Static electroweak properties and partial conservation of axial-vector current in the MIT bag model

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We calculate charge radii, magnetic moments, and axial-vector couplings in a version of the MIT bag model that incorporates the partial conservation of axial-vector current condition. We find a modest improvement over values obtained in the standard bag model.

It has recently been shown that the PCAC (partial conservation of axial-vector current) condition can be incorporated into the MIT bag model in a self-consistent manner without adversely affecting the good spectroscopic results of the standard bag model.[1,2] In this note we present results for static electroweak properties in this same PCAC-extended bag model, which will henceforth be referred to as MITP.

The salient feature of MITP is the introduction of a pion field whose domain excludes the region $r<R$, which is reserved for quarks and gluons. The pion field is determined by the Klein-Gordon equation and an equation of continuity for the normal component of the axial-vector current,

$$a_\lambda^l=\bar{q}\gamma_\lambda\gamma\tfrac{1}{2}\tau^l q+f_\pi\partial_\lambda\phi^l . \qquad (1)$$

The quark field is determined by the well known MIT-bag-model equations,[3] and any influence of the pion field upon the quarks is totally ignored except for the contribution of the pion field energy to the radius of a specific hadron state. We want to make it clear that we are merely presenting the results of a forced marriage between the bag and the PCAC condition. The model is *not* chirally invariant in the limit of vanishing masses. It does, however, satisfy the PCAC condition by virtue of the definition of the axial-vector current and obedience to free-particle wave equations by pions and by quarks.

In this model, the pion field generated by the $a\to b$ transition of an up or down quark is given by

$$\vec{\phi}_{a\to b}=-\frac{m^2}{8\pi f_\pi}\frac{\chi}{2(\chi-1)}\frac{(1+\rho)}{(1+\beta+0.5\beta^2)}\times\frac{\exp(\beta-\rho)}{\rho^2}U_b^\dagger\vec{\sigma}\cdot\hat{r}\vec{\tau}U_a , \qquad (2)$$

where $\chi=2.04$, $\beta=m_\pi R$, and $\rho=m_\pi r$. The spin-isospin functions enter into the up- and down-quark wave functions according to

$$q(r)=\left(\frac{\chi}{8\pi(\chi-1)R^3}\right)^{\frac{1}{2}}j_0(\chi)\begin{pmatrix} ij_0(\chi r/R)U \\ -j_1(\chi r/R)\vec{\sigma}\cdot\hat{r}U\end{pmatrix}, \qquad (3)$$

where the j_l are spherical Bessel functions. We now proceed to the explicit calculation of static electroweak properties of the baryons.

The charge radius is given by the appropriate SU(6) matrix element of the spin-isospin operator

$$\langle r^2\rangle=\int_{r\le R}r^2\rho_q(r)\,d^3r+\int_{r\ge R}r^2\rho_\pi(r)\,d^3r , \qquad (4)$$

in which ρ_q and ρ_π are, respectively, the quark and pion charge densities. Since $\rho_\pi\sim\phi\partial\phi/\partial t$, and in our model the pion field is static, the pion charge density vanishes. The only manifestation of pion-exchange effects in the charge radius is contained in the numerical value of the radius parameter R for a specific hadron state. To a very good approximation, the charge radius scales like R, and our results differ from those of the MIT group by essentially this scale factor. Numerical values for the baryon charge radii using the bag sizes of Ref. 1 are given in Table I.

In contrast to the charge radius, the magnetic moment has an explicit contribution from the pion cloud, which is obtained from

$$\mu_\pi=\int_{r\ge R}\phi T_3L_3\phi\,d^3r , \qquad (5)$$

where T_3 and L_3 are the charge and angular momentum operators

$$(T_3)_{ij}=-i\epsilon_{ij3}\text{ and }L_3=-i\epsilon_{ij3}r_i\partial_j. \qquad (6)$$

We wish to remark on the effect of minimally coupling the electromagnetic field into the boundary condition that determines the pion field. Within the framework of the static cavity approximation, such a coupling vanishes in the gauge in which a uniform magnetic field is described by the vector potential $\vec{A}=(\frac{1}{2})\vec{r}\times\vec{H}$. Evaluating the relevant integrals, we find that we require the SU(6) matrix elements of the operator

$$\mu_\pi=\frac{R}{12\pi}\left(\frac{m_\pi}{f_\pi}\right)^2\left[\frac{\chi}{2(\chi-1)}\right]^2P(\beta)\beta^{-2}(A+B) , \qquad (7)$$

where

$$P(\beta)=(1+0.5\beta)(1+\beta+0.5\beta^2)^{-2} , \qquad (8)$$

$$A=\sum_a(\sigma_3\tau_3)_a , \qquad (9)$$

TABLE I. Charge radii and magnetic moments. The bag radii and the charge radii are given in GeV^{-1} and the magnetic moments in Bohr magnetons. The "MIT" values for the charge radii are our calculation using the MIT formulas and bag radii. For a neutral hadron, the value of $\langle r^2\rangle$ is given by the average of the $\langle r^2\rangle$ of the quarks, weighted by the charges of the quarks; for a charged hadron the weighted average of the charge radii of the quarks divided by the charge of the hadron is used.

State	R_{MIT}	R_{MITP}	Charge radius^2			Magnetic moment		
			MIT	MITP	experiment	MIT	MITP	experiment
p	5.00	6.08	13.2	19.6	20 ± 2	1.90	3.30	2.79
n	5.00	6.08	0.00	0.00	−0.61 ± 0.05	−1.27	−2.53	−1.91
Λ	4.95	5.84	1.14	1.72		−0.49	−0.54	−0.606 ± 0.034
Σ^+	4.95	5.51	14.1	17.6		1.84	2.47	2.83 ± 0.25
Σ^0	4.95	5.51	1.14	1.49		0.59	0.64	
Σ^-	4.95	5.51	11.9	14.7		−0.68	−1.20	−1.48 ± 0.37
Ξ^0	4.91	5.38	2.25	2.82		−1.06	−1.25	
Ξ^-	4.91	5.38	10.6	12.6		−0.44	−0.34	−1.85 ± 0.75

$$B=2\sum_{a\neq b}\left[(\sigma_+\tau_+)_a(\sigma_-\tau_-)_b-(\sigma_+\tau_-)_a(\sigma_-\tau_+)_b\right], \qquad (10)$$

and the summations are only over up and down quarks. The total baryon magnetic moment is the sum of the above and the single-quark moments,[3,4,5]

$$\hat{\mu}_0=\frac{R}{12}\sum_a\left[\frac{4\alpha+2\lambda-3}{\alpha(\alpha-1)+\lambda}Q\sigma_3\right]_a, \qquad (11)$$

where λ and Q are, respectively, the quark mass and charge matrices and $\alpha^2=\chi^2+\lambda^2$.

It is instructive to exhibit the analytic expressions for the proton and neutron moments, which are simple because only massless quarks are involved there. For the proton, we find $\langle A\rangle=\frac{5}{3}$ and $\langle B\rangle=2$, and of course $\langle\Sigma(Q\sigma_3)_a\rangle=1$; the corresponding quantities for the neutron are $-\frac{5}{3}$, -2, and $-\frac{2}{3}$. The nucleon moments are therefore given by

$$2m_p\mu_p=\frac{m_pR}{6}\,\frac{4\chi-3}{\chi(\chi-1)}\left[1+11\Delta(R)/3\right], \qquad (12)$$

$$\frac{\mu_n}{\mu_p}=-\frac{2}{3}\,\frac{1+11\Delta/2}{1+11\Delta/3}, \qquad (13)$$

where

$$\Delta(R)=\left(\frac{m_\pi}{f_\pi}\right)^2\frac{\chi^3}{4\pi(\chi-1)}(4\chi-3)^{-1}\beta^{-2}P(\beta). \qquad (14)$$

The R dependence of the nucleon moments is shown in Fig. 1, where we have used $f_\pi=0.94m_\pi/\sqrt{2}$. From this exercise we see that the nucleon radius parameter must be in the neighborhood of 6 GeV^{-1} in order to have any reasonable agreement with experiment. This conclusion concerning R is independent of the myriad of uncertainties that enter into the spectroscopic fit through which parameters related to R are calculated. We obtain the baryon magnetic moments given in Table I using the R values obtained from our spectroscopic fits.[1]

We now turn to the axial-vector coupling g_A. The static coupling is given by

$$g_A=g_{A0}+g_{A\pi}, \qquad (15)$$

where the first term is the conventional single-quark contribution,

$$g_{A0}=\sum_{i\neq j}\int_{r\leq R}\bar{q}_j\gamma^3\gamma_5\tau_+q_i\,d^3r, \qquad (16)$$

and the pion cloud contributes

$$g_{A\pi}=f_\pi\sum_a\partial^3(\phi_1^a+i\phi_2^a)\,d^3r. \qquad (17)$$

Except for a change in the value of R, pion-exchange effects do not contribute to the couplings associated with $\Delta S\neq 0$ transitions, and since they are relatively insensitive to the precise value of R, we will not discuss them further. Performing the relevant integrations, we find the effective operator for $\Delta S=0$ transitions to be

$$g_A=\frac{\chi}{3(\chi-1)}(1+\delta)\sum_a(\sigma_3\tau_+)_a, \qquad (18)$$

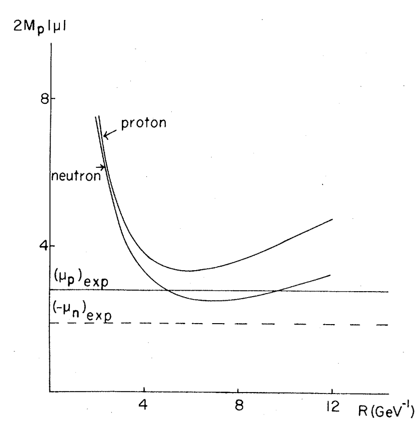

FIG. 1. R dependence of nucleon magnetic moments in the MITP model.

where

$$\delta = \frac{1}{2}\frac{1+\beta}{1+\beta+0.5\beta^2}. \qquad (19)$$

The only $\Delta S=0$ transitions for which data exist are $n\to p$ and $\Sigma\to\Lambda$. The matrix elements of $\sum(\sigma_3\tau_+)_a$ are $\frac{5}{3}$ and $(\frac{2}{3})^{\frac{1}{2}}$ for $n\to p$ and $\Sigma\to\Lambda$, respectively. The experimental couplings for these transitions are

$$g_A^{np}=1.25 \text{ and } (g_A^{\Sigma\Lambda})^{-1}=0.24\pm 0.23\ , \qquad (20)$$

which are to be compared with our theoretical values 1.55 and 1.3, respectively.

We find that PCAC effects make a very substantial contribution to the electroweak static properties of the low-lying baryons. In particular, the proton magnetic moment is raised from 32% below the experimental value to 18% above it by the inclusion of these effects; the neutron moment is shifted from 32% above to 32% below its experimental value. This result is somewhat encouraging, since previously calculated gluon-exchange effects are known to reduce the proton moment by ~10% (Ref. 6), which would come close to bringing that moment in line with experiment. The value of the axial-vector coupling in neutron β decay is increased to a value some 24% above the experimental result of 1.25, which is to be contrasted with the theoretical value of 1.09 in the absence of these effects. Inclusion of nonzero masses for u and d quarks would appear to exacerbate this problem.[7] One might hope, of course, that as in the case of the proton magnetic moment, gluon-exchange effects will produce a modest reduction of g_A^{np}. Inspection of the other magnetic moments listed in Table I shows a significant improvement arising from the PCAC effects. On balance, we conclude that the static electroweak properties of the low-lying baryons are somewhat improved by the inclusion of the pion-exchange current associated with this model. Further improvement may be found by including modifications of the quark wave function that arise in a chirally symmetric model.[8]

Note added. Since the initial preparation of this report, we have become aware of two additional experimental numbers of interest. Settles *et al.*[9] have found $\mu_{\Sigma^+}=2.33+0.13$, and Bunce *et al.*[10] have found $\mu_{\Xi^0}=1.20+0.06$. Both of these numbers are in good agreement with our calculation.

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