PHYSICAL REVIEW D VOLUME 21, NUMBER 7 1 APRIL 1980

Quark-lepton unification in SU($N > 5$)

N. S. Baaklini
International Centre for Theoretical Physics, Trieste, Italy
(Received 14 November 1979; revised manuscript received 26 November 1979)

We discuss a class of flavor-unification models for quarks and leptons based on the unitary groups SU($N>5$). The spontaneous breaking of SU(N) is proposed to go via $SU(3)_c \times [SU(N-3)\times U(1)]_f$, then through successive stages down to $SU(3)_c \times SU(2) \times U(1)$. Our models are anomaly-free and have the distinctive feature of associating several left-handed neutral partners to charged leptons. Quark-lepton assigments, weak currents, and fermion mass generation are discussed for SU(6), SU(7), and SU(8). The embedding of the SU(6) model in E_6 is also indicated. The SU(7) model is noted as the most economical three-generation model (49 chiral fields).

I. INTRODUCTION

The gauge symmetry group $SU(3)_c \times U(1)_{em}$, which governs the strong (color)[1,2] and the electromagnetic interactions of quarks and leptons, is believed to be exact at all energies. On the other hand, known weak interactions are governed[3-5] by the unified electroweak gauge group $SU(2)\times U(1)$, which is spontaneously broken down to $U(1)_{em}$ and is expected to be exact at energies $>10^2$ GeV.

The embedding of $SU(3)_c \times SU(2)\times U(1)$ in the simple group SU(5) has offered[6] an elegant unification scheme for each generation of quarks and leptons. This grand symmetry is expected to be exact at ultrahigh energies $>10^{15}$ GeV. Furthermore, by insisting[7] that SU(5) at its symmetric limit, must be a subgroup of a spontaneously broken grand unification group, for several generations of quarks and leptons, flavor-unification symmetries would be exact at energy scales even higher than 10^{15} GeV. Hence, between 10^2 and 10^{15} GeV, a grand plateau is thus far predicted.

Our purpose in this paper is to consider a class of SU($N>5$) symmetries which can describe several generations of quarks and leptons and which will be broken down, at first stage, to $SU(3)_c \times [SU(N-3)\times U(1)]_f$ rather than passing through an SU(5)-symmetric stage. In this way, we shall interpret the factor subgroup $[SU(N-3) \times U(1)]_f$ as an extended electroweak (flavor) gauge group which would be broken down, through successive stages, to $SU(2)\times U(1)$. Hence, several energy scales and new physical effects will replace[8] the plateau between 10^2 and 10^{15} GeV.

II. GENERAL SCHEME

We shall discuss in the following sections models based on the groups SU(6), SU(7), and SU(8). However, before doing so we list some general features of our scheme.

A. Fermion multiplets

The restriction of color embedding[9] that no representations of $SU(3)_c$ containing other than 3^c (quarks), $\bar{3}^c$ (antiquarks) and 1^c (leptons) should occur leads to the choice of the totally antisymmetric representations $[m] = (N\wedge N\wedge \ldots \wedge N)$ (m N's) for the fermion multiplets.

B. Electric charge operator

Consistently with the choice $[m]$ for the fermion multiplets and requiring that no (exotic) charges other than $\frac{2}{3}$ and $-\frac{1}{3}$ for quarks and 0 and -1 for leptons should occur, the diagonal charge operator can have only the following extension of the SU(5) model,[6] $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1, 0, 0, \ldots, 0)$ ($N-4$ zeros), for its eigenvalues.

In our scheme, we are mainly interested in the left-handed fermion representation $\underline{N}_L$ and the right-handed representations $[2]_R$ and $[3]_R$. The representation $[2]_R$ contains, according to the above choice of the charge operator and for all $N \geq 4$, only one (tricolored) quark of charge $\frac{2}{3}$ (plus antiquark) and $(N-4)$ charge $\frac{1}{3}$ antiquarks. The representation $[3]_R$ contains $(N-4)$ charge $\frac{2}{3}$ quarks (plus antiquarks) and a single charge $-\frac{1}{3}$ quark (plus $\frac{1}{2}(N-4)(N-5)$ antiquarks).

C. Absence of triangle anomalies

The value of the triangle anomaly[10,11] for the representation $[m]$ is given[12] by

$$A([m]) = \frac{(N-2m)(N-3)!}{(N-m-1)!\,(m-1)!}. \tag{1}$$

Hence we obtain

$$\begin{aligned} &A(\underline{N}_L) = -1\,,\\ &A([2]_R) = (N-4)\,,\\ &A([3]_R) = 1/2(N-3)(N-6)\,. \end{aligned} \tag{2}$$

Thus we utilize $(N-4)$ multiplets $\underline{N}_L$ to cancel the anomaly of $[2]_R$ and $\frac{1}{2}(N-3)(N-6)$ of them to cancel that of $[3]_R$. Note that in both cases, the numbers of right-handed charged leptons and charge $\frac{1}{3}$ antiquarks match those of the left-handed parts in the multiplets $\underline{N}_L$ utilized.

D. Spontaneous symmetry breaking

The first stage of symmetry breaking is envisaged to be $SU(N) \to SU(3)_c \times [SU(N-3) \times U(1)]_f$. This breaking is achieved via a Higgs multiplet in the adjoint representation $\underline{(N^2-1)}$. It is clear that the energy scale involved at this stage is >10^{15} GeV, since we must take care of proton decay, just as in the SU(5) model.[6] The remaining $(N-4)$ stages,

$$\begin{aligned} SU(N-3)\times U(1) &\longrightarrow SU(N-4)\times U(1) \\ &\;\;\cdots \\ &\longrightarrow SU(2)\times U(1) \\ &\longrightarrow U(1)_{em}, \end{aligned} \tag{3}$$

would all proceed utilizing complex Higgs multiplets in the fundamental $\underline{N}$ representation. From the electric charges of the fundamental representation, $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1, 0, \ldots, 0)$, we know that there are just enough neutral components which can trigger the above $(N-4)$ breaking stages respectively. The energy scales involved are expected between 10^{15} and 10^2 GeV. Their actual orders of magnitude should be determined from various quark-flavor- and lepton-flavor-changing processes which are highly suppressed relative to ordinary weak processes.

E. Fermion masses

As long as the only surviving symmetry is $SU(3)_c \times U(1)_{em}$, we do not think that there is as yet a fundamental principle which allows a grand unified model of predicting fermion mass relationships. There is nothing fundamental about constraining the choice of the fermion-mass-generating Higgs fields. In our scheme, all the representations which have neutral and color-singlet components and occur in the products of the fermion representations, $\underline{N}\times\underline{N}$, $[2]\times[2]$ (or $[3]\times[3]$), and $[2]\times\overline{\underline{N}}$ (or $[3]\times\overline{\underline{N}}$), can serve to generate fermion masses via Yukawa couplings. All fermions occurring with both chiralities may be given arbitrary masses.

F. Neutral leptons

From the above proposed assignments, it is clear that our $[SU(N-3)\times U(1)]_f$ flavor subgroup associates several left-handed neutral partners to charged leptons. Most of these may acquire mass by conspiring with right-handed neutrals and/or antiparticles, presumably included among the neutrals in the same representation. The number of the remaining massless neutrinos can be arranged so that no conflict arises with the cosmological bounds.[13]

G. Weak currents and transitions

Within the (low-energy) weak phenomenology governed by the standard $SU(2)\times U(1)$, our models contain right-handed $(V+A)$ charged-current transitions such as $x^0 \to l^-$, $\sigma^- \to \nu_\sigma$ and $q(\frac{2}{3}) \to q(\frac{1}{3})$. Here l^- is a charged lepton (e^-, μ^-, τ^-), σ^- and x^0 are heavy leptons, and the q's are heavy quarks. The rest of the $SU(2)\times U(1)$ sector is conventional.

Beyond $SU(2)\times U(1)$, we note that since flavor unification will take place in our scheme via right-handed multiplets, then lepton-flavor-changing transitions are expected to involve right-handed $(V+A)$ currents, while flavor-changing transitions of charge $-\frac{1}{3}$ quarks are $(V-A)$, since the antiquarks occur in the multiplet. However, since the charge $\frac{2}{3}$ quarks occur together with their antiparticles in the multiplet, then both $(V-A)$ and $(V+A)$ currents are involved in their flavor-changing transitions.

The masses of the vector bosons mediating flavor-changing transitions, or rather the energy scales involved in the intermediate symmetry-breaking stages between 10^{15} and 10^2 GeV, could in principle be determined from flavor-changing decays of heavy leptons and hadrons. In this respect, the present experimental situation is rather poor. As far as the leptonic sector is concerned, the decay $\mu \to e+\gamma$ (and consequently $\mu \to ee\bar{e}$) occurs through the one-loop Feinberg graphs,[14] where the mediating particles are a vector boson, say X, and a massive neutral fermion, say f^0. The branching ratio $R = T(\mu \to e\gamma)/T(\mu \to e\nu\bar{\nu})$ is of the order of $10^{-4}(M_W/M_X)^4$. Note that 10^{-4} for $M_X = M_W$ was the expected branching ratio[14] when ν_e and ν_μ were thought to be the same particle, and $\mu \to e\gamma$ would have proceeded via the mediation of ν and W. The present[15] experimental limit on R is $<10^{-11}$. Hence, $M_X > (10^2-10^3)M_W$ or $M_X > 10^4-10^5$ GeV.

Besides $\mu \to e\gamma$, other leptonic decays such as $x^0 \to l^- + e^+ + \nu_e'$ and $\mu \to e + \nu_e + \nu_e'$ are expected as well.

On the other hand, flavor-changing hadronic decays[15,16] which have experimental limits are $K^0 \to e\mu$ and $K^+ \to \pi^+ e^- \mu^+$. These have branching limits $<2\times10^{-9}\%$ and $<5\times10^{-9}\%$, respectively. These limits correspond to energy scales, again of the order of 10^4-10^5 GeV.

H. The weak mixing angle

From the electric-charge and weak-isospin eigenvalues of the fundamental representation, it is clear that the canonical value of the $SU(2)\times U(1)$ mixing angle is essentially the same as in the

SU(5) model,[6] $\sin^2\theta_0 = \mathrm{Tr}(T_3{}^2)/\mathrm{Tr}(Q^2) = \frac{3}{8}$. The renormalized value,[17] in the SU(5) model is $\sin^2\theta \approx 0.2$, which is slightly lower than the present[8] experimental value of ≈ 0.23. Since our scheme of breaking the grand unified symmetry SU($N > 5$) will introduce several mass scales associated with the breaking stages between 10^{15} and 10^2 GeV, the value of the angle is expected[8] to receive further renormalization effects. This delicate problem deserves a careful treatment.

III. THE SU(6) MODEL

A. Lepton-quark assignments

The fermions in this model are represented in two left-handed $\underline{6}_L$ and one right-handed $\underline{15}_R$ (or $[2]_R$). The $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_f$ content of these representations is

$$\underline{6}_L = (\underline{3}, \underline{1})_L + (\underline{1}, \underline{3})_L \,, \tag{4}$$

$$\underline{15}_R = (\underline{\bar{3}}, \underline{1})_R + (\underline{3}, \underline{3})_R + (\underline{1}, \underline{\bar{3}})_R \,. \tag{5}$$

Hence we propose the assignments

$$\underline{6}_L \begin{cases} (\underline{3}, \underline{1})_L = \bar{d}^i_L \quad (i = 1, 2, 3 \text{ is a color index}), \\ (\underline{1}, \underline{3})_L = (e^-, \nu_e; \nu'_e)_L \,, \end{cases} \tag{6}$$

$$\underline{6}'_L \begin{cases} (\underline{3}, \underline{1})'_L = \bar{b}^i_L \,, \\ (\underline{1}, \underline{3})'_L = (\mu^-, \nu_\mu; \nu'_\mu)_L \,, \end{cases} \tag{7}$$

$$\underline{15}_R \begin{cases} (\underline{3}, \underline{1})_R = u^i_R \,, \\ (\underline{3}, \underline{3})_R = (\bar{u}^i, \bar{d}^i; \bar{b}^i)_R \,, \\ (\underline{1}, \underline{\bar{3}})_R = (x^0, \mu^-; e^-)_R \,. \end{cases} \tag{8}$$

The semicolon serves to partition the doublets and the singlets with respect to the SU(2) sector.

The strange and the charm quarks as well as the τ lepton may be included in another generation via the replacements

$$\begin{gathered} u^i \to c^i, \quad d^i \to s^i, \quad b^i \to a^i \,, \\ e \to \tau, \quad \mu \to \sigma, \quad x^0 \to \lambda^0 \,. \end{gathered} \tag{9}$$

Here σ^- is a new lepton and a^i is a new quark of charge $-\frac{1}{3}$.

It is interesting to note that the above fermion representations of SU(6) can be combined into a single representation $\underline{27}_L$ of the exceptional group E_6. Hence, one may embed our SU(6) in E_6 and demand the initial breaking pattern, $\mathrm{E}_6 \to (>10^{15}\ \mathrm{GeV}) \to \mathrm{SU}(2) \times \mathrm{SU}(6) \to$ etc. This initial breaking stage can be achieved by giving a nonvanishing vacuum value to the SU(2) as well as the SU(6) singlet component of a Higgs multiplet in the representation $\underline{650}$ of E_6. In the $\mathrm{SU}(2) \times \mathrm{SU}(6)$ decomposition of E_6, we have

$$\begin{aligned} \underline{27} &= (\underline{1}, \underline{15}) + (\underline{2}, \underline{\bar{6}}) \,, \\ \underline{650} &= (\underline{1}, \underline{1}) + (\underline{1}, \underline{35}) + (\underline{2}, \underline{20}) + (\underline{1}, \underline{189}) \\ &\quad + (\underline{2}, \underline{70}) + (\underline{2}, \underline{\overline{70}}) + (\underline{3}, \underline{35}) \,. \end{aligned} \tag{10}$$

Grand unification in E_6 has been considered[18,19] before. All previous treatments emphasize the initial breaking, $\mathrm{E}_6 \to \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{SU}(3)_c$ and assign different multiplets to the electron and the muon, $M_e = \underline{27}$ and $M_\mu = \underline{27}'$. The distinctive feature of our present scheme is the assignment of the electron and the muon to the same multiplet, hence unifying these leptonic flavors. Moreover, we emphasize the association of two neutrals to these charged leptons within an extended electroweak (flavor) group $\mathrm{SU}(3) \times \mathrm{U}(1)$. Note that $\mathrm{SU}(3) \times \mathrm{U}(1)$ has been considered before (see Ref. 20 and references therein). In these works, a new charged lepton M^- has been introduced in the fundamental triplet, rather than a neutral. That sort of assignment would not fit in the present SU(6) model.

B. Weak currents

Beyond the SU(2) transitions discussed in Sec. II G, Table I classifies the flavor-changing transitions. Note that all our assignments are subject to mixing angles among massive particles of equal charge.

C. Fermion mass generation

Without adding to the general discussion of the point given in Sec. II E, we only indicate the possible fermion-mass-generating Higgs multiplets. From the SU(6) products

$$\begin{aligned} \underline{6} \times \underline{6} &= (\underline{15})_A + (\underline{21})_S \,, \\ \underline{\bar{6}} \times \underline{15} &= \underline{84} + \underline{6} \,, \\ \underline{15} \times \underline{15} &= \underline{105}_A + (\underline{105} + \underline{\overline{15}})_S \,, \end{aligned} \tag{11}$$

we learn that the representations $\underline{84}, \underline{105}_S$ as well as $\underline{21}$, $\underline{6}$, and $\underline{15}$ may serve our purpose via their neutral and color-singlet components. In the $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_f$ decompositions, we obtain

TABLE I. Flavor-changing transitions in the SU(6) model.

Current	$(V-A)$	$(V+A)$
Charged	$l^- \longleftrightarrow \nu'_l$	$\chi^0 \longleftrightarrow e^-$
	$u \longleftrightarrow b$	$\lambda^0 \longleftrightarrow \tau^-$
	$c \longleftrightarrow a$	
Neutral	$\nu_l \longleftrightarrow \nu'_l$	$\mu^- \longleftrightarrow e^-$
	$d \longleftrightarrow b$	$\tau^- \longleftrightarrow \sigma^-$
	$s \longleftrightarrow a$	

$$\begin{aligned}
\underline{21} &= (\underline{6},\underline{1}) + (\underline{3},\underline{3}) + (\underline{1},\underline{6}),\\
\underline{84} &= 2(\underline{3},\underline{3}) + (\underline{8},\underline{\bar{3}}) + (\underline{6},\underline{1}) + (\underline{\bar{3}},\underline{8})\\
&\quad + (\underline{1},\underline{6}) + (\underline{1},\underline{\bar{3}}) + (\underline{\bar{3}},\underline{1}),\\
\underline{105}_S &= (\underline{6},\underline{1}) + (\underline{6},\underline{6}) + (\underline{\bar{3}},\underline{\bar{3}}) + (\underline{8},\underline{3})\\
&\quad + (\underline{3},\underline{8}) + (\underline{1},\underline{6}).
\end{aligned} \tag{12}$$

In the embedding E_6 model, we can use $\underline{27}$ and $\underline{351}_S$ as fermion-mass-generating Higgs multiplets. This can be seen by noting the product,

$$\underline{27}\times\underline{27} = (\underline{27}+\underline{351})_S + (\underline{351})_A, \tag{13}$$

and the SU(2) × SU(6) decompositions of E_6, Eq. (10), and

$$\underline{351}_S = (\underline{1},\underline{105}_S) + (\underline{2},\underline{84}) + (\underline{3},\underline{21}) + (\underline{1},\underline{\overline{15}}). \tag{14}$$

Hence, the SU(6) mass-generating Higgs multiplets $\underline{6}$, $\underline{21}$, $\underline{15}$, $\underline{84}$, and $\underline{105}_S$ are contained in the E_6 representations $\underline{27}$ and $\underline{351}_S$.

IV. THE SU(7) MODEL

A. Lepton-quark assignments

The fermions are represented in two left-handed $\underline{7}_L$ and one right-handed $\underline{35}_R$ (or $[3]_R$). The $SU(3)_c \times SU(4)_f$ decompositions are

$$\begin{aligned}
\underline{7}_L &= (\underline{3},\underline{1})_L + (\underline{1},\underline{4})_L,\\
\underline{35}_R &= (\underline{1},\underline{1})_R + (\underline{\bar{3}},\underline{4})_R + (\underline{3},\underline{6})_R + (\underline{1},\underline{\bar{4}})_R.
\end{aligned} \tag{15}$$

Hence we propose the assignments

$$\underline{7}_L\begin{cases}(\underline{3},\underline{1})_L = \bar{d}^i_L,\\ (\underline{1},\underline{4})_L = (e^-,\nu_e;\nu'_e,\nu''_e)_L,\end{cases} \tag{16}$$

$$\underline{7}'_L\begin{cases}(\underline{3},\underline{1})'_L = \bar{s}^i_L,\\ (\underline{1},\underline{4})'_L = (\mu^-,\nu_\mu;\nu'_\mu,\nu''_\mu)_L,\end{cases} \tag{17}$$

$$\underline{35}_R\begin{cases}(\underline{1},\underline{1})_R = \sigma^+_R,\\ (\underline{\bar{3}},\underline{4})_R = (b_i,t_i;c_i,u_i)_R,\\ (\underline{3},\underline{6})_R = 1/\sqrt{2}\begin{bmatrix}0 & \bar{t}_i & \bar{u}_i & \bar{c}_i\\ -\bar{t}_i & 0 & \bar{d}_i & \bar{s}_i\\ -\bar{u}_i & -\bar{d}_i & 0 & \bar{b}_i\\ -\bar{c}_i & -\bar{s}_i & -\bar{b}_i & 0\end{bmatrix}_R\\ (\underline{1},\underline{\bar{4}})_R = (\nu_\sigma,\sigma^-;\mu^-,e^-)_R.\end{cases} \tag{18}$$

B. Weak currents

Note that within the SU(2) × U(1) sector, the third charged lepton σ^- and its associated neutral ν_σ, as well as the third-generation quarks t_i and b_i, have right-handed $(V+A)$ charged currents. This makes us wonder whether the recently discovered τ lepton[21] must have a $(V+A)$ weak current. In that case, it could well be identified with σ of our model. The analysis of the experimental τ data seems[22,23] to indicate a $(V-A)$ structure. However, taking into account the possibility of a massive associated neutrino, the determination of a $(V-A)$ structure from the present data may be less conclusive. In our model, the right-handed neutral ν_σ may conspire with one of the primed neutrals in Eqs. (16) and (17) associated with e^- and μ^- to produce a massive particle.

Beyond SU(2) × U(1), flavor-changing transitions are summarized in Table II.

C. Fermion mass generation

From the SU(7) products

$$\begin{aligned}
\underline{7}\times\underline{7} &= (\underline{21})_A + (\underline{28})_S,\\
\underline{\bar{7}}\times\underline{35} &= \underline{224} + \underline{21},\\
\underline{35}\times\underline{35} &= (\underline{490}+\underline{140})_S + (\underline{588}+\underline{7})_A,
\end{aligned} \tag{19}$$

and the $SU(3)_c \times SU(4)_f$ decompositions

$$\begin{aligned}
\underline{28} &= (\underline{6},\underline{1}) + (\underline{3},\underline{4}) + (\underline{1},\underline{10}),\\
\underline{21} &= (\underline{\bar{3}},\underline{1}) + (\underline{3},\underline{4}) + (\underline{1},\underline{6}),\\
\underline{140} &= (\underline{3},\underline{6}) + (\underline{8},\underline{\bar{4}}) + (\underline{6},\underline{1}) + (\underline{3},\underline{4}) + (\underline{\bar{3}},\underline{15})\\
&\quad + (\underline{1},\underline{20}) + (\underline{1},\underline{\bar{4}}) + (\underline{\bar{3}},\underline{1}),\\
\underline{224} &= (\underline{3},\underline{4}) + (\underline{8},\underline{6}) + (\underline{6},\underline{\bar{4}}) + (\underline{\bar{3}},\underline{\bar{4}}) + (\underline{1},\underline{4}) + (\underline{3},\underline{15})\\
&\quad + (\underline{\bar{3}},\underline{20}) + (\underline{1},\underline{10}) + (\underline{3},\underline{1}) + (\underline{1},\underline{6}),\\
\underline{490} &= (\underline{6},\underline{20}) + (\underline{8},\underline{20}) + (\underline{\bar{6}},\underline{10}) + (\underline{3},\underline{20}) + (\underline{\bar{3}},\underline{15})\\
&\quad + (\underline{1},\underline{\overline{10}}) + (\underline{3},\underline{6}) + (\underline{\bar{3}},\underline{4}) + (\underline{1},\underline{4}) + (\underline{1},\underline{1}),
\end{aligned} \tag{20}$$

we learn that the representations $\underline{21}$, $\underline{28}$, $\underline{140}$, $\underline{224}$, and $\underline{490}$ may serve as fermion-mass-generating Higgs multiplets via their neutral and color-singlet components.

TABLE II. Flavor-changing transitions in the SU(7) model.

Current	$(V-A)$	$(V+A)$
Charged	$l^- \longleftrightarrow \nu'_e, \nu''_e$ $t \longleftrightarrow d, s, b$ $u, c \longleftrightarrow b$	$\nu_\sigma \longleftrightarrow l^-$ $(u,c,t) \leftrightarrows (d,s,b)$
Neutral	$\nu_l \leftrightarrow \nu'_l \leftrightarrow \nu''_l \leftrightarrow \nu_l$ $d \leftrightarrow s \leftrightarrow b \leftrightarrow d$ $u \leftrightarrow c \leftrightarrow t \leftrightarrow u$	$u \leftrightarrow c \leftrightarrow t \leftrightarrow u$ $e^- \leftrightarrow \mu^- \leftrightarrow \sigma^- \leftrightarrow e^-$

V. THE SU(8) MODEL

A. Lepton-quark assignments

The fermions are represented in five left-handed $\underline{8}_L$ and one right-handed $\underline{56}_R$ (or $[3]_R$), which make a total of 96 chiral fields. The $SU(3)_c \times SU(5)_f$ decompositions are

$$\begin{aligned}\underline{8}_L &= (\underline{3},\underline{1})_L + (\underline{1},\underline{5})_L\,,\\ \underline{56}_R &= (\underline{1},\underline{1})_R + (\underline{\bar{3}},\underline{5})_R + (\underline{3},\underline{10})_R + (\underline{1},\underline{\overline{10}})_R\,.\end{aligned} \tag{21}$$

Hence we give the assignments

$$\underline{8}_L \quad \begin{aligned}&(\underline{3},\underline{1})_L = \bar{d}^i_L\,,\\ &(\underline{1},\underline{5})_L = (e^-, \nu_e;\ \nu'_e, \nu''_e, \nu'''_e)_L\,.\end{aligned} \tag{22}$$

Four additional $\underline{8}_L$ are assigned to μ^-, τ^-, σ^-, and ω^- charged leptons and their associated neutrals, as well as $\bar{s}^i_L$, $\bar{b}^i_L$, $\bar{m}^i_L$, and $\bar{n}^i_L$ charge $\frac{1}{3}$ antiquarks, respectively:

$$\underline{56}_R \begin{cases}(\underline{1},\underline{1})_R = \gamma^+_R\\ (\underline{\bar{3}},\underline{5})_R = (y^i, x^i; t^i, c^i, u^i)_R\\ (\underline{3},\underline{10})_R = \dfrac{1}{\sqrt{2}}\begin{pmatrix}0 & \bar{x}^i & \bar{u}^i & \bar{c}^i & \bar{t}^i\\ -\bar{x}^i & 0 & \bar{d}^i & \bar{s}^i & \bar{b}^i\\ -\bar{u}^i & -\bar{d}^i & 0 & \bar{y}^i & \bar{m}^i\\ -\bar{c}^i & -\bar{s}^i & -\bar{y}^i & 0 & \bar{n}^i\\ -\bar{t}^i & -\bar{b}^i & -\bar{m}^i & -\bar{n}^i & 0\end{pmatrix}_R,\\ (\underline{1},\underline{\overline{10}})_R = \dfrac{1}{\sqrt{2}}\begin{pmatrix}0 & x^0 & \nu_\gamma & \nu_\sigma & \nu_\omega\\ -x^0 & 0 & \gamma^- & \sigma^- & \omega^-\\ -\nu_\gamma & -\gamma^- & 0 & e^- & \mu^-\\ -\nu_\sigma & -\sigma^- & -e^- & 0 & \tau^-\\ -\nu_\omega & -\omega^- & -\mu^- & -\tau^- & 0\end{pmatrix}_R.\end{cases} \tag{23}$$

B. Weak currents

We have in this SU(8) model five sequential charged leptons (e, μ, τ, σ, and ω) each with four associated left-handed neutrals. Whereas e^-, μ^-, and τ^- have only left-handed $(V-A)$ SU(2) weak currents, σ^- and ω^- have both $(V-A)$ and $(V+A)$ SU(2) weak currents. There is a sixth charged lepton γ^- which has only a $(V+A)$ SU(2) weak current. Besides the sequential quark doublets (u^i, d^i), (c^i, s^i), and (t^i, b^i) which have $(V-A)$ currents, there are a doublet (x^i, y^i) and two charge $-\frac{1}{3}$ singlets m^i and n^i with $(V+A)$ currents.

The right-handed neutrals x^0, ν_γ, ν_σ, and ν_ω may conspire with four left-handed primed neutrals to produce massive particles.

With respect to the flavor-changing transitions beyond $SU(2)\times U(1)$, they are qualitatively similar to those indicated in the previous SU(6) and SU(7) models and discussed in Sec. II G.

C. Fermion mass generation

From the SU(8) products

$$\begin{aligned}&\underline{8}\times\underline{8} = (\underline{28})_A + (\underline{36})_S\,,\\ &\underline{\bar{8}}\times\underline{56} = \underline{420} + \underline{28}\,,\\ &\underline{56}\times\underline{56} = (\underline{1176}+\underline{420})_S + (\underline{1512}+\underline{\overline{28}})_A\,,\end{aligned} \tag{24}$$

and the $SU(3)_c \times SU(5)_f$ decompositions

$$\begin{aligned}\underline{28} &= (\underline{\bar{3}},\underline{1}) + (\underline{3},\underline{5}) + (\underline{1},\underline{10})\,,\\ \underline{36} &= (\underline{6},\underline{1}) + (\underline{3},\underline{5}) + (\underline{1},\underline{15})\,,\\ \underline{420} &= (\underline{3},\underline{10}) + (\underline{8},\underline{\overline{10}}) + (\underline{6},\underline{\bar{5}}) + (\underline{1},\underline{5}) + (\underline{3},\underline{24}) + (\underline{\bar{3}},\underline{45})\\ &\quad + (\underline{1},\underline{40}) + (\underline{3},\underline{1}) + (\underline{1},\underline{\overline{10}}) + (\underline{\bar{3}},\underline{\bar{5}})\,,\\ \underline{1176} &= (\underline{1},\underline{1}) + (\underline{6},\underline{15}) + (\underline{6},\underline{50}) + (\underline{1},\underline{\overline{10}}) + (\underline{1},\underline{50})\\ &\quad + (\underline{\bar{3}},\underline{5}) + (\underline{3},\underline{10}) + (\underline{3},\underline{75}) + (\underline{\bar{3}},\underline{45}) + (\underline{8},\underline{40})\,,\end{aligned} \tag{25}$$

we learn that the representations $\underline{28}$, $\underline{36}$, $\underline{420}$, and $\underline{1176}$ may serve as fermion-mass-generating Higgs multiplets.

VI. CONCLUSIONS

The SU(5) model[6] of grand unification, which applies to one generation of quarks and leptons (15 chiral fields), is a special case of the class of $SU(N \geq 5)$ models discussed in this paper. It corresponds to the case when the number of left-handed neutral partners to charged leptons is just one (the neutrino). The existence of several neutral partners to charged leptons in the framework of an extended electroweak (flavor) gauge group is the possibility proposed by our models for $N > 5$. Consequently, the principles of quark-lepton symmetry and grand unification lead to the extension of the quark sector and the grand unified group.

An essential element of our approach is that flavor unification should come earlier on the energy scale than grand unification. This leads to the desirable expectations of several physical effects between 10^2 and 10^{15} GeV, rather than a grand plateau.

The SU(6) model applies to (nearly) two generations of quarks and leptons (27 chiral fields), in which the electron, the muon, and neutrinos are unified with the u, d, and b quarks. This SU(6) model can be embedded in E_6. Our approach gives to a certain extent a new scheme for grand unification within E_6.

The SU(7) model which unifies 49 chiral fields would have been an extremely attractive three-generation model. The crucial point is whether the third-generation leptons (τ, ν_τ) and quarks

(t, b) have $(V+A)$ weak SU(2) currents rather than $(V-A)$.

The SU(8) model manages to describe three generations of sequential quarks and leptons, plus several others of $(V+A)$ currents within 96 chiral fields.

There is a seemingly unpleasant feature of our scheme, which is the usage of several multiplets in the fundamental representation $\underline{N}_L$, rather than avoiding the occurrence of this representation more than once. There is actually no fundamental reason for constraining the occurrence of any particular representation more than once. Perhaps flavor symmetries should not act on these recurring left-handed representations, but rather on the right-handed fields occurring in $[2]_R$ or $[3]_R$, as indicated in our models. Moreover, perhaps further consolidation of these models should take place in the framework of larger symmetries at energies $>10^{15}$ GeV. The E_6 symmetry is such an example with respect to our SU(6) model. For the other models, we do not know the consolidating symmetries. Perhaps space-time symmetries and gravitational interactions must enter the game in an ultimate consolidation at energies of the order of the Planck mass.

Finally, it is worth remarking that if the τ generation turns out to be $(V+A)$, our SU(7) model, being the most economical three-generation model, should receive greatest support.

Notes added in proof. The following remarks supplement the considerations of this paper and will be discussed in detail in a forthcoming article:

(1) Following standard renormalization-group calculations,[17] we have found that the breaking heirarchy suggested in the paper is too simplistic to satisfy the observed values of the weak and the strong coupling constants as well as the weak mixing angle. Hence we have considered the generalized pattern

$$\begin{aligned} \mathrm{SU}(N) &\longrightarrow \mathrm{SU}(X_1)_c \times \mathrm{SU}(Y_1)_f \times \mathrm{U}(1)_1 \\ &\cdots \\ &\longrightarrow \mathrm{SU}(3)_c \times \mathrm{SU}(2) \times \mathrm{U}(1)\,, \end{aligned}$$

where each intermediate stage is characterized by a pair of numbers $(X_1, Y_1) \geq (X_i, Y_i) \geq (3, 2)$, $X_1 + Y_1 = N$, and in each breaking step either X_i or Y_i decreases by one unit, with the breaking and the appearance of the associated U(1) factors. This generalized breaking hierarchy utilizes the same set of Higgs multiplets suggested in the paper. Letting the ratio of two successive intermediate mass scales be 10^{λ_i} and with $(N-4)\lambda^{\mathrm{av}} = \sum_{i=1}^{N-4}\lambda_i$, we find from the observed values $\alpha_s(M_W) = 0.15$, $\alpha(M_W) = \frac{1}{128}$, and $\sin^2\theta(M_W) = 0.23$, the condition $\sum_{i=1}^{N-4}(X_i - Y_i) \approx 1.37(N-4)$. This condition serves to select the most favorable hierarchy of all the available ones. For instance, SU(7) has four possible breaking patterns with $\sum_i (X_i - Y_i) = 0$, 2, 4, and 6. The selected pattern is, by virtue of $1.37 \times 3 \approx 4.11$,

$$\begin{aligned} \mathrm{SU}(7) &\longrightarrow \mathrm{SU}(4)_c \times \mathrm{SU}(3) \times \mathrm{U}(1)'' \\ &\longrightarrow \mathrm{SU}(4)_c \times \mathrm{SU}(2) \times \mathrm{U}(1)' \\ &\longrightarrow \mathrm{SU}(3)_c \times \mathrm{SU}(2) \times \mathrm{U}(1)\,, \end{aligned}$$

which has $\sum_i (X_i - Y_i) = 4$. Further refinement of this technique attributes the remaining difference to deviations from λ^{av} of separate average values associated with intermediate color and flavor breakings.

(2) Perfect accord with the observed values of $\alpha_s(M_W)$, $\alpha(M_W)$, and $\sin^2\theta(M_W)$ would dictate the grand unification mass $M_X \sim 10^{12} M_W$ independently of $N>5$. This value is greater by a factor of more than 10 relative to the SU(5) value. However, taking care not to get in conflict with the stringent experimental limit on proton decay, we can demonstrate possibilities of suppressing this decay from low-order transitions. These possibilities depend on allowing the leptoquarks to transform doublets and singlets from different conventional generations.

(3) We note also the interesting anomaly-free models

$$\underline{63} = \underline{35} + \underline{\overline{21}} + \underline{7} \quad \text{in} \quad \mathrm{SU}(7)$$

and

$$\underline{92} = \underline{56} + \underline{\overline{28}} + \underline{\overline{8}} \quad \text{in} \quad \mathrm{SU}(8)\,.$$

ACKNOWLEDGMENT

I am grateful to Professor Abdus Salam for hospitality at the International Centre for Theoretical Physics, Trieste, Italy.

[1]S. Weinberg, Phys. Rev. Lett. 31, 494 (1973).
[2]H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973).
[3]S. L. Glashow, Nucl. Phys. 22, 579 (1961).
[4]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
[5]A. Salam, in *Elementary Particle Theory; Relativistic Groups and Analyticity* (*Nobel Symposium No. 8*) edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.
[6]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
[7]H. Georgi, Nucl. Phys. B156, 126 (1979).

[8]A. Salam, talk at the concluding session of the European Physical Society Conference, Geneva, 1979 (unpublished).
[9]M. Gell-Mann, P. Ramond, and R. Slansky, Rev. Mod. Phys. 50, 721 (1978).
[10]S. L. Adler, Phys. Rev. 177, 2466 (1969).
[11]J. S. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969).
[12]J. Banks and H. Georgi, Phys. Rev. D 14, 1159 (1976).
[13]S. Weinberg in *Neutrinos—78,* proceedings of the International Conference on Neutrino Physics and Astrophysics, Purdue Univ., edited by E. C. Fowler (Purdue Univ. Press, West Lafayette, Indiana, 1978), p.1; D. Schramm, *ibid.*, p. 87.
[14]G. Feinberg, Phys. Rev. 110, 1482 (1958).
[15]Particle Data Group, Phys. Lett. 75B, 1 (1978).
[16]A. W. Hendry and D. B. Lichtenberg, Rep. Prog. Phys. 41, 1707 (1978).
[17]H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).
[18]F. Gursey, P. Ramond, and P. Sikivie, Phys. Lett. 60B, 177 (1976).
[19]Y. Achiman and B. Stech, Phys. Lett. 77B, 389 (1978).
[20]B. W. Lee and S. Weinberg, Phys. Rev. Lett. 38, 1237 (1977).
[21]M. L. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975).
[22]G. J. Feldman, Comm. Nucl. Part. Phys. 9, 23 (1979).
[23]W. Bacino *et al.*, Phys. Rev. Lett. 41, 13 (1978); 42, 749 (1979).