdot\epsilon$ to the extreme right, we obtain:

$$\begin{aligned} -N_2+N_3+N_4 = {} & 2i(\epsilon\cdot\epsilon^*)\bar{u}[\gamma\cdot(p_2+k_B)(\bar{t}_B\bar{t}_A)-\gamma^0\bar{t}_B\gamma^0m\bar{t}_A]u-2i(k_B\cdot\epsilon)\bar{u}[\gamma\cdot\epsilon^*(\bar{t}_B\bar{t}_A)]u\\ & -2i(k_A\cdot\epsilon^*)\bar{u}[\gamma^0(\bar{t}_B\bar{t}_A)\gamma^0\gamma\cdot\epsilon]u+2i(p_1\cdot\epsilon)\bar{u}[\gamma\cdot\epsilon^*(\bar{t}_A\bar{t}_B)]u+2i(p_2\cdot\epsilon^*)\bar{u}[\gamma^0(\bar{t}_A\bar{t}_B)\gamma^0\gamma\cdot\epsilon]u\\ & -i\bar{u}[\gamma\cdot\epsilon^*\gamma\cdot(p_1-k_B)\gamma^0(\bar{t}_B\bar{t}_A)\gamma^0\gamma\cdot\epsilon]u-i\bar{u}[\gamma\cdot\epsilon^*(\bar{t}_Am\gamma^0\bar{t}_B)\gamma^0\gamma\cdot\epsilon]u. \end{aligned} \quad (2.34)$$

Note that the $1/\mu_G{}^2$ term in N_4, arising from the projection operator Π_{pq} (I-3.26), cancels a similar term in N_3, a cancellation required by gauge invariance. In establishing this result, we require the identity

$$\begin{aligned} (\gamma^0\Gamma_q)[(\bar{\theta}_B\lambda\bar{\theta}_A)+(\bar{\theta}_A\lambda\bar{\theta}_B)]_q &= -\{[\bar{t}_B,[\bar{t}_A,\gamma^0(\lambda,\Gamma)]]+(A\leftrightarrow B)\}\\ &= -[\bar{t}_B,[\bar{t}_A,\gamma^0m]]-[\bar{t}_A,[\bar{t}_B,\gamma^0m]], \end{aligned} \quad (2.35)$$

where use has been made of (2.11), (2.13), and (2.20). We also need the identity

$$(\gamma_0\Gamma_q)\sum_G(\bar{\theta}_G\lambda)_q(\bar{\theta}_G\lambda)_p[(\bar{\theta}_B\lambda\bar{\theta}_A)+(\bar{\theta}_A\lambda\bar{\theta}_B)]_p = i\sum_G[\bar{t}_G,\gamma^0m]\bar{C}_{ABG}(\mu_B{}^2-\mu_A{}^2)/\mu_G{}^2, \quad (2.36)$$

which is verified by manipulations similar to those used in proving (I-3.30).

Equation (2.34) is particularly useful in computing the sense-nonsense and nonsense-nonsense matrix elements, since

$$\left.\begin{aligned} (\gamma\cdot\epsilon)u&=0\\ \text{and}\quad &\\ \bar{u}(\gamma\cdot\epsilon^*)&=0 \end{aligned}\right\} \text{ for } \textit{nonsense} \text{ states}. \quad (2.37)$$

As a result of (2.37), only the first line of (2.34) is needed in the evaluation of the nonsense-nonsense helicity matrix—a considerable simplification. Other useful identities are

$$\begin{aligned} p_1\cdot\epsilon^{(\pm)}&=p_2\cdot\epsilon^{(\pm)*}=0,\\ p_1\cdot\epsilon^{(0)}&=WK_A/\mu_A, \end{aligned} \quad (2.38)$$

and

$p_2 \cdot \epsilon^{(0)*} = WK_B/\mu_B$,

where $\epsilon^{(\pm)}$ and $\epsilon^{(0)}$ denote the helicity $= \pm 1$ and 0 vector-meson wave functions, respectively. Equations (2.38) enter the computation of the sense-nonsense and sense-sense matrix elements of (2.34). The calculation of the kinematical-singularity-free helicity amplitudes is presented in Appendix B.

III. HELICITY AMPLITUDES AND THE FACTORIZATION CONDITION

At $J=\frac{1}{2}$, where the fermion-vector partial-wave amplitudes satisfy equations which admit solutions without Castillejo-Dalitz-Dyson poles, the Reggeization requirement is equivalent to the factorization condition[3]

$$f_{ss} = f_{sn} f_{nn}^{-1} f_{ns}\,; \tag{3.1}$$

where the f's are the coefficients of $\delta_{J,1/2}$, $1/(J-\frac{1}{2})^{1/2}$, $1/(J-\frac{1}{2})$, respectively, in the partial-wave projections of the sense-sense ($|\lambda|$, $|\lambda'|=\frac{1}{2}$), sense-nonsense ($|\lambda|=\frac{1}{2}$, $|\lambda'|=\frac{3}{2}$), and nonsense-nonsense ($|\lambda|=|\lambda'|=\frac{3}{2}$) helicity amplitudes. [See (I-2.9).] Equivalently they are the coefficients of z^0, $1/z$, $1/z$ (for large z) of the helicity amplitudes themselves. Our task is to compute these amplitudes in the kinematical-singularity-free, right-signature, parity combinations described in Appendix A. In checking the necessary conditions for Reggeization of the fermions, we have deemed it sufficient to look at sense states with only transverse mesons, helicity ± 1. This leads to appreciable simplification of the algebra and is not expected to cause any loss of generality in the final results. The details of the computation of the corresponding helicity amplitudes are given in Appendix B. We denote the resulting coefficients of z^0 in the amplitudes $\tilde{f}^{(\epsilon',\epsilon)}_{1/2,1/2}$ by f_{ss}, and of $1/z$ in $\tilde{f}^{(\epsilon',\epsilon)}_{1/2,3/2}$, $\tilde{f}^{(\epsilon',\epsilon)}_{3/2,1/2}$, and $\tilde{f}^{(\epsilon',\epsilon)}_{3/2,3/2}$ by f_{sn}, f_{ns}, and f_{nn}, respectively, and observe that each of these can be written as matrices in parity space:

$$f = \begin{pmatrix} f^{++} & f^{+-} \\ f^{-+} & f^{--} \end{pmatrix}, \tag{3.2}$$

with

$$f_{nn} = \mathfrak{N}\tilde{T}_B \mathcal{W} T_A \mathfrak{N}\,, \tag{3.3a}$$

$$f_{ns} = \mathfrak{N}(\tilde{T}_B \mathcal{W} T_A + 2\tilde{T}_B T_A \underline{\mathcal{E}})\mathfrak{N}\,, \tag{3.3b}$$

$$f_{sn} = \mathfrak{N}(2\underline{\mathcal{E}}\tilde{T}_B T_A + \tilde{T}_B \mathcal{W} T_A)\mathfrak{N}\,, \tag{3.3c}$$

$$f_{ss} = -4\mathfrak{N}\underline{\mathcal{E}}\tilde{T}_B \mathcal{W}^{-1} T_A \underline{\mathcal{E}}\mathfrak{N}\,, \tag{3.3d}$$

and the following matrices defined in parity space:

$$T = \frac{1}{2}\begin{pmatrix} \bar{t}^R + \bar{t}^L & \bar{t}^R - \bar{t}^L \\ -\bar{t}^R + \bar{t}^L & -\bar{t}^R - \bar{t}^L \end{pmatrix}, \tag{3.4a}$$

$$\tilde{T} = \frac{1}{2}\begin{pmatrix} \bar{t}^R + \bar{t}^L & -\bar{t}^R + \bar{t}^L \\ \bar{t}^R - \bar{t}^L & -\bar{t}^R - \bar{t}^L \end{pmatrix}, \tag{3.4b}$$

$$\mathcal{W} = \begin{pmatrix} W-m & 0 \\ 0 & W+m \end{pmatrix}, \tag{3.5}$$

$$\underline{\mathcal{E}} = \begin{pmatrix} \mathcal{E}-m & 0 \\ 0 & \mathcal{E}+m \end{pmatrix}, \tag{3.6}$$

$$\mathfrak{N} = [2m(E+m)]^{1/2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.7}$$

The *individual* entries in the above definitions are *matrices* in *fermion* internal-symmetry space, labeled by vectors $\psi^{(i)} = \psi^{(i)L} \otimes \psi^{(i)R}$ with $m = m_{ji}$, $\bar{t}^R \pm \bar{t}^L \equiv (t^R \otimes 1^L \pm 1^R \otimes t^L)_{ji}$, and $\mathcal{E} = \mathcal{E}_i \delta_{ji}$. According to the unitarity-analyticity method for computing the S matrix in the J plane, the sense-sense elements of the analytically continued S matrix near $J=\frac{1}{2}$ are given from (3.1) by

$$\lim_{J\to 1/2} f_{sn}(J-\tfrac{1}{2}-\mathcal{K}f_{nn})^{-1}\mathcal{K}f_{ns}\,, \tag{3.8}$$

where $\mathcal{K}$ is essentially an integral over phase space and the matrix multiplication is understood to mean

$$\sum_{Ck,\,C'k'} (f)_{Bj,Ck}(J-\tfrac{1}{2}-\mathcal{K}f)^{-1}{}_{Ck,C'k'}(\mathcal{K}f)_{C'k',Ai}\,. \tag{3.9}$$

The resulting expression is to be compared to the sense-sense amplitude evaluated directly at $J=\frac{1}{2}$. Using equations (3.3), (3.9) can be written as

$$\lim_{J\to 1/2} 4\mathfrak{N}\underline{\mathcal{E}}\tilde{T}T(J-\mathcal{K}\tilde{T}\mathcal{W}T)^{-1}\mathcal{K}\tilde{T}T\underline{\mathcal{E}}\mathfrak{N}\,, \tag{3.10}$$

plus terms which are regular, and should be compared with that part of f_{ss} which goes as z^0 in D_2, D_3, and D_4. The crucial point is that f_{ss}, corresponding to the s-channel term D_1, has poles in W at the masses of the fermions, and the necessary condition for Reggeization is that $f_{sn}(J-\frac{1}{2}-\mathcal{K}f_{nn})^{-1}\mathcal{K}f_{ns}$ should have the same structure at $J=\frac{1}{2}$. Therefore, a necessary condition for Reggeization is that in this limit we should have

$$(4\mathfrak{N}\underline{\mathcal{E}}\tilde{T}_B\mathcal{W}^{-1}T_A\underline{\mathcal{E}}\mathfrak{N})_{ji} = \lim_{\alpha\to 0}\,[4\mathfrak{N}\underline{\mathcal{E}}\tilde{T}T(\tilde{T}\mathcal{W}T-\alpha)^{-1}\tilde{T}T\underline{\mathcal{E}}\mathfrak{N}]_{Bj,Ai} \tag{3.11}$$

for every initial and final vector meson A, B and every initial and final fermion i, j. We have defined, for the above equation

$$\alpha=(J-\tfrac{1}{2})\mathcal{K}^{-1}, \qquad (3.12)$$

since the phase-space integral $\mathcal{K}$ is nonsingular. Thus, since the *kinematical* factors on both sides of (3.11) are identical, we must investigate under what conditions the following equation holds:

$$\tilde{T}\mathcal{W}^{-1}T=\lim_{\alpha\to 0}\tilde{T}T(\tilde{T}\mathcal{W}T-\alpha)^{-1}\tilde{T}T. \qquad (3.13)$$

The relation (3.13) is immediately satisfied in those cases where the inverse exists in the $\alpha\to 0$ limit. Since $\mathcal{W}$ is not singular for arbitrary values of the energy (which are not eigenvalues of $\pm m$), the existence of the inversion is determined by the singular nature of the matrices $T, \tilde{T}$. Even when these matrices are singular, (3.13) may be satisfied if the kernel of $\tilde{T}\mathcal{W}T$ is annihilated by T and/or $\tilde{T}$. We proceed to investigate this situation in more detail. We define X to be the inverse of $(\tilde{T}\mathcal{W}T-\alpha)$ (which exists for $\alpha\neq 0$) so that

$$[(\tilde{T}\mathcal{W}T-\alpha)X]_{Bj,Ai}=\delta_{BA}\delta_{ji}. \qquad (3.14)$$

Multiplying on the left by T_B and on the right by $\tilde{T}_A$, and summing over all the vector-meson states, we have

$$(T\tilde{T}\mathcal{W}-\alpha)TX\tilde{T}=T\tilde{T}. \qquad (3.15)$$

We denote

$$C=\sum_A T_A\tilde{T}_A\equiv T\tilde{T}, \qquad (3.16)$$

so that the condition (3.13) becomes a *relation between matrices in fermion space*,

$$\tilde{T}_B\mathcal{W}^{-1}T_A=\lim_{\alpha\to 0}\tilde{T}_B(C\mathcal{W}-\alpha)^{-1}CT_A. \qquad (3.17)$$

The significance of C is that it is a matrix of the Casimir operators of the fermion representation

$$C=\begin{pmatrix} C_R+C_L & -C_R+C_L \\ -C_R+C_L & C_R+C_L \end{pmatrix}, \qquad (3.18)$$

where

$$C_R=\sum_A \bar{t}^R_A\bar{t}^R_A, \qquad (3.19)$$

$$C_L=\sum_A \bar{t}^L_A\bar{t}^L_A. \qquad (3.20)$$

The condition (3.17) must hold for all meson states A, B if *all* fermions are to Reggeize. It is convenient to go to a representation (in parity space) where C is diagonal:

$$C'=\begin{pmatrix} C_R & 0 \\ 0 & C_L \end{pmatrix}. \qquad (3.21)$$

Then in this representation,

$$\mathcal{W}'=\begin{pmatrix} W & -m \\ -m & W \end{pmatrix},$$

$$T'=\begin{pmatrix} 0 & \bar{t}^R \\ \bar{t}^L & 0 \end{pmatrix}, \quad \tilde{T}'=\begin{pmatrix} 0 & \bar{t}^L \\ \bar{t}^R & 0 \end{pmatrix}, \qquad (3.22)$$

as can be easily verified. The factorization condition is immediately satisfied in *case I*: C is nonsingular, $\det(C_LC_R)\neq 0$, i.e., both the left- and right-handed fermions belong to nonsinglet representations. In this case all fermions lie on Regge trajectories.

We now discuss case II, when $C_L\equiv 0$, i.e., $\bar{t}^L_A=0$ for all A, $\det C_R\neq 0$, i.e., all fermions are right-handed and belong to nonsinglet representations:

$$C'\mathcal{W}'-\alpha=\begin{pmatrix} C_RW-\alpha & -C_Rm \\ 0 & -\alpha \end{pmatrix}, \qquad (3.23)$$

$$(C'\mathcal{W}'-\alpha)^{-1}=\begin{pmatrix} (C_RW-\alpha)^{-1} & -\dfrac{1}{\alpha}(C_RW-\alpha)^{-1}C_Rm \\ 0 & -1/\alpha \end{pmatrix}, \qquad (3.24)$$

and

$$\tilde{T}'(C'\mathcal{W}'-\alpha)^{-1}C'T'=\begin{pmatrix} 0 & 0 \\ 0 & \bar{t}^R(C_RW-\alpha)^{-1}C_R\bar{t}^R \end{pmatrix}, \qquad (3.25)$$

while

$$\tilde{T}'\mathcal{W}'^{-1}T'=\begin{pmatrix} 0 & 0 \\ 0 & \bar{t}^RW(W^2-m^2)^{-1}\bar{t}^R \end{pmatrix}. \qquad (3.26)$$

In the limit $\alpha\to 0$ the factorization condition can hold only if $t^R_Bm^2t^R_A=0$, or multiplying by $\bar{t}^R_B$ on the left and $\bar{t}^R_A$ on the right and summing, $C_Rm^2C_R=0$. Since C_R is nonsingular we conclude that $m^2=0$. Therefore for *case II*: If all fermions belong to (nonsinglet) purely right-handed (or purely left-handed) representations of the gauge group, they will all Reggeize only if they are all massless.

We consider next the *general case*, without special restrictions on C_R or C_L. By direct computation, using (3.21) and (3.22), we find that

$$\tilde{T}\mathcal{W}^{-1}T=\begin{pmatrix} \bar{t}^L\dfrac{W}{W^2-m^2}\bar{t}^L & -\bar{t}^L\dfrac{m}{W^2-m^2}\bar{t}^R \\ -\bar{t}^R\dfrac{m}{W^2-m^2}\bar{t}^L & \bar{t}^R\dfrac{W}{W^2-m^2}\bar{t}^R \end{pmatrix} \qquad (3.27)$$

and

$$\tilde{T}'(C'\mathcal{W}'-\alpha)^{-1}C'T'=\begin{pmatrix}\bar{t}^{L}DC_{L}\bar{t}^{L} & \bar{t}^{L}EC_{R}\bar{t}^{R}\\ \bar{t}^{R}FC_{L}\bar{t}^{L} & \bar{t}^{R}BC_{R}\bar{t}^{R}\end{pmatrix}, \tag{3.28}$$

where the matrices in (3.28) are

$$B=[(C_RW-\alpha)-C_Rm(C_LW-\alpha)^{-1}C_Lm]^{-1}, \tag{3.29}$$

$$F=[(C_RW-\alpha)^{-1}C_Rm]\times[-C_Lm(C_RW-\alpha)^{-1}C_Rm+(C_LW-\alpha)]^{-1}, \tag{3.30}$$

$$D=[(C_LW-\alpha)-C_Lm(C_RW-\alpha)^{-1}C_Rm]^{-1}, \tag{3.31}$$

and

$$E=[(C_LW-\alpha)^{-1}C_Lm]\times[-C_Rm(C_LW-\alpha)^{-1}C_Lm+(C_RW-\alpha)]^{-1}, \tag{3.32}$$

which gives four conditions to be satisfied if the factorization constraint (3.17) is to hold. First consider

$$\bar{t}^R_B\frac{W}{W^2-m^2}\bar{t}^R_A \overset{?}{=}\lim_{\alpha\to 0}\bar{t}^R_B[(C_RW-\alpha)-C_Rm(C_LW-\alpha)^{-1}C_Lm]^{-1}C_R\bar{t}^R_A \tag{3.33a}$$

$$=\lim_{\alpha\to 0}\bar{t}^R_BW\left[C_R(W^2-m^2)-\alpha-C_Rm\frac{\alpha}{C_LW-\alpha}m\right]^{-1}C_R\bar{t}^R_A. \tag{3.33b}$$

Now,

$$\lim_{\alpha\to 0}\alpha(C_LW-\alpha)^{-1}=-\hat{P}_L, \tag{3.34}$$

where $\hat{P}_L$ ($\hat{P}_R$) is the projection operator onto the null space of C_L (C_R), so that

$$\hat{P}_LC_L=C_L\hat{P}_L=0 \quad\text{and}\quad \hat{P}_RC_R=C_R\hat{P}_R=0. \tag{3.35}$$

Thus, we have

$$\bar{t}^R_B\frac{W}{W^2-m^2}\bar{t}^R_A\overset{?}{=}\lim_{\alpha\to 0}\bar{t}^R_BW[C_R(W^2-m^2)-\alpha+C_Rm\hat{P}_Lm]^{-1}C_R\bar{t}^R_A. \tag{3.36}$$

Since (3.16) and (3.35) imply that

$$\hat{P}_R\bar{t}^R=\bar{t}^R\hat{P}_R=0, \tag{3.37}$$

the factorization condition (3.36) requires

$$C_Rm\hat{P}_Lm\bar{t}^R_A=0,\quad\text{for any } A. \tag{3.38}$$

Multiplying by $\bar{t}^R_A$ on the right and summing on A gives

$$C_Rm\hat{P}_LmC_R=0. \tag{3.39}$$

Since $\hat{P}_L{}^2=\hat{P}_L$ and C_R, m, and $\hat{P}_L$ are Hermitian, this simplifies to

$$C_Rm\hat{P}_L=0, \tag{3.40}$$

or equivalently

$$\bar{t}^R_Am\hat{P}_L=0,\quad\text{for all vector-meson states } A. \tag{3.41}$$

From (3.27), (3.29), and (3.32) we obtain the analogous result

$$\bar{t}^L_Am\hat{P}_R=0,\quad\text{for all } A. \tag{3.42}$$

Finally, the off-diagonal elements of (3.27) and (3.28) automatically satisfy the factorization condition (3.36), once (3.40) and (3.41) are imposed. In establishing this final result, one must also make use of (3.37). In summary, we have found the following criteria: In the *general case*, necessary conditions for all fermions to lie on a Regge trajectory are

$$\left.\begin{aligned}&\bar{t}^L_Am\hat{P}_R=0\\ \text{and}\quad&\\ &\bar{t}^R_Am\hat{P}_L=0\end{aligned}\right\},\quad\text{for all } A. \tag{3.43}$$

Equation (3.42) obviously subsumes the special cases discussed above. Conditions (3.43) are trivially satisfied if $\hat{P}_R=0$ and $\hat{P}_L=0$, which can only occur if there are no right-handed or left-handed singlet fermion representations of $\mathcal{G}$. If singlet representations do occur, the above conditions become constraints on the fermion mass matrix. In the next section we turn to applications of our results.

IV. APPLICATIONS

In Sec. III we derived a necessary condition for the Reggeization of the $J=\frac{1}{2}$ fermions in an arbitrary gauge theory. One requires

$$\left.\begin{aligned}&\bar{t}^L_Am\hat{P}_R=0\\ \text{and}\quad&\\ &\bar{t}^R_Am\hat{P}_L=0\end{aligned}\right\},\quad\text{for all } A \tag{4.1}$$

where the projection operators $\hat{P}$ satisfy

$$\hat{P}_LC_L=C_L\hat{P}_L=0 \quad\text{and}\quad \hat{P}_RC_R=C_R\hat{P}_R=0. \tag{4.2}$$

From the definition of the fermion Casimir operators, one also obtains slightly weaker conditions

$$C_Lm\hat{P}_R=0 \tag{4.3a}$$

and

$$C_R m\hat{P}_L = 0 \tag{4.3b}$$

from (4.1). It is sometimes convenient to work in the representation given by Eqs. (2.4)–(2.7). From the properties of the set of orthonormal vectors $\{C_{\alpha A}\}$ given in (I-3.19)–(I-3.20), one can write

$$C_R = \sum_A \bar{t}^R_A \bar{t}^R_A = t^R_\alpha t^R_\alpha$$

and (4.4)

$$C_L = t^L_\alpha t^L_\alpha .$$

Similarly, our criteria become

$$\left.\begin{aligned} t^L_\alpha m\hat{P}_R &= 0 \\ t^R_\alpha m\hat{P}_L &= 0 \end{aligned}\right\}, \quad \text{for all } \alpha \qquad (4.5a),\ (4.5b)$$

in this basis. We also remark that (4.1) or (4.5) need not be evaluated in the basis in which the fermion mass matrix is diagonal; the choice of fermion basis is a matter of convenience. By using (2.11), (2.15), and (2.20) we can rewrite (4.5) in terms of the Yukawa couplings, i.e.,

$$t^L_\alpha m\hat{P}_R = mt^R_\alpha \hat{P}_R - (\lambda\theta_\alpha \Gamma^{LR})\hat{P}_R = -(\lambda\theta_\alpha\Gamma^{LR})\hat{P}_R = 0$$

and (4.6)

$$t^R_\alpha m\hat{P}_L = -(\lambda\theta_\alpha\Gamma^{RL})\hat{P}_L = 0 .$$

We now consider specific models, with emphasis on models where some fermions fail to Reggeize.

We begin with a model which has no parity violation in order to demonstrate that a central issue of fermion Reggeization is whether fermions belong to singlet representations or not, with parity violation not directly relevant. Consider the gauge group $\mathcal{G} = \mathrm{SU}(2)$, with fermions transforming as

$$\left.\begin{aligned} &\psi \sim 3 \\ \text{and } &l \sim 1 \end{aligned}\right\} \text{-dimensional representations of } \mathcal{G} \tag{4.7}$$

and scalars transforming

$$\phi_p \sim 3\text{-dimensional representation of } \mathcal{G};\ p = 1, 2, 3 . \tag{4.8}$$

The Lagrangian of this model is given by

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4}(\mathcal{F}^a_{\mu\nu})(\mathcal{F}^a_{\mu\nu}) , \tag{4.9}$$

where

$$\mathcal{F}^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon_{abc}A^a_\mu A^c_\nu ; \tag{4.10}$$

$$\begin{aligned}\mathcal{L}_{\text{fermion}} = {} & i\bar{\psi}\gamma^\mu D_\mu \psi - \bar{\psi}m_0\psi + i\bar{l}\gamma^\mu\partial_\mu l - m_0\bar{l}l \\ & - G_1 i\epsilon_{abc}(\bar{\psi}_a\psi_b)\phi_c \\ & - G_2[(\bar{\psi}_a l)\phi_a + (\bar{l}\psi_a)\phi_a] , \end{aligned} \tag{4.11}$$

where

$$(D_\mu\psi)_a = \partial_\mu\psi_a - g\epsilon_{abc}\psi_b A^c_\mu ; \tag{4.12}$$

and

$$\mathcal{L}_{\text{boson}} = \tfrac{1}{2}(D_\mu\phi, D^\mu\phi) - V(\phi) , \tag{4.13}$$

where

$$(D_\mu\phi)_a = \partial_\mu\phi_a - g\epsilon_{abc}\phi_b A^c_\mu . \tag{4.14}$$

(Note that we have chosen identical bare-mass terms for the fermions for simplicity.) It is possible to find a $V(\phi)$ such that

$$\langle\phi_1\rangle = \langle\phi_2\rangle = 0$$

and (4.15)

$$\langle\phi_3\rangle = v \neq 0 .$$

That is,

$$\langle\phi_a\rangle = \delta_{a3}v .$$

By translating the boson field $\phi_p = \phi'_p + \delta_{p3}v$, we obtain the following mass terms in the Lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{mass}} = {} & \tfrac{1}{2}(g^2v^2)(\delta_{ba} - \delta_{3b}\delta_{3a})A^a_\mu A^b_\mu - m_0(\bar{\psi}_a\psi_a + \bar{l}l) \\ & - iG_1 v[(\bar{\psi}_1\psi_2) - (\bar{\psi}_2\psi_1)] - G_2 v[(\bar{\psi}_3 l) + (\bar{l}\psi_3)] . \end{aligned} \tag{4.16}$$

Let us concentrate on the fermion mass matrix. In this basis the mass matrices are

$$M_1 = \begin{array}{cc} (\psi_1) & (\psi_2) \\ \multicolumn{2}{c}{\begin{pmatrix} m_0 & iG_1 v \\ -iG_1 v & m_0 \end{pmatrix}} \end{array} , \tag{4.17a}$$

with eigenvalues $m_0 \pm G_1 v$, and

$$M_2 = \begin{array}{cc} (\psi_3) & (l) \\ \multicolumn{2}{c}{\begin{pmatrix} m_0 & G_2 v \\ G_2 v & m_0 \end{pmatrix}} \end{array} , \tag{4.17b}$$

with eigenvalues $m_0 \pm G_2 v$. The fermion mass matrix is diagonal in the basis defined by

$$\begin{aligned} E_\pm &= \frac{1}{\sqrt{2}}(\psi_1 \pm i\psi_2) , \\ E_0 &= \frac{1}{\sqrt{2}}(\psi_0 + l_0) , \\ e_0 &= \frac{1}{\sqrt{2}}(\psi_0 - l_0) , \end{aligned} \tag{4.18}$$

leading to

$$\begin{aligned}\mathcal{L}_{\text{mass}} = {} & -(m_0 + G_1 v)\bar{E}_+E_+ - (m_0 - G_1 v)\bar{E}_-E_- \\ & -(m_0 + G_2 v)\bar{E}_0 E_0 - (m_0 - G_2 v)\bar{e}_0 e_0 . \end{aligned} \tag{4.19}$$

Consider vector-fermion scattering in this model. The relevant interaction term in the Lagrangian is

$$\begin{aligned} \mathcal{L}' = & -g(\bar{E}_+\gamma^\mu E_+ - \bar{E}_-\gamma^\mu E_-)Z_\mu \\ & \times \frac{-g}{\sqrt{2}}\,[(\bar{E}_0\gamma^\mu E_- + \bar{e}_0\gamma^\mu E_- \\ & \qquad - \bar{E}_+\gamma^\mu E_0 - \bar{E}_+\gamma^\mu e_0)W^+_\mu + \text{H.c.}]\,, \end{aligned} \tag{4.20}$$

where we have defined

$$\begin{aligned} & A^3_\mu = Z_\mu\,, \\ & \frac{1}{\sqrt{2}}(A^1_\mu \pm iA^2_\mu) = W^\pm_\mu\,. \end{aligned} \tag{4.21}$$

We now show that the singlet-triplet mixing of fermions spoils the Reggeization of the E^0 and e^0 fermions, a result which we establish in two different ways. Consider the coupled processes

$$\begin{array}{ccccc} W^+ + E^- & \searrow & & \nearrow & W^+ + E^- \\ & & (E^0, e^0) & & \\ W^- E^+ & \nearrow & & \searrow & W^- + E^+ \end{array}\,, \tag{4.22}$$

which are the only channels which can contribute to the Reggeization of the E^0 and e^0 fermions, as can be seen from (4.20). By direct computation it is easy to show that the nonsense-nonsense matrix for the process (4.22) is

$$(f_{nn})_{Bi;Aj} = -g^2(W - m_0)\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{4.23}$$

in the basis in which both the vector-meson and fermion mass matrices are diagonal. Since

$$\operatorname{rank} f_{nn} = 1\,, \tag{4.24}$$

there can be *at most* one Regge trajectory in the (E^0, e^0) sector;[1] not enough to Reggeize both fermions. Further,

$$f_{nn}(W = m_0) = 0\,, \tag{4.25}$$

so that this trajectory does not pass through either fermion. By comparison, the sense-sense matrix for (4.22) behaves as

$$f_{ss} \sim \frac{g^2}{W - m_{E^0}} + \frac{g^2}{W - m_{e^0}}\,, \tag{4.26}$$

where obvious coupling matrices have been omitted for simplicity. Thus in (4.22) neither E^0 or e^0 lies on a Regge trajectory. If there had been no mixing between the singlet and triplet fermions, then it is straightforward to verify that ψ_3 would lie on a Regge trajectory, but l would not.

The Reggeization of the $E_\pm$ fermions requires study of the coupled processes

$$\begin{array}{ccccc} Z + E_+ & \searrow & & \nearrow & Z + E_+ \\ W^+ + E_0 & \longrightarrow & E_+ & \longrightarrow & W^+ + E_0\,. \\ W^+ + e_0 & \nearrow & & \searrow & W^+ + e_0 \end{array} \tag{4.27}$$

A direct study of f_{nn} would now require extraction of the eigenvalues of a 3×3 matrix. Therefore, the complexity of the brute force analysis of f_{nn} increases rapidly with the number of coupled channels. For these more difficult situations, our general criteria (4.1)–(4.6) then become a powerful tool. We now turn to these techniques.

Since parity is conserved in this example, the left and right representations are isomorphic, and we may write our criteria as

$$\begin{aligned} & t_\alpha m\hat{P} = 0\,, \text{ for all } \alpha \\ & \text{with} \\ & \hat{P}C = 0 \end{aligned} \tag{4.28}$$

for the model described above in the representation D_F given by (4.9)–(4.18). Then,

$$t_a = \begin{array}{c} \begin{array}{cc} (\psi) & (l) \end{array} \\ \begin{bmatrix} (t_a)_{bc} & 0 \\ 0 & 0 \end{bmatrix} \end{array}, \quad a, b, c = 1, 2, 3\,, \tag{4.29}$$

where $(t_a)_{bc} = ig\epsilon_{abc}$,

$$C = t_\alpha t_\alpha = \begin{bmatrix} 2g^2\delta_{ab} & 0 \\ 0 & 0 \end{bmatrix}, \quad a, b = 1, 2, 3\,, \tag{4.30}$$

and

$$\hat{P} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.31}$$

in this basis. It should be emphasized that Eqs. (4.29)–(4.31) are 4×4 matrices in D_F, due to the reducible representation content of the fermions. Finally, combining (4.17) with (4.18) we have

$$m = \begin{array}{c} \begin{array}{cccc} (\psi_1) & (\psi_2) & (\psi_3) & (l) \end{array} \\ \begin{bmatrix} m_0 & iG_1 v & 0 & 0 \\ -iG_1 v & m_0 & 0 & 0 \\ 0 & 0 & m_0 & G_2 v \\ 0 & 0 & G_2 v & m_0 \end{bmatrix} \end{array}. \tag{4.32}$$

Returning to the Reggeization of the $E_\pm$ fermions, we observe that (4.28) is trivially satisfied, since $\hat{P} = 0$ on the subspace defined by (4.27). Therefore, the factorization condition (I-2.15) is satisfied for the coupled-channel problem (4.27), and the $E_\pm$ lie on a Regge trajectory. The ease with which we obtained this conclusion should be compared with a direct analysis of the 3×3 matrix f_{nn} for the amplitudes (4.27).

It is also trivial to analyze (4.22) by these methods. Let us consider the condition

$$Cm\hat{P}=0 \tag{4.33}$$

on the subspace defined by (4.22). Then the necessary condition for factorization of (4.22) is

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_0 & G_2 v \\ G_2 v & m_0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \overset{?}{=} 0\,, \tag{4.34a}$$

i.e.,

$$\begin{bmatrix} 0 & G_2 v \\ 0 & 0 \end{bmatrix} \overset{?}{=} 0\,. \tag{4.34b}$$

Therefore, factorization fails for (4.22) and both E^0 and e^0 fail to Reggeize in this model. The role of mixing of ψ_3 with l is clearly evident in (4.34). Again, the efficiency of this method of verifying factorization is apparent.

The next example we consider is the Weinberg-Salam model. For simplicity we concentrate on a single generation of leptons and of quarks. Conclusions are easily drawn for more complicated cases without further computation. The relevant part of the lepton Lagrangian for our purposes is

$$\begin{aligned} \mathcal{L}' = {} & g(\bar{L}\gamma^\mu \tfrac{1}{2}\tau_a L)A^a_\mu \\ & - g'[(\bar{e}_R\gamma^\mu e_R) + \tfrac{1}{2}(\bar{e}_L\gamma^\mu e_L) + \tfrac{1}{2}(\bar{\nu}_L\gamma^\mu \nu_L)]B_\mu \\ & - m_e(\bar{e}_R e_L + \bar{e}_L e_R)\,, \end{aligned} \tag{4.35}$$

with

$$L=\begin{bmatrix} \nu_L \\ e_L \end{bmatrix},$$

where A^a_μ and B_μ are the gauge fields of the SU(2) and U(1) gauge subgroups, respectively. It is more convenient to work in this basis for the vector bosons, rather than in the representation in which the vector-meson mass matrix is diagonal. Therefore we will be interested in examining the factorization criteria in the form (4.4) and (4.5). From (4.35) we have

$$t^L_\alpha = \tfrac{1}{2} g\tau_\alpha\,, \quad \text{for } \alpha=1,2,3$$

and (4.36)

$$t^L_\alpha = -\tfrac{1}{2} g' \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{for } \alpha=4$$

while

$$t^R_\alpha = 0\,, \quad \text{for } \alpha=1,2,3$$

and (4.37)

$$t^R_\alpha = -g' \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{for } \alpha=4\,.$$

Therefore,

$$\begin{aligned} C_L = t^L_\alpha t^L_\alpha = \tfrac{1}{4}(3g^2+g'^2) & \begin{array}{cc} (\nu_L) & (e_L) \end{array} \\ & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ = \tfrac{1}{4} g^2 (3+\tan^2\theta_W) & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \tag{4.38}$$

and

$$C_R = (g')^2 \begin{array}{c} (\nu_R)\;(e_R) \\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \end{array} = g^2\tan^2\theta_W \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{4.39}$$

which implies

$$\hat{P}_L = 0$$

and (4.40)

$$\hat{P}_R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Finally, the mass matrix is

$$m = \begin{array}{c} \begin{array}{cccc} (\nu_L) & (e_L) & (\nu_R) & (e_R) \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_e \\ 0 & 0 & 0 & 0 \\ 0 & m_e & 0 & 0 \end{bmatrix} \end{array}. \tag{4.41}$$

Since

$$\begin{aligned} m\hat{P} & \equiv \begin{bmatrix} 0 & m^{LR} \\ m^{RL} & 0 \end{bmatrix} \begin{bmatrix} P_L & 0 \\ 0 & P_R \end{bmatrix} \\ & = \begin{bmatrix} 0 & m^{LR}\hat{P}_R \\ m^{RL}\hat{P}_L & 0 \end{bmatrix} \equiv 0\,, \end{aligned} \tag{4.42}$$

Eq. (4.5) is trivially satisfied for all α, and the electron and neutrino both Reggeize. More generally, if the lepton sector is described by several generations of left-handed doublets and right-handed singlets, with couplings as in (4.35), and all neutrinos are massless, then all charged leptons and neutrinos Reggeize. For more general models, one proceeds similarly. The contrast between the SU(2) model analyzed in (4.7)–(4.34) and the SU(2) × U(1) model of Weinberg-Salam is striking. In particular, the presence of the U(1) subgroup is crucial to the Reggeization of both the neutrinos and charged leptons, since this means that $C_R \neq 0$, and $\hat{P}_R \neq 1$. The additional fermion-boson channels provided by the B_μ boson increases the rank of f_{nn} so as to allow the Reggeization. The benefits of coupling additional channels in the Reggeization program has been noted in earlier work.[6]

Let us now turn to the quark sector of the Weinberg-Salam model, with just one generation of quarks for simplicity. Then the important part of the interaction Lagrangian is

$$\mathcal{L}' = g(\bar{L}\gamma^\mu \tfrac{1}{2}\tau_a L)A^a_\mu + g'(\bar{u}_R\gamma^\mu u_R + \tfrac{1}{2}\bar{d}_L\gamma^\mu d_L)B_\mu - m_u(\bar{u}_R u_L + \bar{u}_L u_R) - m_d(\bar{d}_R d_L + \bar{d}_L d_R)\,, \quad (4.43)$$

where

$$L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}.$$

Now

$$t^L_\alpha = \tfrac{1}{2} g\tau_\alpha\,, \quad \text{for } \alpha = 1,2,3\,,$$
$$t^L_\alpha = \tfrac{1}{2} g' \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{for } \alpha = 4\,, \quad (4.44)$$

and

$$t^R_\alpha = 0\,, \quad \text{for } \alpha = 1,2,3\,,$$
$$t^R_\alpha = g' \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \text{for } \alpha = 4\,. \quad (4.45)$$

Thus,

$$C_L = \tfrac{1}{4} g^2 \left[3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \tan^2\theta_W \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right] \quad (4.46)$$

and

$$C_R = g^2 \tan^2\theta_W \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\,, \quad (4.47)$$

with

$$\hat{P}_L = 0$$

and

$$\hat{P}_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\,, \quad \text{for } \theta_W \neq 0\,. \quad (4.48)$$

Equations (4.3b) and (4.5b) are trivially satisfied, while (4.5a) requires

$$(t^L_\alpha) \begin{pmatrix} 0 & 0 \\ 0 & m_d \end{pmatrix} \stackrel{?}{=} 0 \text{ for all } \alpha\,, \quad (4.49)$$

which cannot be satisfied for any α, given (4.44), unless $m_d = 0$, which is not the case. Therefore the quarks do *not* Reggeize in the Weinberg-Salam theory.

As our last example consider the "standard model" with gauge group $SU(2)\times U(1)\times SU(3)$, where we will see immediately that the additional couplings of the SU(3) of color enable *all* fermions to Reggeize. The reason is that for the quarks we now have

$$\det C_R \neq 0\,, \qquad \det C_L \neq 0\,, \quad (4.50)$$

and

$$\hat{P}_L = \hat{P}_R = 0\,, \quad (4.51)$$

due to the enlargement of the gauge group. This now allows (4.5) to be satisfied for the quarks without constraint on the flavor content or mass spectrum of the quark sector. Furthermore, the analysis of the lepton sector is identical to that of (4.35)–(4.42), since the leptons are colored singlets. More generally, we see that if we encounter a situation in which some fermions do not Reggeize, if one can enlarge the gauge group so that $\hat{P}_L = \hat{P}_R = 0$ due to the additional gauge couplings, then Reggeization will be achieved. It should be remarked that once the massless colored gluons are considered, then one encounters possible infrared singularities in the Regge trajectory functions.[1,3,7] We conjectured in I that these infrared singularities could be removed if *inclusive* processes were considered in this program.

In a GU theory[4] such as SU(5), where one has the option of assigning each generation of fermions to the $5+\overline{10}$ representation of the gauge group, all fermions Reggeize since $\det C_R \neq 0$, $\det C_L \neq 0$, and $\hat{P}_R = \hat{P}_L = 0$. Therefore in this version of grand unified theories, *all* fermions and *all* gauge bosons lie on Regge trajectories.[8] Although these conclusions may not have any immediate practical consequences, they have a great deal of aesthetic appeal. Other GU theories, with their particular fermion representation content, may also be analyzed by the methods of this paper.

The most immediate consequence would occur if perturbation theory breaks down above 1 TeV in the unified electroweak theory. Then one will expect to see $J=\frac{3}{2}$ recurrences of the elementary leptons in the 2-to-4-TeV region. However, the widths of these lepton resonances are expected to be enormous due to the large available phase space. Therefore, it remains a challenge to find a way in which the possible lepton and intermediate-vector-boson Regge recurrences make their presence known above 1 TeV.

ACKNOWLEDGMENTS

The research of M.T.G. was supported in part by the National Science Foundation under Grant No. PHY78-07757. The work of H.J.S. was supported in part by the U.S. Department of Energy under Contract No. E(11-1).

APPENDIX A

This Appendix is devoted to an extension, for the case when P and CP conservation is *not* assumed, of the formalism of Gell-Mann, Goldberger, Low, Marx, and Zachariasen (GGMLZ)[3] for defining signatured, natural- and unnatural-parity helicity amplitudes free of kinematical singularities. We follow closely their procedure and obtain results which reduce to theirs if P and CP are conserved.

We begin by defining two-body helicity states $|J,\lambda_i\rangle \equiv |J,M,\lambda_1,\lambda_2\rangle$ of given total J, J_z and particle helicities λ_1,λ_2. Under the parity operation we have

$$P|J,\lambda_i\rangle = \eta\,|J,-\lambda_i\rangle\,, \tag{A1}$$

where $\eta=\eta_1\eta_2(-1)^{J-s_1-s_2}$ and s_1, s_2 are the spins of the two particles. The helicity scattering amplitude is defined by

$$f_{\lambda'_i,\lambda_j}=\frac{1}{\sqrt{KK'}}\sum_J (2J+1)\langle\lambda'_i|F^J|\lambda_j\rangle d^J_{\lambda\lambda'}(\theta)\,, \tag{A2}$$

where $\langle\lambda'_i|F^J|\lambda_j\rangle=\langle J,\lambda'_i|S-1|J,\lambda_j\rangle$, K (K') is the initial (final) center-of-mass momentum, and $\lambda=\lambda_1-\lambda_2$, $\lambda'=\lambda'_1-\lambda'_2$.

We define the parity combinations

$$|J,\lambda_i\rangle_\pm=\frac{1}{\sqrt{2}}\left[\,|J,\lambda_i\rangle\pm\bar\eta\,|J,-\lambda_i\rangle\right]\,, \tag{A3}$$

with $\bar\eta=\eta_1\eta_2(-1)^{-v+s_1+s_2}$, $v=0$ or $\frac{1}{2}$ if J is integer or half-integer, and the amplitudes

$$F^{J(\epsilon',\epsilon)}_{\lambda'_i,\lambda_j}={}_{\epsilon'}\langle\lambda'_i|F^J|\lambda_j\rangle_\epsilon\,,\quad \epsilon,\epsilon'=\pm\,. \tag{A4}$$

For example,

$$\begin{aligned}F^{J(+,+)}_{\lambda'_i,\lambda_j}=\langle\lambda'_i|F^J|\lambda_j\rangle&+\bar\eta\langle\lambda'_i|F^J|-\lambda_j\rangle\\&+\bar\eta'\langle-\lambda'_i|F^J|\lambda_j\rangle+\bar\eta\bar\eta'\langle-\lambda'_i|F^J|-\lambda_j\rangle\,.\end{aligned} \tag{A5}$$

Note that if parity is conserved the terms on the right-hand side of (A5) are pairwise equal, and $F^{+-}=F^{-+}=0$. After some algebra we can express the scattering amplitude as

$$\begin{aligned}f_{\lambda'_i\lambda_j}=\frac{1}{\sqrt{KK'}}\sum_J(2J+1)[F^{J++}&+F^{J+-}\\&+F^{J-+}+F^{J--}]d^J_{\lambda'\lambda}(\theta)\,.\end{aligned} \tag{A6}$$

The usual kinematical-singularity-free amplitude is

$$\begin{gathered}\tilde f_{\lambda'_i\lambda_j}=(c)^{-|\lambda+\lambda'|}(s)^{-|\lambda-\lambda'|}f_{\lambda'_i\lambda_j}\,,\\ c=\sqrt{2}\cos\tfrac{1}{2}\theta\,,\quad s=\sqrt{2}\sin\tfrac{1}{2}\theta\,.\end{gathered} \tag{A7}$$

It is convenient, however, to define "parity" amplitudes similar to those of GGLMZ. They are given by the following combinations:

$$f^{(\epsilon',\epsilon)}_{\lambda'_i\lambda_j}=[\,\tilde f_{\lambda'_i\lambda_j}+\epsilon\epsilon'(-1)^{\lambda'-\lambda}\bar\eta\,\bar\eta'\tilde f_{-\lambda'_i-\lambda_j}]+\epsilon'(-1)^{\lambda+\lambda_m}\bar\eta'[\,\tilde f_{-\lambda'_i\lambda_j}+\epsilon\epsilon'(-1)^{\lambda'-\lambda-2\lambda_m}\bar\eta\,\bar\eta' f_{\lambda'_i-\lambda_j}]\,, \tag{A8}$$

where $\lambda_m=\max(|\lambda|,|\lambda'|)$. They can be expressed as in GGLMZ, as an expansion in terms of the functions $e^{J\pm}_{\lambda\lambda'}$ that they introduce:

$$f^{\pm\pm}_{\lambda'\lambda}=\frac{1}{\sqrt{KK'}}\sum(2J+1)[F^{J\pm\pm}_{\lambda'\lambda}e^{J+}_{\lambda\lambda'}(z)+F^{J\mp\mp}_{\lambda'\lambda}e^{J-}_{\lambda\lambda'}(z)]\,, \tag{A9a}$$

$$f^{\pm\mp}_{\lambda'\lambda}=\frac{1}{\sqrt{KK'}}\sum(2J+1)[F^{J\pm\mp}_{\lambda'\lambda}e^{J+}_{\lambda\lambda'}(z)+F^{J\mp\pm}_{\lambda'\lambda}e^{J-}_{\lambda\lambda'}(z)]\,, \tag{A9b}$$

with an inversion formula exactly as in GGLMZ:

$$F^{J\epsilon'\epsilon}=\frac{\sqrt{KK'}}{2}\int_{-1}^{1}dz\,[c^{J+}_{\lambda\lambda'}(z)f^{\epsilon'\epsilon}_{\lambda'\lambda}(z)+c^{J-}_{\lambda\lambda'}(z)f^{-\epsilon'-\epsilon}_{\lambda'\lambda}(z)]\,. \tag{A10}$$

If parity is conserved $f^{\pm\mp}=0$. In the form of Eq. (A9), the high-energy behavior of $f^{\epsilon'\epsilon}$ is determined by Regge poles in $F^{J\epsilon'\epsilon}$.

APPENDIX B

In this Appendix we give details of the calculation of helicity amplitudes, in Born approximation, for vector-spinor scattering.

We work in the center of mass, with the definitions

$$\begin{aligned}k_A&=(E_A,0,0,K_A)\,,\\ k_B&=(E_B,K_By,0,K_Bz)\,,\\ p_1&=(\mathcal{E}_i,0,0,-K_A)\,,\\ p_2&=(\mathcal{E}_j,-K_By,0,-K_Bz)\,,\end{aligned} \tag{B1}$$

for the momenta of vectors and spinors, respectively, with $z=\cos\theta$, $y=\sin\theta$. The total center-of-mass energy is $W=\mathcal{E}_i+E_A=\mathcal{E}_j+E_B$.

We use the following expressions for the helicity wave functions:

$$\begin{aligned}
\epsilon^{\lambda}(K_A) &= -\frac{\lambda}{\sqrt{2}}(0,1,i\lambda,0), \quad \lambda=\pm 1, \\
&= \frac{1}{\mu_A}(K_A,0,0,E_A), \quad \lambda=0, \\
\epsilon^{\lambda'}(K_B) &= \frac{\lambda'}{\sqrt{2}}(0,z,i\lambda,-y), \quad \lambda'=\pm 1, \\
&= \frac{1}{\mu_B}(K_B, yE_B, 0, zE_B), \quad \lambda'=0,
\end{aligned} \tag{B2}$$

$$u^{\mu}(p_1) = [2m_i(\mathcal{E}_i+m_i)]^{-1/2}\begin{pmatrix}\mathcal{E}_i+m_i \\ \mu K_A\end{pmatrix}\otimes \chi_{-\mu}, \quad \mu=\pm\tfrac{1}{2},$$

$$\bar{u}^{\mu'}(p_2) = [2m_j(\mathcal{E}_j+m_j)]^{-1/2}\begin{pmatrix}\mathcal{E}_j+m_j \\ -\mu K_B\end{pmatrix}\otimes \bar{\chi}_{-\mu'}, \quad \mu'=\pm\tfrac{1}{2}, \tag{B3a}$$

$$\chi_- = \begin{pmatrix}0\\1\end{pmatrix}, \quad \chi_+ = \begin{pmatrix}1\\0\end{pmatrix},$$

$$\chi'_- = \begin{pmatrix}-\sin\frac{1}{2}\theta \\ \cos\frac{1}{2}\theta\end{pmatrix}, \quad \chi'_+ = \begin{pmatrix}\cos\frac{1}{2}\theta \\ \sin\frac{1}{2}\theta\end{pmatrix}. \tag{B3b}$$

Our γ matrices are such that

$$\gamma\cdot v = \begin{pmatrix} v_0 & -\sigma\cdot v \\ \sigma\cdot v & -v_0\end{pmatrix}, \quad \gamma_5 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}. \tag{B4}$$

We note the following useful relations

$$\gamma\cdot\epsilon^{\lambda}u^{\mu}=0, \quad \bar{u}^{\mu'}\gamma\cdot\epsilon^{\lambda'*}=0, \tag{B5}$$

for (λ,μ) or (λ',μ') in the nonsense combinations $(1,-\frac{1}{2})$ and $(-1,\frac{1}{2})$. They imply in particular, that only the first term in Eq. (2.34) contributes to the nonsense-nonsense amplitude. We also have

$$p_1\cdot\epsilon^{\pm}(K_A) = p_2\cdot\epsilon^{\pm}(K_B) = 0. \tag{B6}$$

Further relations we shall use are

$$\epsilon^{\lambda}\cdot\epsilon^{\lambda'*} = -\tfrac{1}{2}\lambda\lambda'(\lambda\lambda'+z), \tag{B7}$$

for $\lambda,\lambda'=\pm 1$, and

$$\begin{aligned}
\bar{u}^{\mu'}u^{\mu} &= N[(\mathcal{E}+m)(\mathcal{E}'+m') - \mu\mu'KK']\bar{\chi}_{-\mu'}\chi_{-\mu}, \\
\bar{u}^{\mu'}\gamma_0 u^{\mu} &= N[(\mathcal{E}+m)(\mathcal{E}'+m') + \mu\mu'KK']\bar{\chi}_{-\mu'}\chi_{-\mu}, \\
\bar{u}^{\mu'}\gamma_5 u^{\mu} &= N[(\mathcal{E}'+m')\mu K - (\mathcal{E}+m)\mu'K']\bar{\chi}_{-\mu'}\chi_{-\mu}, \\
\bar{u}^{\mu'}\gamma_0\gamma_5 u^{\mu} &= N[(\mathcal{E}'+m')\mu K + (\mathcal{E}+m)\mu'K']\bar{\chi}_{-\mu'}\chi_{-\mu}, \\
\bar{u}^{\mu'}\gamma\cdot\epsilon^{\lambda}u^{\mu} &= \sqrt{2}\lambda N[(\mathcal{E}'+m')\mu K \\
&\qquad + (\mathcal{E}+m)\mu'K']\bar{\chi}_{-\mu'}\sigma^{\lambda}\chi_{-\mu}, \\
\bar{u}^{\mu'}\gamma\cdot\epsilon^{\lambda}\gamma_5 u^{\mu} &= \sqrt{2}\lambda N[(\mathcal{E}'+m')(\mathcal{E}+m) \\
&\qquad + \mu\mu'KK']\bar{\chi}_{-\mu'}\sigma^{\lambda}\chi_{-\mu},
\end{aligned} \tag{B8}$$

for $\lambda=\pm 1$, with $\sigma^{\lambda}=\sigma_x+i\lambda\sigma_y$ and $N=[4mm'(\mathcal{E}+m)(\mathcal{E}'+m')]^{-1/2}$.

Because of simplifications in the algebra we shall compute the helicity amplitudes only for transverse meson helicity states. We begin with the nonsense-nonsense amplitudes, $\lambda=\pm 1$, $\mu=\mp\frac{1}{2}$; $\lambda'=\pm 1$, $\mu'=\mp\frac{1}{2}$ for which, as mentioned above, only the first term in Eq. (2.34) contributes. (The term D_1 in (2.33) contributes only to the sense-sense amplitudes, since it does not contain a z^{-1} factor.) We write

$$\bar{t} = \tfrac{1}{2}(1+\gamma_5)\bar{t}^R + \tfrac{1}{2}(1-\gamma_5)\bar{t}^L, \tag{B9}$$

and find that for large z the relevant part of D in (2.33) is

$$T = \frac{i\epsilon\cdot\epsilon^*}{2K_AK_Bz}\{\bar{u}\gamma\cdot(p_1+k_A)[(\bar{t}_B^R\bar{t}_A^R + \bar{t}_B^L\bar{t}_A^L) + \gamma_5(\bar{t}_B^R\bar{t}_A^R - \bar{t}_B^L\bar{t}_A^L)]u + \bar{u}[(\bar{t}_B^R m\bar{t}_A^L + \bar{t}_B^L m\bar{t}_A^R) - \gamma_5(\bar{t}_B^R m\bar{t}_A^L - \bar{t}_B^L m\bar{t}_A^R)]u\}. \tag{B10}$$

In the center of mass $\gamma\cdot(p_1+k_A)=\gamma_0 W$, so that we can read the various matrix elements from Eq. (B8). We evaluate the expression in Eq. (B9) for the various helicity configurations so as to obtain the quantities

$$f_{++}=f_{+1,-1/2;+1,-1/2}, \quad f_{+-}=f_{+1,-1/2;-1,+1/2}, \quad f_{-+}=f_{-1,+1/2;+1,-1/2}, \quad f_{--}=f_{-1,-1/2;-1,-1/2},$$

remove kinematical singularities according to Eq. (A7), and define parity combinations according to (A8). We find, for *the coefficient of* z^{-1},

$$\begin{aligned}
f^{++}_{3/2,3/2} &= \bar{N}(\mathcal{E}_j+m_j)(\mathcal{E}_i+m_i)[W(\bar{t}_B^R\bar{t}_A^R + \bar{t}_B^L\bar{t}_A^L) - (\bar{t}_B^R m\bar{t}_A^L + \bar{t}_B^L m\bar{t}_A^R)], \\
f^{--}_{3/2,3/2} &= \bar{N}K_BK_A[W(\bar{t}_B^R\bar{t}_A^R + \bar{t}_B^L\bar{t}_A^L) + (\bar{t}_B^R m\bar{t}_A^L + \bar{t}_B^L m\bar{t}_A^R)], \\
f^{-+}_{3/2,3/2} &= \bar{N}K_B(\mathcal{E}_i+m_i)[W(\bar{t}_B^R\bar{t}_A^R - \bar{t}_B^L\bar{t}_A^L) - (\bar{t}_B^R m\bar{t}_A^L - \bar{t}_B^L m\bar{t}_A^R)], \\
f^{+-}_{3/2,3/2} &= \bar{N}(\mathcal{E}_j+m_j)K_A[W(\bar{t}_B^R\bar{t}_A^R - \bar{t}_B^L\bar{t}_A^L) + (\bar{t}_B^R m\bar{t}_A^L - \bar{t}_B^L m\bar{t}_A^R)],
\end{aligned} \tag{B11}$$

where

$$\bar{N} = i2\sqrt{2}[2m_j(\mathcal{E}_j+m_j)2m_i(\mathcal{E}_i+m_i)]^{-1/2}(2K_AK_B)^{-1}. \tag{B12}$$

We compute next the sense-nonsense amplitudes for final transverse mesons, $\lambda=\pm 1$, $\mu=\mp\frac{1}{2}$; $\lambda'=\pm 1$,

$\mu' = \pm\frac{1}{2}$. We still have $\gamma\cdot\epsilon u = 0$, but $\bar{u}\gamma\cdot\epsilon^* \neq 0$. We decompose t as in (B9) and find that the relevant part of the amplitude is

$$T' = T - ik_B\cdot\epsilon\bar{u}[(\bar{t}_B^R\bar{t}_A^R + \bar{t}_B^L\bar{t}_A^L) + \gamma_5(\bar{t}_B^R\bar{t}_A^R - \bar{t}_B^L\bar{t}_A^L)]\gamma\cdot\epsilon^* u\,, \tag{B13}$$

where T is given by Eq. (B10). Using Eq. (B8) and the definitions in Eqs. (A7) and (A8), we find for the coefficient of z^{-1}:

$$\begin{aligned}
\tilde{f}^{++}_{1/2,3/2} &= \bar{N}(\mathcal{E}_j + m_j)(\mathcal{E}_i + m_i)[\{W - 2(\mathcal{E}_j - m_j)\}(\bar{t}_B^R\bar{t}_A^R + \bar{t}_B^L\bar{t}_A^L) - (\bar{t}_B^R m\bar{t}_A^L + \bar{t}_B^L m\bar{t}_A^R)]\,,\\
\tilde{f}^{--}_{1/2,3/2} &= -\bar{N}K_BK_A[\{W - 2(\mathcal{E}_j + m_j)\}(\bar{t}_B^R\bar{t}_A^R + \bar{t}_B^L\bar{t}_A^L) + (\bar{t}_B^R m\bar{t}_A^L + \bar{t}_B^L m\bar{t}_A^R)]\,,\\
\tilde{f}^{-+}_{1/2,3/2} &= \bar{N}(\mathcal{E}_i + m_i)K_B[\{W - 2(\mathcal{E}_j + m_j)\}(\bar{t}_B^L\bar{t}_A^L - \bar{t}_B^L\bar{t}_A^L) - (\bar{t}_B^R m\bar{t}_A^L - \bar{t}_B^L m\bar{t}_A^R)]\,,\\
\tilde{f}^{+-}_{1/2,3/2} &= -\bar{N}(\mathcal{E}_j + m_j)K_A[\{W - 2(\mathcal{E}_j - m_j)\}(\bar{t}_B^R\bar{t}_A^R - \bar{t}_B^L\bar{t}_A^L) + (\bar{t}_B^R m\bar{t}_A^L - \bar{t}_B^L m\bar{t}_A^R)]\,.
\end{aligned} \tag{B14}$$

Finally we need the sense-sense amplitude for $\lambda = \pm 1$, $\mu = \pm\frac{1}{2}$; $\lambda' = \pm 1$, $\mu' = \pm\frac{1}{2}$. Rather than compute the whole amplitude, we restrict ourselves to the part which actually exhibits the direct (s) channel fermion pole, as given by D_1:

$$\begin{aligned}
T'' = -\tfrac{1}{2}i\bar{u}\gamma\cdot\epsilon^*\Big[\gamma\cdot(p_1 + k_A)\Big(\bar{t}_B^R\frac{1}{s-m^2}\bar{t}_A^R + \bar{t}_B^L\frac{1}{s-m^2}\bar{t}_A^L\Big) - \gamma_5\Big(\bar{t}_B^R\frac{1}{s-m^2}\bar{t}_A^R - \bar{t}_B^L\frac{1}{s-m^2}\bar{t}_A^L\Big)\\
+ \Big(\bar{t}_B^R\frac{m}{s-m^2}\bar{t}_A^L + \bar{t}_B^L\frac{m}{s-m^2}\bar{t}_A^R\Big) + \gamma_5\Big(\bar{t}_B^R\frac{m}{s-m^2}\bar{t}_A^L - \bar{t}_B^L\frac{m}{s-m^2}\bar{t}_A^R\Big)\Big]\gamma\cdot\epsilon u\,.
\end{aligned} \tag{B15}$$

We replace $\gamma\cdot(p + k_A)$ by $\gamma_0 W$, and using Eqs. (B8), (A7), and (A8) find

$$\begin{aligned}
\tilde{f}^{++}_{1/2,1/2} &= 4\bar{N}(\mathcal{E}_j + m_j)(\mathcal{E}_i + m_i)\left[W\left(\bar{t}_B^R\frac{1}{s-m^2}\bar{t}_A^R + \bar{t}_B^L\frac{1}{s-m^2}\bar{t}_A^L\right) + \left(\bar{t}_B^R\frac{m}{s-m^2}\bar{t}_A^L + \bar{t}_B^L\frac{m}{s-m^2}\bar{t}_A^R\right)\right],\\
\tilde{f}^{--}_{1/2,1/2} &= 4\bar{N}K_BK_A\left[W\left(\bar{t}_B^R\frac{1}{s-m^2}\bar{t}_A^R + \bar{t}_B^L\frac{1}{s-m^2}\bar{t}_A^L\right) - \left(\bar{t}_B^R\frac{m}{s-m^2}\bar{t}_A^L + \bar{t}_B^L\frac{m}{s-m^2}\bar{t}_A^R\right)\right],\\
\tilde{f}^{-+}_{1/2,1/2} &= 4\bar{N}(\mathcal{E}_i + m_i)K_B\left[W\left(\bar{t}_B^R\frac{1}{s-m^2}\bar{t}_A^R - \bar{t}_B^L\frac{1}{s-m^2}\bar{t}_A^L\right) + \left(\bar{t}_B^R\frac{m}{s-m^2}\bar{t}_A^L - \bar{t}_B^L\frac{m}{s-m^2}\bar{t}_A^R\right)\right],\\
\tilde{f}^{+-}_{1/2,1/2} &= 4\bar{N}K_A(\mathcal{E}_j + m_j)\left[W\left(\bar{t}_B^R\frac{1}{s-m^2}\bar{t}_A^R - \bar{t}_B^L\frac{1}{s-m^2}\bar{t}_A^L\right) - \left(\bar{t}_B^R\frac{m}{s-m^2}\bar{t}_A^L - \bar{t}_B^L\frac{m}{s-m^2}\bar{t}_A^R\right)\right].
\end{aligned} \tag{B16}$$

Equations (B11), (B14), and (B16) can then be rewritten in the form of Eqs. (3.2)–(3.7) of the main text.

---

[1]M. T. Grisaru and H. J. Schnitzer, Phys. Rev. D 20, 784 (1979). This paper, denoted as I, contains extensive references to earlier work on this subject.

[2]D. A. Dicus and V. S. Mathur, Phys. Rev. D 7, 3111 (1973); M. Veltman, Acta Phys. Pol. B8, 475 (1977); Phys. Lett. 70B, 253 (1977); B. W. Lee *et al.*, Phys. Rev. Lett. 38, 883 (1977); Phys. Rev. D 16, 1519 (1977); M. Green, Nucl. Phys. B153, 187 (1979); C. E. Vayonakis, Lett. Nuovo Cimento 17, 383 (1976).

[3]M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964); M. Gell-Mann, M. L. Goldberger, F. E. Low, V. Singh, and F. Zachariasen, *ibid.* 133, B161 (1964); S. Mandelstam, *ibid.* 137, B944 (1965); E. Abers and V. L. Teplitz, *ibid.* 158, 1365 (1967); M. T. Grisaru, H. J. Schnitzer, and H.-S. Tsao, *ibid.* 8, 4498 (1973).

[4]H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974); H. Georgi, H. Quinn, and S. Weinberg, *ibid.* 33, 451 (1974); A. J. Buras *et al.*, Nucl. Phys. B135, 66 (1978); H. Georgi and D. V. Nanopoulos, Phys. Lett. 82B, 95 (1979); M. Gell-Mann, P. Ramond, and R. Slansky, Rev. Mod. Phys. 50, 721 (1978).

[5]S. Weinberg, Phys. Rev. D 7, 1068 (1973); 7, 2887 (1973); 8, 605 (1973); 8, 4482 (1973).

[6]M. T. Grisaru, H. J. Schnitzer, and H.-S. Tsao, Phys. Rev. D 9, 2864 (1974).

[7]A. N. Chester, Phys. Rev. 143, 1275 (1966); G. M. Cicuta, Phys. Rev. Lett. 43, 826 (1979).

[8]Reggeization of the gauge mesons in a general case has also been discussed by L. Łukaszuk and L. Symanowski, Nucl. Phys. B159, 316 (1979). See also L. Łukaszuk, Phys. Lett. 64B, 311 (1976).